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**David G. Luenberger: Information Science**

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# 7 MARKETS

Information has both micro and macro manifestations. At the micro level are bits and bytes, codes and errors, capacity and compression. As shown in previous chapters, the concept of entropy provides a basis for analysis of many of these micro issues.

At the macro level are information products and services: books and movies, software and music, art and theater, telephone service and cable, consulting and research, insurance and guarantees. At this level, a high degree of organization and order is more important than randomness, and quality is not measured in bits, but more often in economic terms. It is products and services that people pay for.

From an economic perspective information products are fundamentally different from most other commodities because it is easy to produce them in large quantity from an initial version. In fact, in many cases the cost of producing additional copies is essentially zero. Economists say that the marginal cost of an additional copy is small.

Additionally, most information products can be used repeatedly. Books can be read more than once, music CDs can be played often, and software can be used everyday. Furthermore, most information products, including books, music, movies, and computer files, can be easily duplicated without authorization and passed on to other people. Yet the original creation of these products may entail considerable expense. How then, with all these difficulties, can such products survive in a free market? How can the creators be properly compensated?

We know that compensation does occur in practice, for these products and services are available in the market. Sometimes compensation rests on copyright protection, which essentially grants monopoly rights to the creator. There are other methods as well; but one may well ask whether existing methods of distribution and compensation are socially efficient. Perhaps the exclusive right granted by copyright leads to excessive prices, denying access to people who could benefit from the information. Or perhaps the opposite holds, and copyright laws do not provide enough incentive to produce some socially valuable materials.

Basic economic theory provides a framework for addressing these questions, allowing one to systematically trace economic value delivered to consumers, firms, and society as a whole. This chapter introduces this framework and applies it to some of the questions raised in the previous paragraphs. The analysis provides general conclusions, some of which may be surprising at first, and provides a stepping stone for analysis of the innovative methods that have evolved for marketing information products.

## 7.1 Demand

Consider a single individual contemplating the purchase of some item. Our interest is primarily directed toward information items, but here the item can be anything that is sold. Assume that the individual, referred to as a **consumer**, assigns a value $w$, termed the **willingness-to-pay**, to the item. This value is the maximum amount the consumer would pay to obtain the item. If the item is priced at or below $w$, the consumer will purchase the item. If the price is higher than $w$, he or she will not purchase it. This is the basic model of consumer behavior used in many economic analyses. We state it here.

**Consumer rule.** A consumer has a willingness-to-pay (WTP) $w$ for an item. If the price $p$ of the item satisfies $p \leq w$, then the consumer purchases the item. If $p > w$, the consumer does not purchase it.

Often it makes sense for a consumer to purchase more than one unit of the same item. This is true of tomatoes, cans of Coca-Cola, or bags of cement. In some cases it can be thought of as being true of books and CDs as well. For instance, someone may wish to get several identical CDs for gifts or several copies of a software package for a group.

When a consumer might possibly purchase more than one unit, he or she is considered to have a willingness-to-pay for each successive unit of that item. That is, there is a willingness-to-pay $w_1$ for the first unit, $w_2$ for the second, $w_3$ for the third, and so forth. Usually, successive values decrease since each one assumes that previous units have already been obtained.[1] Under this assumption, the willingness-to-pay for the second unit is less than the first, the third less than the second, and so forth. The consumer is therefore characterized by the decreasing sequence of willingness-to-pay values $w_1, w_2, \ldots$, as shown in figure 7.1. If the price per unit is $p$, the consumer will purchase the number $i$ of units such that $w_i \geq p$ and $w_{i+1} < p$.

Now return to the case where a consumer will acquire at most one unit of the item, but consider several consumers, each with perhaps different willingness-to-pay values. If these willingness-to-pay values are rank-ordered from greatest to smallest and plotted, the result will again look something like that of figure 7.1. Hence the aggregate willingness to pay is similar to that of an individual who might purchase several units of the item.

[1] In some cases, the willingness-to-pay may increase after a consumer has experienced the product. This is why, for example, some firms offer low-cost trials.

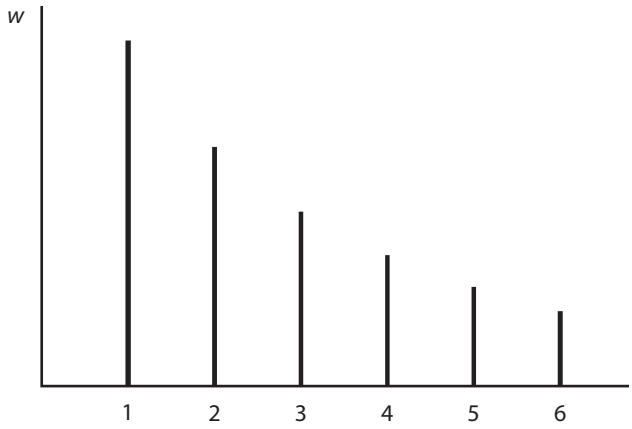


**FIGURE 7.1 Willingness-to-pay.** The willingness-to-pay usually decreases with each additional unit.

## Continuous Approximation

When the item under consideration is a commodity, such as sugar, that can be divided into arbitrarily fine units, it is assumed that the consumer assigns a willingness-to-pay value for each tiny quantity increment $\Delta q$. Letting $\Delta q$ go to zero, a **marginal willingness-to-pay function** $w(q)$ is defined. At quantity $q$, the willingness-to-pay for an additional amount $\Delta q$ is $w(q)\Delta q$. Figure 7.2 shows a typical marginal willingness-to-pay function.

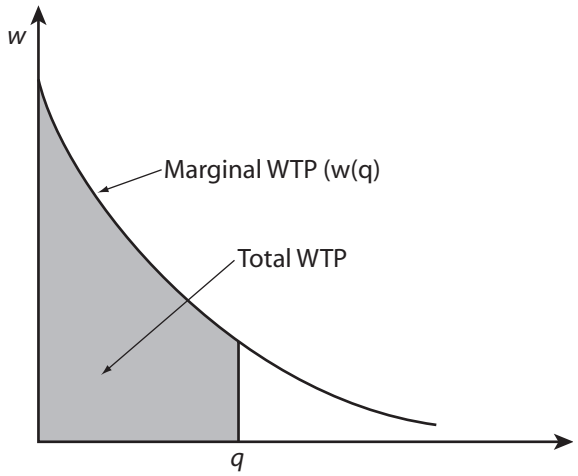


**FIGURE 7.2 Marginal willingness-to-pay.** The marginal willingness-to-pay function $w(q)$ is defined so that $w(q)\Delta q$ is the WTP for adding $\Delta q$ to $q$. The total WTP for an amount $q$ is the area under the $w(q)$ curve.

