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VI.60 William Burnside

b. London, 1852; d. West Wickham, England, 1927
Theory of groups; character theory; representation theory

Burnside's mathematical abilities first showed themselves at school. From there he won a place at Cambridge, where he read for the Mathematical Tripos and graduated as 2nd Wrangler in 1875. For ten years he remained in Cambridge as a Fellow of Pembroke College, coaching student rowers and mathematicians. In 1885, having published three very short papers, he was appointed professor at the Royal Naval College, Greenwich. He married in 1886 and the next year, at the age of thirty-five, he embarked on his career as a productive mathematician. He was elected as a Fellow of the Royal Society in 1893 on the basis of his contributions in applied mathematics (statistical mechanics and hydrodynamics), geometry, and the theory of functions. Although he continued to contribute to these areas throughout his working life, and added probability theory to his fields of interest during World War I, he turned to the theory of groups in 1893, and it is for his discoveries in this subject that he is remembered.

Burnside treated every aspect of the theory of finite groups. He was much concerned with the search for finite simple groups, and made the famous conjecture, finally proved by Walter Feit and John Thompson in 1962, that there are no simple groups of odd composite order (see THE CLASSIFICATION OF FINITE SIMPLE GROUPS [V.7]). He helped to develop character theory, which had been created by FROBENIUS [VI.58] in 1896, into a tool for proving theorems of pure group theory, using it in 1904 to spectacular effect when he proved his so-called $p^\alpha q^\beta$-theorem: the theorem that groups whose orders are divisible by at most two different prime numbers are soluble. By asking, in effect, whether a group all of whose elements have finite order and which is generated by finitely many elements must be finite, he launched the huge area of research which for much of the twentieth century was known as the Burnside problem (see GEOMETRIC AND COMBINATORIAL GROUP THEORY [IV.10 §5.1]).

Although CAYLEY [VI.46] and the Reverend T. P. Kirkman had written about groups before him, he was the only British mathematician to work in group theory until Philip Hall started his mathematical career in 1928. Burnside's influential book *Theory of Groups of Finite Order* (1897) was written in the hope of "arousing interest among English mathematicians in a branch of pure mathematics which becomes the more interesting the more it is studied." Its influence in his own country was minimal, however, until several years after his death. It went to a second edition in 1911 (reprinted 1955), which differs from the first in that it has been substantially revised and, in particular, it includes chapters about the character theory of finite groups and its applications—mathematics which had been much developed by Frobenius, Burnside, and Schur over the fifteen years following the invention of character theory in 1896.

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Peter M. Neumann

VI.61 Jules Henri Poincaré

b. Nancy, France, 1854; d. Paris, 1912
Function theory; geometry; topology; celestial mechanics; mathematical physics; foundations of science

Educated at the École Polytechnique and the École des Mines in Paris, Poincaré began his teaching career at the University of Caen in 1879. In 1881 he took up an appointment at the University of Paris where, from 1886, he held successive chairs until his death in 1912. He was of a retiring nature and did not attract graduate students, but his lecture courses provided the basis for a number of treatises, mostly in mathematical physics.

Poincaré came to international prominence in the early 1880s when, fusing ideas from complex function theory, group theory, non-Euclidean geometry, and the theory of ordinary linear differential equations, he identified an important class of automorphic functions. Named Fuchsian functions, in honor of the mathematician Lazarus Fuchs, they are defined on a disk and

Jules Henri Poincaré

remain invariant under certain discrete groups of transformations. Soon after, he identified the related but more complicated Kleinian functions, which are automorphic functions without a limit circle. His theory of automorphic functions was the first significant application of non-Euclidean geometry. It led to his discovery of the disk model of the hyperbolic plane and later inspired the UNIFORMIZATION THEOREM [V.34].

During the same period Poincaré began pioneering work on the qualitative theory of differential equations, motivated in part by an interest in some of the fundamental questions of mechanics, notably the problem of the stability of the solar system. What was new and important was his idea of thinking of the solutions in terms of curves rather than functions, i.e., thinking geometrically rather than algebraically, and it was this that marked a departure from the work of his predecessors, whose research had been dominated by power-series methods. From the mid 1880s he began applying his geometric theory to problems in celestial mechanics. His memoir on THE THREE-BODY PROBLEM [V.33] (1890) is famous both for providing the basis for his acclaimed treatise, *Les Méthodes Nouvelles de la Mécanique Céleste* (1892–99), and for containing the first mathematical description of CHAOTIC BEHAVIOR [IV.14 §1.5] in a dynamical system. Stability was also at the heart of his investigation into the forms of rotating fluid masses (1885). This work, which contained the discovery of new, pear-shaped figures of equilibrium, aroused considerable attention because of its important implications for cosmogony in relation to the evolution of binary stars and other celestial bodies.

Poincaré's work on Fuchsian functions and on the qualitative theory of differential equations led him to recognize the importance of the topology (or, as it was then called, *analysis situs*) of MANIFOLDS [I.3 §6.9]. And in the 1890s he began to study the topology of manifolds as a subject in its own right, effectively creating the powerful independent field of ALGEBRAIC TOPOLOGY [IV.6]. In a series of memoirs published between 1892 and 1904, the last of which contains the hypothesis known today as THE POINCARÉ CONJECTURE [IV.7 §2.4], he introduced a number of new ideas and concepts, including Betti numbers, THE FUNDAMENTAL GROUP [IV.6 §2], HOMOLOGY [IV.6 §4], and torsion.

