APPENDIX 1

1. Pretend, for a moment, that x has a fixed value. Let's denote this fixed value by x_i ("i" for "initial"). Let's label the corresponding y-value y_i. The linear equation then reads

$$y_i = mx_i + b.$$

Now, let's add 1 to the x-value. We do this by replacing x_i in the equation by $x_i + 1$. Let's call the new y-value y_f ("f" for "final"). The new linear equation is

$$y_f = m(x_i + 1) + b.$$

We can distribute the m on the right-hand side of this equation: $m(x_i + 1) = mx_i + m$. This simplifies the equation to

$$y_f = mx_i + m + b.$$

Now look closely: the right-hand side is just m plus $mx_i + b$ (the order of the terms doesn't matter). But since $y_i = mx_i + b$, we can substitute this in to get

$$y_f = y_i + m.$$

Okay, let's recap what happened. After increasing the x-value by one unit (from x_i to $x_i + 1$), the new y-value (y_f) ended up being the initial y-value (y_i) plus the slope m. If m is positive, then y_f is bigger than y_i (the y-value has increased) whereas if it's negative, y_f is smaller than y_i (the y-value has decreased). That's the generalized slope interpretation I italicized on page 6.

2. Let's solve $4x + 370 \leq 400$ using algebra. (Since there aren't any negative numbers in our inequality the inequality sign gets treated the same as an equals sign.) Ready? Let's begin.

1. Here's the starting inequality: $4x + 370 \leq 400$
2. Now subtract 370 from both sides: $4x \leq 30$
3. Finally, divide both sides by 4: $x \leq \frac{30}{4} = 7.5$

3. Here's how you would "mathematize" this problem. Let p be the total grams of protein eaten in a day, c the total grams of carbs, and f the total grams of fat. The Atwater general factor system tells us that the protein contains $4p$ calories, the carbs $4c$ calories, and the fat $9f$ calories. The total calories eaten, T, is then

$$T = 4p + 4c + 9f.$$

This equation is an example of a *multilinear* function. We'll discuss these in more detail in the next section. For now note that we can go through the same analysis of capping the total calories, T, to a certain number and then solving for the grams of each macronutrient. For example, capping T at 1,000 yields the inequality

$$4p + 4c + 9f \leq 1{,}000.$$

If we know two of three variables in this equation we can solve for the remaining variable. For example, if you wanted to stick to a diet low in carbs (say, $c = 150$) and fat (say, $f = 20$), then your diet would have at most 55 grams of protein (i.e., $p \leq 55$).

4. The full RMR_m equation involves four variables. To graph it would require a four-dimensional graph, which we can't visualize. But if I plug in a height, say, $h = 67$, we get an equation with three variables:

$$\text{RMR}_\text{m} = 4.5w - 5a + 1{,}070.3. \tag{A1.1}$$

This equation requires a three-dimensional graph. But that's okay; we graph in 3D just like we graph in 2D. We first draw the xy-plane on the

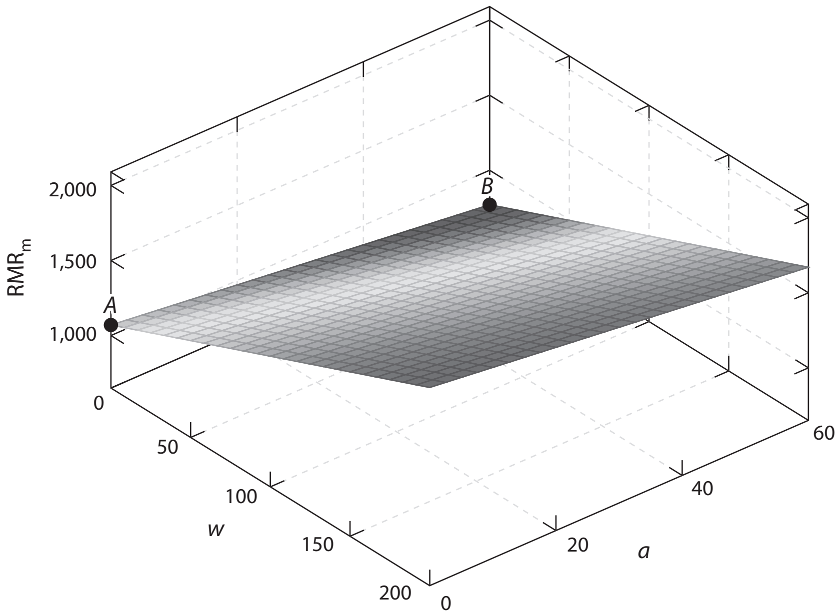

Figure A1.1. The 3D graph of equation (A1.1) for weight values w between 0 and 200 and age values a between 0 and 60.