Again, a similar diagram applies when there are many consumers, even if each would acquire at most a single unit. In that case the scale of $q$ is considered to be large compared to one unit, so that a single unit is essentially a tiny quantity. For example, the available quantity of a certain CD album might be measured in tens or hundreds of thousands, with each consumer buying at most one. The **total willingness-to-pay** for an amount $q$ is the sum of the willingness-to-pay values for every unit acquired. In the continuous approximation, it is equal to the area under the marginal willingness-to-pay curve, as shown in the figure. In equation form,

$$\text{Total WTP}(q) = \int_0^q w(s)\,\mathrm{d}s. \tag{7.1}$$

The relation between several individuals' marginal willingness-to-pay functions and that of the corresponding aggregate group is a bit complex, but there is a simple case. Suppose that $n$ people in a group all have identical marginal willingness-to-pay functions $w_i(q)$. Then the marginal willingness-to-pay for the group of these $n$ people is the function[2] $w_g(q) = w_i(q/n)$ because an amount $q$ for the group translates into $q/n$ for each person.

## Demand Curve

If a fixed price $p$ is established for all units of a commodity, then in aggregate the group of consumers will purchase the quantity $q$, satisfying $w(q) = p$, where $w(q)$ is the aggregate willingness-to-pay. If the price is varied from a high price downward, the total amount bought will vary as well, with more being purchased as the price is lowered. The relation $w(q) = p$ is termed the **demand curve**, since implicitly it gives the quantity purchased as a function of price.

## Consumer Surplus

Suppose a fixed price $p$ is set per unit and that $w(q)$ is the aggregate marginal willingness-to-pay function. As stated above, the amount $q$ will be sold that satisfies $w(q) = p$. For most of the units purchased the marginal willingness-to-pay will be higher than $p$. Hence, consumers as a group pay less than it is worth to them. For instance, many consumers might be willing to pay \$30 for a new CD; if it is priced at \$12, those consumers get \$18 in extra value. This extra value is termed **consumer surplus**. In terms of the marginal willingness-to-pay curve, the total consumer surplus is equal to the area under the curve and above the horizontal line at $p$, as illustrated in figure 7.3. As a general rule, **consumer surplus** is the difference between the total willingness-to-pay for a quantity and the amount paid.

[2] It is *not* equal to $nw_i(q/n)$. We have $\text{WTP}_g(q) = \int_0^q w_g(q')\mathrm{d}q' = \int_0^{q/n} nw_i(q')\mathrm{d}q'$. Differentiation gives $w_g(q) = w_i(q/n)$.

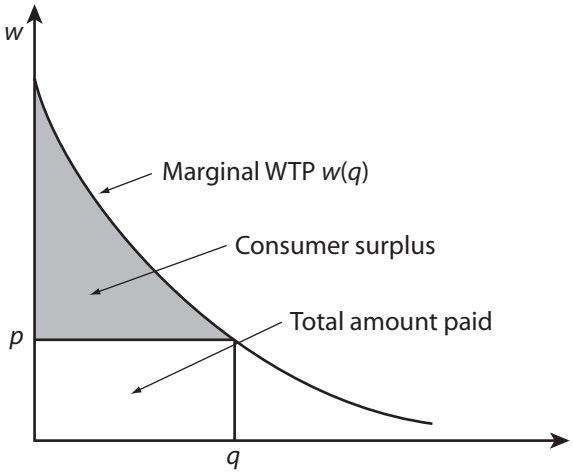


**FIGURE 7.3 Consumer surplus.** The extra value to consumers, above the price paid, is the consumer surplus.

## 7.2 Producers

Producers, too, are concerned with value. However, they are on the opposite side of the economic equation, seeking profit that depends on payments from consumers and on production costs. Accordingly, to analyze producers' actions, one must characterize their costs.

The cost of production of an item is a function of how many units are produced. A typical cost function for information products is shown in figure 7.4. There is **fixed cost** associated with merely getting ready for production. In book publication, for example, the fixed cost includes the cost of manuscript preparation, editing, art work, typographical composition, and press setup. It may also include marketing costs. Once production is begun, there are additional costs that depend on how many units

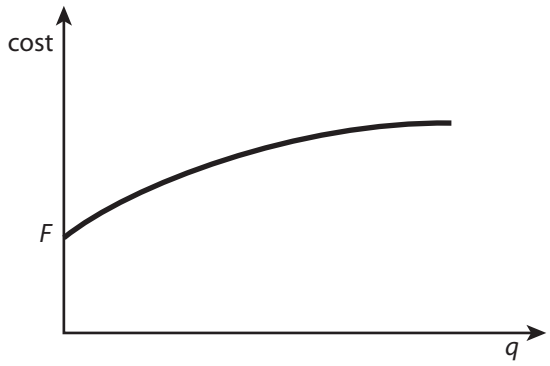


**FIGURE 7.4 Cost function.** Total cost increases with the quantity $q$. There is a fixed cost that must be paid at any nonzero production level. Then costs tend to rise smoothly.

are produced. For a book these are press charges, paper costs, bindery costs, and shipping. Total cost increases as additional units are produced.

The incremental cost associated with increasing production by a small amount is termed the **marginal cost**, and is usually denoted $m(q)$. Formally, if production is increased from the level $q$ to $q + \Delta q$, the increase in cost is $m(q)\Delta q$.

The marginal cost function $m(q)$ can be viewed as defining the cost of each successive unit. In book production, the cost of the first copy is likely to be very large even after the fixed setup costs, while the cost of a second copy may be smaller; the cost of a third copy smaller yet. Generally the cost of an additional copy (the marginal cost) decreases as the number of units increases, finally reaching a relatively small value associated with large production runs and efficient distribution. Indeed, information products are typically characterized by high fixed cost followed ultimately by low marginal cost at high production levels. Decreasing marginal cost is reflected in figure 7.4 by the flattening of the cost curve as $q$ is increased.

Marginal cost is the slope of the total cost curve $c(q)$. That is,

$$m(q) = c'(q). \tag{7.2}$$

This can be turned around to obtain

$$c(q) = F + \int_0^q m(s)\mathrm{d}s. \tag{7.3}$$

The fixed cost $F$ is $c(0)$.