A deep interest in physical problems lay behind Poincaré's achievements in mathematical physics. His work in potential theory forms a bridge between that of Carl Neumann on boundary-value problems and that of FREDHOLM [VI.66] on integral equations. He introduced a technique known as the "méthode de balayage" ("sweeping-out method") for establishing the existence of solutions to the DIRICHLET PROBLEM [IV.12 §1] (1890); and he had the idea that the Dirichlet problem itself should give rise to a sequence of EIGENVALUES AND EIGENFUNCTIONS [I.3 §4.3] (1898). In developing the theory for functions of several variables he was led to the discovery of new results in complex function theory. In *Électricité et Optique* (1890, revised 1901), which derived from his university lectures, he gave an authoritative account of the electromagnetic theories of Maxwell, Helmholtz, and Hertz. In 1905 he responded to Lorentz's new theory of the electron, coming close to anticipating Einstein's theory of SPECIAL RELATIVITY [IV.13 §1], thereby provoking controversy among later writers about the question of priority. And in 1911 he attended the first Solvay Conference on quantum theory, publishing an influential memoir (1912) in its favor.

As Poincaré's career developed, so too did his interest in the philosophy of mathematics and science. His ideas became widely known through four books of essays: *La Science et l'Hypothèse* (1902), *La Valeur de la Science* (1905), *Science et Méthode* (1908), and *Dernières Pensées* (1913). As a philosopher of geometry he was a proponent of the view, known as conventionalism, that it is not an objective question which model of geometry best fits physical space but is rather a matter of which

model we find most convenient. By contrast, his position on arithmetic was intuitionist. On the question of foundational issues, he was largely critical. Although sympathetic to the goals of set theory, he attacked what he perceived as its counterintuitive results. (See THE CRISIS IN THE FOUNDATIONS OF MATHEMATICS [II.7 §2.2] for further discussion.)

Poincaré's visionary geometric style led him to new and brilliant ideas, which frequently connected different branches of mathematics, but lack of detail often made his work hard to follow. At times his approach was censured for imprecision; it was in marked contrast to that of HILBERT [VI.63], his German counterpart, whose work was rooted in algebra and rigor.

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VI.62 Giuseppe Peano

b. Spinetta, Italy, 1858; d. Turin, 1932

Analysis; mathematical logic; foundations of mathematics

Known above all for his (and DEDEKIND's [VI.50]) axiom system for the natural numbers, Peano made important contributions to analysis, logic, and the axiomatization of mathematics. He was born in Spinetta (Piedmont, Italy) as the son of a peasant, and from 1876 studied at the University of Turin, taking his doctoral degree in 1880. He remained there until his death in 1932, becoming full professor in 1895.

During the 1880s Peano worked in analysis, achieving what are generally considered to be his most important results. Particularly noteworthy are the continuous space-filling *Peano curve* (1890), the notion of *content* (a precedent of MEASURE THEORY [III.55]) developed independently by JORDAN [VI.52], and his theorems on the existence of solutions for differential equations of the first order (1886, 1890). The textbook he published in 1884, *Calcolo Differentiale e Principii di Calcolo Integrale*, partly based on lectures by his teacher Angelo Genocchi, was noteworthy for its rigor and critical style, and is counted among the very best nineteenth-century treatises.

The years 1889–1908 saw Peano dedicating himself intensively to symbolic logic, axiomatization, and producing the encyclopedic *Formulaire de Mathématiques* (1895–1908, five volumes). This ambitious assembly of mathematical results, compactly presented in the symbols of mathematical logic, was given completely without proofs. This was by no means standard at the time, but it shows what Peano expected from logic: it was supposed to bring precision of language and brevity, but not a greater level of rigor (something that was, by contrast, crucial for FREGE [VI.56]). In 1891, together with some colleagues, he founded the journal *Rivista di Matematica*, gathering around him an important group of followers.

Peano was an accessible man, and the way he mingled with students was regarded as "scandalous" in Turin. He was a socialist in politics, and a tolerant universalist in all matters of life and culture. In the late 1890s Peano became increasingly interested in elaborating a universal spoken language, "Latino sine flexione"; the last edition of the *Formulario* (1905–8) appeared in this language.

Peano followed closely the work of German mathematicians such as Hermann Grassmann, Ernst Schröder, and Richard Dedekind; for example, the 1884 textbook defined the real numbers by Dedekind cuts, and in 1888 he published *Calcolo Geometrico Secondo l'Ausdehnungslehre di H. Grassmann*. In 1889 there appeared (notably in Latin) a first version of the famous PEANO AXIOMS [III.67] for the set of natural numbers, which he refined in volume 2 of the *Formulaire* (1898). It aimed at filling the most significant gap in the foundations of mathematics at a time when the *arithmetization* of analysis had essentially been completed. It is no coincidence that other mathematicians (Frege, Charles S. Peirce, and Dedekind) published similar work in the same decade. Peano's attempt is better rounded than Peirce's, but simpler and framed in more familiar terms than those of Frege and Dedekind; because of this, it has been more popular.

Peano's work on the natural numbers was at the crossroads of his diverse mathematical contributions, linking naturally his previous research in analysis with his later work on logical foundations, and being a necessary prerequisite for the *Formulaire* project. Actually, *Arithmetices Principia* can be regarded as a simplification, refinement, and translation into logical language (the "nova methodo" in its title) of Grassmann's *Lehrbuch der Arithmetik* (1861). Grassmann had striven to elaborate a stern deductive structure, stressing proofs by mathematical induction and recursive definitions. But curiously, unlike Peano, he did not postulate an axiom of induction; thus, Peano presented the basic assumptions much more clearly, bringing