bottom (like a floor) and then add a third axis going up. Then we plot a bunch of points relative to the origin (defined to be where the upward axis intersects the plane) and connect the dots. Figure A1.1 shows the graph of (A1.1) (called a *plane*). Planes are multilinear functions (note the lines that make up the edges of the plane in the figure). To illustrate that, notice that setting $w = 0$ gives $\mathrm{RMR_m} = -5a + 1{,}070.3$. This is the downward sloping line (the slope is -5) connecting the points labeled A and B in the figure.

5. Starting from $20 = 0.15r - 8.85$, we . . .

a. Add 8.85 to both sides: $0.15r = 28.85$,

b. Divide both sides by 0.15: $r = \frac{28.85}{0.15} = 192.3 \approx 192$.

Here $\approx$ means "approximately." (I've put a list of the mathematical symbols in the Glossary of Mathematical Symbols in Appendix A.)

TABLE A1.1.
The general forms of polynomials of degree 0 to 3, along with their names and specific examples.

n	Polynomial	Name	Example
0	a_0	Constant function	$y = 2$
1	$a_1x + a_0$	Linear function	$y = 4x + 370$
2	$a_2x^2 + a_1x + a_0$	Quadratic function	$y = -0.007x^2 + 192$
3	$a_3x^3 + a_2x^2 + a_1x + a_0$	Cubic function	$y = x^3 - 2x^2 + 1$

6. Every polynomial has the form

$$y = a_nx^n + a_{n-1}x^{n-1} + \ldots + a_1x + a_0$$

for some numbers a_0, a_1, up to a_n (we assume $a_n \neq 0$), and some nonnegative whole number n. The number n in this equation is called the *degree* of the polynomial; it's the highest power of x present. Table A1.1 gives the general form of polynomials of degree 0 to 3, along with their names and concrete examples.

7. To find the answer we set MHR = MHR_{pop}:

$$220 - a = 192 - 0.007a^2.$$

Adding $0.007a^2$ to both sides and subtracting 192 from both sides yields

$$0.007a^2 - a + 28 = 0.$$

The fastest way to solve this is to use the *quadratic formula*, which says that the solutions to $Ax^2 + Bx + C = 0$ are

$$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}.$$

The $\pm$ symbol means "plus or minus" (see the Glossary of Mathematical Symbols in Appendix A). It tells us to write down two solutions: one that uses the + sign and another that uses the − sign. Comparing $Ax^2 + Bx + C = 0$ to $0.007a^2 - a + 28 = 0$, we see that $A = 0.007$,

$B = -1$, and $C = 28$ (and $x = a$). The quadratic formula then gives the two solutions

$$a = \frac{20(25 - 3\sqrt{15})}{7} \approx 38.2, \qquad a = \frac{20(25 + 3\sqrt{15})}{7} \approx 104.6.$$

The first solution is the age (a-value) of the visible intersection point in Figure 1.2(b); the other solution corresponds to the other intersection point (not shown on the graph).

8. I'll show you how to mathematize this using Jason's ACB equation. Let t be the number of minutes it takes him to burn c calories. This means that

$$\text{ACB} = \frac{c}{t},$$

since ACB is the aerobic caloric burn *per minute*. This, together with Jason's ACB equation implies that

$$\frac{c}{t} = 0.15r - 8.85.$$

Since Jason's MHR is about 192 bpm, then x% of that is $\frac{192x}{100}$. (For example, to find 50% of his MHR we'd first divide 50 by 100 and then multiply the result by 192.) Thus, Jason will be exercising at this heart rate:

$$r = \frac{192x}{100}.$$

Inserting this into the previous equation yields

$$\frac{c}{t} = 0.15\left(\frac{192x}{100}\right) - 8.85 \Rightarrow \frac{c}{t} = 0.228x - 8.85.$$

To solve for t we take the reciprocal of both sides (the reciprocal of $\frac{a}{b}$ is $\frac{b}{a}$) and then multiply both sides by c:

$$t = \frac{c}{0.288x - 8.85}.$$

For example, if Jason wanted to burn 400 calories ($c = 400$) by exercising at 70% ($x = 70$) of his MHR, this analysis estimates it would take him about $t \approx 46$ minutes.