Decreasing marginal cost is shown more explicitly by the **marginal cost curve**, illustrated in figure 7.5. The total cost is the fixed cost plus the integral from 0 to $q$ of the marginal cost, as expressed in (7.3).

Production processes for many traditional commodities, such as wheat, are characterized by marginal cost curves that increase rather than decrease, because at high levels of production less efficient resources (such as poor land) must be employed. This difference in the nature of marginal cost curves is one reason that information products are economically distinct from many traditional commodities.

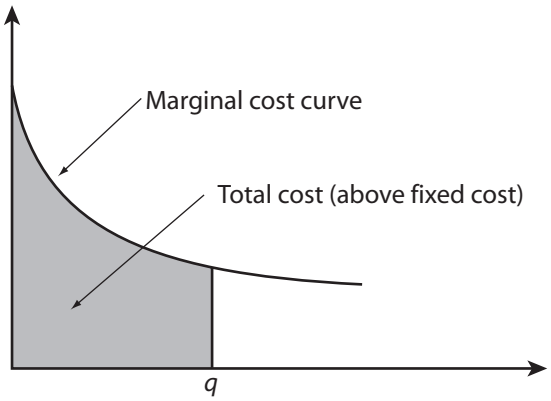


**FIGURE 7.5 Cost.** The marginal cost is the cost for an additional unit. The total cost is the sum, or integral, of all marginal costs plus the fixed cost.

### Constant Marginal Cost

Cost functions of information products frequently can be approximated by **constant marginal cost**, written as $m(q) = m$. In that case the total cost function is

$$c(q) = F + m \cdot q. \tag{7.4}$$

Because production of many information products entails extremely low marginal cost, it is sometimes assumed as an approximation that $m = 0$. For example, the marginal cost of a CD is a fraction of a cent, and that of a hardback book is a few dollars. The marginal cost of an Internet message is practically zero.

### Profit and Producer Surplus

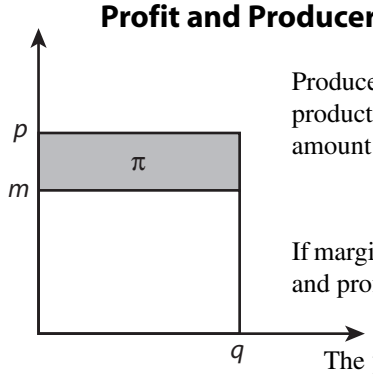


**FIGURE 7.6 Profit.** Producer surplus is the shaded area representing the price minus the marginal cost times the quantity.

Producers are interested in profit: the amount of money received minus the cost of production. If the total quantity sold is $q$ and all units are sold at price $p$, then the total amount received is $p \cdot q$. Hence the total profit is

$$\text{Profit} = p \cdot q - c(q).$$

If marginal cost is constant, as is typical of information products, then $c(q) = F + m \cdot q$, and profit is

$$p \cdot q - F - m \cdot q = (p - m) \cdot q - F.$$

The **producer surplus** $\pi$ is the part of profit that does not include the fixed cost $F$. In the case of constant marginal cost the producer surplus is $\pi = (p - m)q$. This portion of the profit is shown as the shaded area in figure 7.6. It is the total amount received by producers beyond what they would receive at marginal cost.

Typically, both consumers and producers try to maximize their respective surplus measures, and of course these objectives are to a large extent conflicting.

## 7.3 Social Surplus

Consumer surplus is value to consumers, and producer surplus is value to producers. The sum of these is termed **social surplus**; it is total value to all.[3] This definition of social surplus is general in the sense that it is independent of how transactions are carried out: through competition, monopoly, or individual negotiation. Social surplus is associated with each transaction and then summed, exactly the way the consumer and producer surpluses are associated with each transaction and summed.

The most important case is when all items are sold at a common price $p$. Consumer and producer surplus are then equal to the areas defined by the marginal willingness-to-pay curve and marginal cost curve. Figure 7.7 shows the situation when marginal cost is a constant $m$.

The quantity sold at price $p$ is determined by the demand curve (identical to the marginal willingness-to-pay curve). Consumer surplus is therefore the area of the

[3] If payments are made to the government (as from taxes), these would be added to the definition of social surplus.

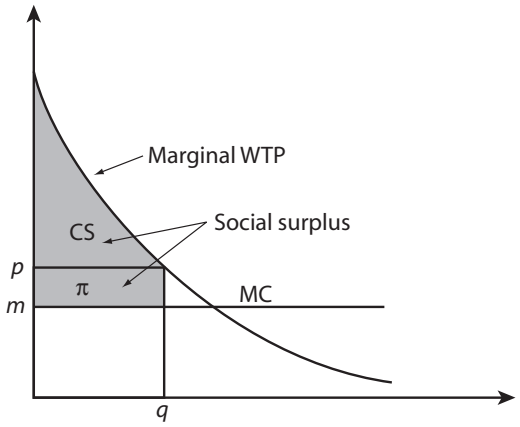


**FIGURE 7.7 Social surplus.** At a fixed price $p$, consumers purchase a quantity $q$ such that $p =$ marginal WTP; that is, $p$ is determined by the demand curve. The associated consumer surplus is the area between that curve and the horizontal line at $p$. Producer surplus $\pi$ is equal to the area of the rectangle of height $p - m$ and width $q$. Social surplus is the sum of these two.

shaded triangular-shaped region above the horizontal line at $p$ and under the demand curve. Producer surplus $\pi = p \cdot q - m \cdot q$ is the area of the shaded rectangle. Hence social surplus is the total of the two shaded areas.

From the perspective of society, a transaction is valuable if the associated social surplus is positive. It does not matter whether the value accrues to consumers or to producers since (1) firms are owned by individuals and firms' profits are distributed to the owners, and (2) value to either consumers or producers can, at least in theory, be redistributed through tax mechanisms. On the other hand, from the same perspective, a product is socially valuable if the associated social surplus is greater than the fixed cost, for then the net gain to society is positive.

## 7.4 Competition

Most goods and services are sold through competition, but competition is an exceedingly complex process, taking many forms. In order to analyze economic issues, economists idealize the complex process, simplifying it while capturing its primary characteristics.

In an idealized form of competition, termed **perfect competition**, there is a single price $p$ for all items of a given kind. This price is determined in ways that are difficult or impossible to model, but for purposes of analysis economists often simply assume that there a common price determined by market forces.

It is also assumed that each producer's and consumer's market participation is small compared to the size of the overall market, and there is no collusion. It follows that each party is a **price taker** in the sense that any individual's or firm's market action does not affect the price.

For our purposes, we also assume that all producers have identical cost structure, with constant marginal cost.

These idealizing assumptions are sufficient to draw a significant conclusion about the nature of the competitive price $p$.

**Marginal cost pricing.** Under perfect competition, if a product is produced, its price is equal to its marginal cost.

***Proof:*** As stated, marginal cost is assumed to be a constant $m$, although the conclusion is true more generally. Suppose first that $p < m$. Then no producer will produce because to do so would entail an immediate loss that could be avoided by not producing. Hence, it follows that $p \geq m$.

Suppose next that $p > m$. Then any producer could offer the item at a lower price $p' < p$ and virtually all consumers would move their business to that producer, increasing that producer's profit enormously. Other producers would react by lowering their price below $p'$, and this would continue until $p = m$. ∎

Marginal cost pricing has important implications for information products. If price equals marginal cost, producer surplus is zero, which means that actual profit, accounting for the fixed cost, is negative. Hence, a typical information product sold in a competitive environment will garner strictly negative profit. Who would want to produce under these conditions?

Competition *is* viable for goods with increasing marginal cost, such as wheat, because profits can be made. But, when marginal cost is constant (or worse yet, decreasing), as is the case for most information products, the overall profit associated with marginal cost pricing is negative.

## 7.5 Optimality of Marginal Cost Pricing

Competition, with its attendant marginal cost pricing, is not desirable from a producer's perspective, but it is highly desirable from a social perspective. When a product is sold at a common fixed price $p$ per unit, setting that price equal to marginal cost yields the maximum possible social surplus. This optimality result can be established algebraically or by the simple graphical analysis shown in figure 7.8.

The graphical analysis is explained in the figure caption. The algebraic derivation is also straightforward. If consumers select $q$ such that $w(q) = p$, it follows that

$$\begin{aligned} \mathrm{CS}(q) &= \int_0^q (w(s) - w(q))\mathrm{d}s \\ \pi(q) &= (w(q) - m)q \\ \mathrm{SS}(q) &= \int_0^q w(s)\mathrm{d}s - mq. \end{aligned}$$

Setting the derivative of the social surplus with respect to $q$ to zero produces

$$w(q) = m.$$

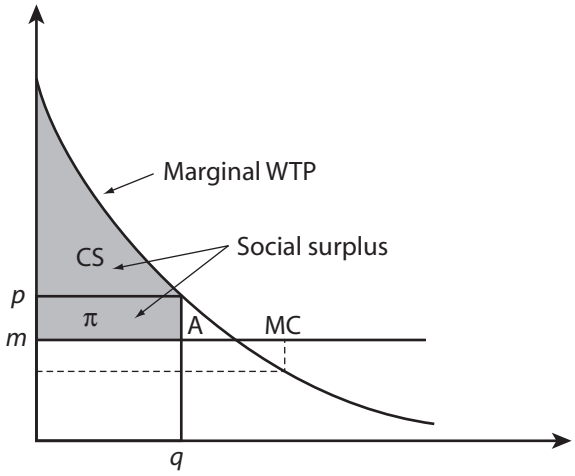


**FIGURE 7.8 Maximum social surplus.** If the price $p$ in the figure is reduced to $m$, the associated social surplus will increase by the area of the small triangular region A. If the price is reduced further, to the level of dashed line, consumer surplus will increase, but producer surplus will be negative and equal in magnitude to the area defined by the dashed rectangle. The net effect is to reduce social surplus. The maximum social surplus is obtained at $p = m$, although producer surplus is zero at that point.

Since $p = w(q)$, the price is $p = m$, which is marginal cost pricing. We reach the following conclusion about optimal pricing.

**Optimality of marginal cost pricing.** In a single-price system, social surplus is maximized when price is equal to marginal cost.

It is clear that there is an economic dilemma associated with constant (or decreasing) marginal cost. Perfect competition is optimal from a social perspective, but yields negative profit to producers. How then are books, music, motion pictures, software, or the vast assortment of other information products to be sold?

## 7.6 Linear Demand Curves

For purposes of analysis, it is convenient to use demand curves—the marginal cost curves—that are downward sloping straight lines. A straight line is a reasonable approximation in many cases, and it occurs naturally in some situations.

Consider a product that is either purchased as a unit or not at all. Nobody purchases more than one unit. For instance, the product may be a book, a magazine subscription, or a software program. With reference to this product, a consumer is characterized by his or her single willingness-to-pay value. The collection of all consumers is accordingly characterized by the distribution of these WTP values.

Assume that there are many consumers; so many that it is reasonable to describe their WTP values as distributed continuously on a value axis. A special case of such a distribution is shown in figure 7.9(a). This figure implies that the WTP values are

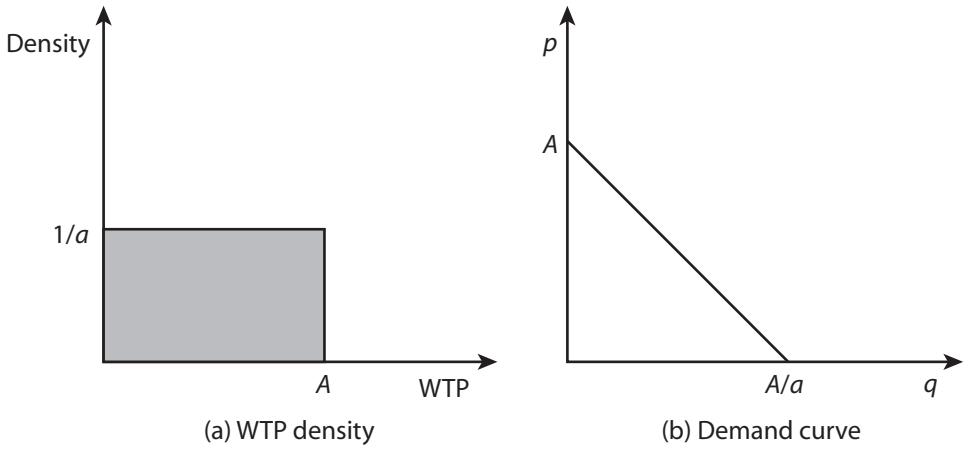


**FIGURE 7.9 Linear demand from uniform density.** A uniform density of WTP values implies a linear demand curve.

distributed uniformly between 0 and $A$. For instance, one-fourth of the people have WTP less than or equal to $A/4$. The total number of people in any $\Delta W$ range of WTP is $\Delta W/a$. The total number of people is $A/a$. For example, the WTP for a book may range between \$0 and $A = \$40$, and the total number of people considered might be $A/a = 40/a = 1$ million, and hence $a = 4 \times 10^{-5}$.

The uniform distribution of WTP values leads to the linear demand curve of figure 7.9(b). To see that, consider first the lowest price that can be set such that no one will buy the item. Clearly, that price is $A$, for no one has a WTP figure higher than that. Hence the demand curve intersects the price axis at $p = A$. Next, it is easy to see that the maximum quantity that can be sold (at $p = 0$) is $A/a$ since that is the total number of people in the market.

The demand curve moves down linearly between the $p = A$ and $q = A/a$ units. This is because at, say $p = A/4$, one-fourth of the people will have WTP greater than or equal to $p$ and hence, exactly $A/4a$ people will buy.

## 7.7 Copyright and Monopoly

Before the advent of the printing press, copying of manuscripts was tedious but rarely of legal concern. However, in 567 an Irish monk copied from a neighboring monastery the abbot's Psalter without permission. When the monk refused to return the copy, the abbot appealed to the king, who ordered the copy returned.

The printing press was introduced into England in 1476 and that same year a law was established that required printers to license books they printed. This law effectively granted a monopoly to printers, not to authors.

The first real copyright law was the Statute of Anne of 1710, which established the principles that the copyright belongs to the author and that the term of protection is limited.

These principles were embodied in the U.S. Constitution with the phrase, "the Congress shall have power . . . to promote the progress of science and useful arts, by securing for limited times to authors and inventors the exclusive right to their respective writings and discoveries." The term of copyright was originally set by Congress in 1790 as 14 years plus a possible renewal of 14 years. U.S. copyright law has subsequently been modified several times, but the latest version is embodied in the Sonny Bono Copyright Term Extension Act, which extended copyright protection to life of the author plus 70 years, and for works made for hire to 95 years (which many say was so that the Disney corporation could maintain control of Mickey Mouse).

Copyright law is intended to increase social welfare (that is, social surplus) by enabling authors and producers to make profit sufficient to cover the fixed costs of socially desirable products. It is fair to ask, however, to what extent such law resolves the dilemma inherent in information products: that competition leads to zero producer surplus and hence no incentive to expend initial resources for creation of an information product. Fortunately, there is a nice answer to this question (at least for the single-period case), which can be deduced by simple graphical reasoning or by elementary calculation.

A monopolist has the power to set the price of a product, rather than being forced to accept the price set by the market. An astute monopolist will therefore set price to maximize profit. To see what this entails, first assume, for simplicity, that demand is linear, as shown in figure 7.10. Also assume that marginal cost is a constant, $m$. The monopolist can select any price, and the corresponding quantity will be

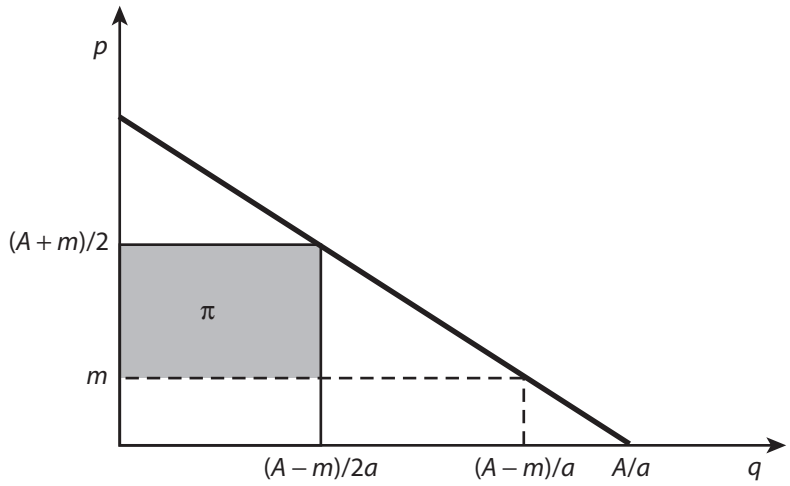


**FIGURE 7.10 Monopoly pricing.** To maximize profit when the demand curve is linear, a monopolist with constant marginal cost sets the quantity at one-half the quantity associated with marginal cost pricing. The producer surplus is then one-half of the maximum possible social surplus. The total social surplus at that price is three-fourths of the maximum possible. However, only half as many consumers are served as would be under perfect competition.

determined by the demand line. The associated producer surplus $\pi$ is the area of the rectangular region bounded by the vertical axis, the marginal cost line, and the demand curve.

The quantity that maximizes profit is one-half the quantity associated with marginal cost pricing. This choice is shown in the figure.

Using calculus, it is easy to prove that the halfway point is best. Let the demand curve be $p = A - aq$, which means that $A$ is the largest price point (with $q = 0$) and $A/a$ is the largest quantity point (with $p = 0$). The quantity associated with perfect competition is the quantity where $p = m$, leading to $m = A - aq$, or equivalently, $q = (A - m)/a$.

At an arbitrary price $p$ the producer surplus is

$$\pi = (p - m)q = (A - m - aq)q.$$

Setting the derivative of this to zero yields

$$A - m - 2aq = 0,$$

or equivalently

$$q = (A - m)/(2a),$$

which is the halfway point on the $q$ axis to the quantity associated with marginal cost pricing. The corresponding $p$ is $p = A - \frac{1}{2}(A - m) = \frac{1}{2}(A + m)$.

The producer surplus associated with this solution is exactly one-half the area under the demand curve and above the marginal cost line. Hence the producer surplus is one-half the maximum possible social surplus. The consumer surplus is half of the producer surplus. The total social surplus at this solution is therefore three-fourths of the maximum possible. The quantity produced is only one-half of what would be produced under perfect competition. In the case where each consumer buys at most one unit, this means that only half as many people are served as would be under perfect competition.

This result has important social implications. A product is socially desirable if the potential social surplus exceeds the fixed cost. From an economic perspective, society should encourage the production of such products. Competition does not provide that encouragement because marginal cost pricing yields zero producer surplus and hence negative net profit.

If producers are granted monopoly rights, they can earn producer surplus equal to one-half the potential social surplus. If fixed costs are less than this, producers will produce. However, if fixed costs are greater than one-half the potential social surplus, a monopolist will have no incentive to produce, even if the potential social surplus exceeds the fixed cost. Only products with fixed cost less than one-half the potential social surplus will be produced. The granting of monopoly rights is therefore only

a partial solution to the problem of encouraging production of useful products. This result is summarized below.

**Monopoly profit.** When demand is linear and marginal cost is constant, a monopolist can collect producer surplus equal to one-half of the potential social surplus. Only one-half of the quantity that would be sold under marginal cost pricing will be sold under monopoly.

## Nonlinear Demand

The results about monopoly can be generalized to demand curves that are nonlinear but convex as in figure 7.11: starting out nearly vertical and gradually becoming more horizontal. Many real demand curves are of this shape.

The optimal combination of price and quantity is some point on the curve, and the producer surplus is equal to the area of the rectangle defined by that and the marginal cost line. Suppose at that point a line tangent to the curve is constructed as shown in the figure. From the analysis for a linear demand curve, we know that the area of the rectangle is at most one-half the area of the triangle defined by the tangent line down to the marginal cost. It is easily seen that the area of this triangle is at most equal to the area under the curve above the marginal cost. Hence, producer surplus is no more than one-half the potential social surplus, just as in the linear case.

This general result implies that other methods should be sought to encourage the development of information products that are beneficial to society but which have large fixed cost.

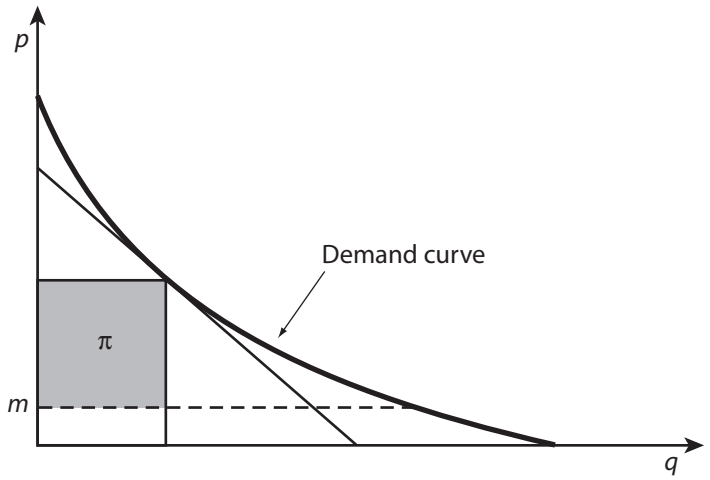


**FIGURE 7.11 Monopoly profit.** When the demand curve is convex, a monopolist can earn at most one-half the available consumer surplus, and the quantity sold is less than one-half that which would be sold under marginal cost pricing.

## 7.8 Other Pricing Methods

A variety of pricing methods have been proposed to encourage the production of socially beneficial products that could not otherwise withstand the pressure of competition. One alternative is for the government to fund the development of these products and make them available at their marginal cost. This funding can take the form of government laboratories, university research grants, or business tax credits for research and development. Two other useful methods are discussed in this section.

### Regulated Monopolies

On the one hand, competition is infeasible as a practical matter when marginal costs are constant or decreasing. On the other hand, monopoly does not yield maximum social surplus. An alternative lying between these two extremes is the **regulated monopoly**. In this arrangement a regulating agency explicitly sets price to maximize social surplus subject to the constraint that fixed costs are recovered.

The regulated solution reduces the price to the point where the producer surplus is just equal to the fixed cost $F$. This maximizes consumer surplus while providing an incentive to produce.

Regulated monopolies are common arrangements for postal services and energy production, but are rare in information products, partly because it is difficult to determine demand curves and costs.

### Voluntary Payment

Some information products such as public television, church services, street performances, and some teaching are supported by voluntary contributions. To see how this method of support fits into the general framework, suppose for simplicity that marginal cost is zero and that all consumers of a product voluntarily contribute an amount equal to one-half of their willingness-to-pay. The resulting producer surplus is then equal to the shaded triangular region indicated on the diagram of figure 7.12. It is easy to see that this producer surplus is one-half of the maximum possible social surplus. This is exactly the profit that a monopolist would obtain. However, with voluntary contributions, the total social surplus is the entire area below the demand curve. This is better than under monopoly, where the social surplus is only three-fourths this amount. Furthermore, everyone is served under a voluntary system. Hence, if the same profit can be generated under either system, the voluntary system is preferable to a monopoly.

In practice, of course, consumers may contribute less than half their consumer surplus, owing to the **free rider** effect: that is, without contributing themselves, consumers can obtain the benefit of everyone else's contributions. Low contributions considerably reduce the producer surplus, but the social surplus is still maximal provided that contributions are sufficient to cover fixed cost.

Other useful mechanisms for pricing information products are discussed in the next chapter.

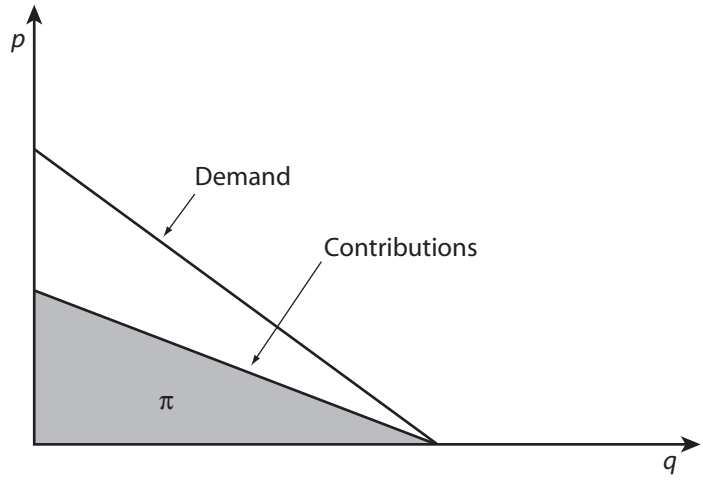


**FIGURE 7.12 Voluntary contributions.** The diagram assumes that marginal cost is zero. When contributions are voluntary, everyone is served, and total social surplus is equal to the maximum possible. If the average rate of contribution is one-half of a consumer's willingness-to-pay, the producer surplus is equal to that which would be obtained by a monopolist.

## 7.9 Oligopoly

There are situations intermediate between perfect competition (with many producers) and monopoly (with one producer). A market in which there are a limited number of producers is an **oligopoly**. The simplest is a **duopoly**, where there are two producers, and we shall study that form first.

Assume that there are two producers, each with marginal cost $m$. If they produce quantities $q_1$ and $q_2$, respectively, their corresponding producer surpluses are

$$\pi_1 = p(q_1 + q_2)q_1 - mq_1 \tag{7.5a}$$

$$\pi_2 = p(q_1 + q_2)q_2 - mq_2. \tag{7.5b}$$

The profit to each producer depends, therefore, not only on its own production level but also on that of the other producer. Neither producer can maximize its producer surplus without knowing what the other will do.

One way they might proceed is by **collusion**, agreeing on a total quantity and then dividing that among themselves. The best total quantity would be the monopoly quantity. The oil cartel OPEC attempts to operate this way. However, within a regulated economy such collusion is not allowed.

A more realistic assumption is that suggested by the economist Augustin Cournot in 1838. In a **Cournot equilibrium** each producer maximizes its own profit assuming that the quantity produced by its competitor is fixed, but fixed at what would be that competitor's optimal quantity. In other words, the producers maximize separately but simultaneously.

To work this out in detail, assume a linear demand function $p(q) = A - aq$. The producer surpluses (7.5) are then

$$\pi_1 = (A - a(q_1 + q_2))q_1 - mq_1 \tag{7.6a}$$
$$\pi_2 = (A - a(q_1 + q_2))q_2 - mq_2. \tag{7.6b}$$

The first of these is maximized by setting its derivative with respect to $q_1$ to zero. Likewise, the second is maximized by setting its derivative with respect to $q_2$ to zero. The result is the two simultaneous equations

$$A - m - 2aq_1 - aq_2 = 0 \tag{7.7a}$$
$$A - m - 2aq_2 - aq_1 = 0. \tag{7.7b}$$

These are easily solved by noting that symmetry implies $q_1 = q_2$. Thus

$$q_1 = \frac{A - m}{3a}, \qquad q_2 = \frac{A - m}{3a}.$$

The corresponding price can be found to be

$$p^* = \frac{A + 2m}{3}.$$

This solution is illustrated in figure 7.13.

Analysis of an oligopoly with $n$ identical competing firms is similar to that for a duopoly. With the same demand function as above and marginal cost $m$, the optimal quantity for each firm is

$$q_i = \frac{A - m}{(n + 1)a},$$

with corresponding price

$$p^* = A - \frac{n(A - m)}{n + 1}.$$

Notice that $p^* \to m$ as $n \to \infty$, which is marginal cost pricing, corresponding to perfect competition.

The concept of a Cournot equilibrium was extended by the brilliant mathematician John Nash[4] to a basic result in game theory involving several players. The players each have payoff functions that may depend on the actions of all players. The Nash

[4]Nash became schizophrenic while quite young, but he was later awarded a Nobel Prize for his work in game theory. His life is reported in the best-selling biography and popular motion picture *A Beautiful Mind.*

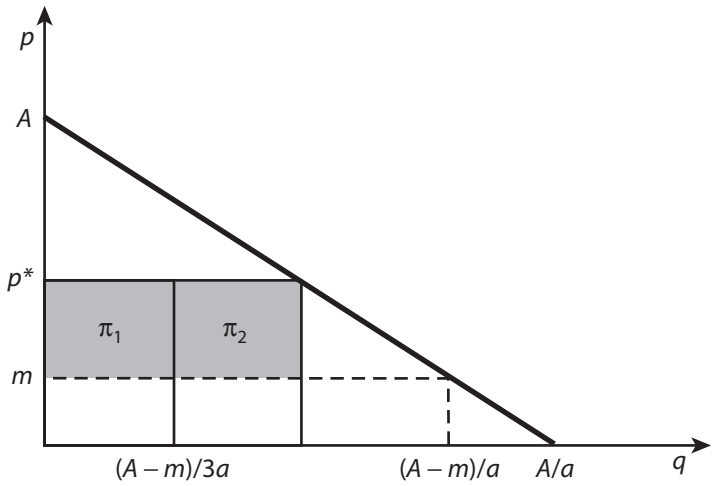


**FIGURE 7.13 Duopoly.** In a duopoly of identical producers, the equilibrium price $p^*$ is lower than under monopoly. Hence the overall producer surplus is less than under monopoly, but social surplus is greater.

equilibrium, like its predecessor the Cournot equilibrium, is the result of simultaneous individual optimization.

**Example 7.1 (Entry blocking).** Suppose a monopolist in a certain product is challenged by a potential entrant to the market. How can the monopolist discourage such entry? One strategy is for the monopolist to lower price, so that entry will be less attractive.

The monopolist will not want to reduce his or her profit below that which would hold under duopoly, but that is a good benchmark for the strategy. Assume a linear demand function $p(q) = A - aq$ and zero marginal cost. The price $p^*$ under duopoly would be $p^* = A/3$, and the quantity produced by each producer would be $A/3a$. Hence the producer surplus to the firm under duopoly would be $\pi_1 = A^2/9a$.

As a monopolist, the firm can achieve this same surplus at a $q$ satisfying

$$\pi = p(q)q = (A - aq)q = Aq - aq^2 = A^2/9a.$$

The solution to this quadratic equation is

$$q = \frac{1}{2a}\left[A \pm \sqrt{A^2 - 4A^2/9}\right] = \frac{A[3 \pm \sqrt{5}]}{6a}.$$

Of the two solutions, the one with a plus sign is the one needed for blocking (why?). This gives $q = .873A/a$. The corresponding price is $p = .127A$ as compared to $p^* = .33A$ under duopoly. This low price may discourage the potential entrant.

## 7.10 EXERCISES

1. (Sharing the cost* ) Two people with identical preferences for e-services decide that they will each purchase individual levels of service, then place a joint order for the total and share the cost equally (as is often done when dining out with friends). Specifically, each person has marginal willingness-to-pay of $w(x) = 10 - x$ for service level $x$. There is a fixed unit cost of \$6.00 per unit level of service. The marginal production cost of the service is \$5.00. (By symmetry, each person purchases the same amount $x$.)
   (a) If the two people did not share the cost, but each paid for his or her own order, how much would each purchase? What is the consumer surplus for each person? What is the profit to the service provider for each person? What is the total social surplus per person?
   (b) Under the sharing arrangement, how much will each purchase?
   (c) What is the consumer surplus of each person in this arrangement?
   (d) What is the profit to the service provider for each person, and what is the total social surplus per person?
2. (Superior product) Currently, the industry for delivering complex financial data is highly competitive with a marginal cost of $M$. A firm has just devised a new technology for delivering this data at a marginal cost of $m < M$. The firm can either enter the market directly on its own, or license the technology by charging other firms a fixed fee for each sale they make. Show that the firm can make at least as much by licensing as by entering the market on its own.
3. (Two markets) A book publisher sells in two distinct markets: A and B. The marginal cost of books is essentially zero. The demand functions in the two markets are different. They are $p_A = 600 - 3q_A$ and $p_B = 400 - 2q_B$, respectively.
   (a) If the publisher uses the same price in each market, what is the effective demand function for the total? Restrict attention to $p_T < 400$. (Hint: it is of the form $p_T = T - tq_T$.)
   (b) Under the conditions of part (a), what is the total profit of the publisher?
   (c) Now assume that the publisher is able to discriminate by charging different prices in the two markets A and B. What is the maximum profit to the publisher?
4. (Taxes) Suppose that an information product is produced with zero marginal cost and sold with perfect monopoly power. Each consumer purchases either one or zero units of the product. The overall demand curve is $p = A - aq$. However, the government taxes the product an amount $t$ dollars per unit. The tax is paid by the producer, so that $p$ is the actual price paid by consumers.
   (a) Find the amount sold and the selling price.
   (b) Find the producer surplus, consumer surplus, and total tax revenue received by the government. Call the sum of these $T$.
   (c) How does $T$ compare with the total social surplus $S$ that would be obtained without taxes?
5. (Copyright term*) Recently the Supreme Court upheld Congress's right to extend the term of basic copyright protection to 95 years. There has been a great deal of debate about whether this serves the public welfare. This exercise suggests a (highly simplified) model of the issue.

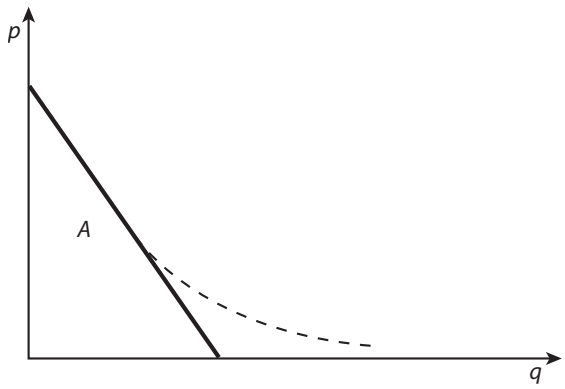


**FIGURE 7.14 Yearly rate of demand for copyrighted material.**

Suppose that the yearly rate of demand for a creative work is constant and follows the linear curve shown in figure 7.14. The area in the triangular region is $A$. The marginal cost of production is zero.

It follows that the yearly rate of producer surplus, if the product is actually produced, is $\pi = A/2$, and if the producer values future surpluses by discounting at the rate $r$, the total surplus to time $T$ is

$$\pi = \int_0^T \pi(t)e^{-rt}\mathrm{d}t = \frac{A}{2r}\left(1 - e^{-rT}\right).$$

The originator will produce the item only if the producer surplus exceeds the fixed cost $F$ of creation. Assume that over the vast assortment of information products with the demand rate of the figure, the cost $F$ is uniformly distributed between 0 and some maximum $M$. Hence the probability that a given work will be created and marketed is proportional to the total available producer surplus $\pi$. Likewise, the expected value $\overline{F}$ of the fixed cost given that this cost is less than $\pi$ is $\pi/2$. Argue that the expected net benefit to society is proportional to

$$B = \{(\text{SS over } 0 \le t \le T) + (\text{SS over } t > T) - \overline{F}\} \cdot \text{prob}(\pi \ge F).$$

(a) Assuming that society uses the same discount rate $r$, give an explicit expression for $B$ (to within a constant multiple).
(b) Show that there is no finite time that maximizes the social objective.
(c) Suppose that the actual demand curve slopes outward at the lower end as indicated by the dashed curve so that the total area under the curve is $\rho A$ where $\rho > 1$. Find the optimal value of $T$.
(d) Find $T$ when $\rho = 1.2$ and $r = .05$.
(e) What is the limit of $T$ as $\rho \to \infty$ in terms of $r$? Compare with the original 14-year term set by Congress in 1790.

6. (Asymmetric duopoly) The demand curve for a certain product is

$$p = 10 - q.$$

Two firms operate as a duopoly in this market. Firm 1 has constant marginal cost $m_1 = 2$ and firm 2 has constant marginal cost $m_2 = 3$. If the firms operate as a Cournot equilibrium (each firm maximizing profit while assuming the other firm's output is fixed), what are their respective output levels $q_1$ and $q_2$?

7. (Cartel cheating) Suppose that a product has a demand curve $p = A - aq$ and the marginal cost is zero. Suppose also that a cartel of two identical firms controls this product.
   (a) If the two firms act together and divide the market equally, what is the maximum profit (ignoring fixed costs) that each firm can obtain, and the corresponding production quantities?
   (b) Suppose that one firm faithfully produces the quantity of part (a). Show that the other firm has an incentive to break the agreement by producing more than agreed to. What is the maximum profit this second firm can obtain?

8. (Convex demand*) Suppose the demand function for a certain information product is

$$p = 10 - 7q + q^2.$$

   However, there is a maximum quantity $q = 2$ that the market can absorb. The product is manufactured with zero marginal cost.
   (a) What is the quantity that would be sold under perfect competition?
   (b) What is the quantity that would be sold if the firm had a monopoly in that product?

## 7.11 Bibliography

Most of the material in this chapter is included in the subject of intermediate microeconomics. Three texts on the subject (in order of increasing difficulty) are [1], [2], [3]. A theory of the economics of information, including the material on the advantage of copyrights, is presented in [4]. [5] is a biography of John Nash.

### References

[1] Pindyke, Robert S., and Daniel L. Rubinfeld. *Microeconomics*. 2nd ed., New York: Macmillan, 1992.
[2] Varian, Hal R. *Intermediate Microeconomics*. New York: Norton, 1987.
[3] Luenberger, David G. *Microeconomic Theory*. New York: McGraw-Hill, 1995.
[4] Bell, Hanan S. "A New Approach to Incentives for Information Creation and Distribution." Ph.D. diss. Department of Engineering-Economic Systems, Stanford University, May 1989.
[5] Nasar, Sylvia. *A Beautiful Mind*. New York: Touchstone, 1998.