Preface

In the United States, a graduate course in classical electromagnetism is often seen as a vehicle for learning mathematical methods of physics. Students mostly perceive the subject as hand-to-hand combat with the ∇ symbol, with endless nights spent gradding, divvying, and curling. Instructors have a more benign view, but many of them too regard the subject as a way of teaching the classical repertoire of Green functions, Sturm-Liouville theory, special functions, and boundary value problems. Perhaps this viewpoint is traditional. In the 1907 preface to the first edition of his classic text *Electricity and Magnetism*, James Jeans writes:

> Maxwell's treatise was written for the fully equipped mathematician: the present book is written more especially for the student, and for the physicist of more limited mathematical attainments.

The emphasis on mathematics continues throughout the preface: words derived from *mathematics* appear ten times in the single page, while derivatives of *physics* appear just five times.

To me, this point of view is both a puzzle and a pity, for while the mathematics of electrodynamics is indeed very beautiful, the physics is much more so. And so, while this book does not shy away from mathematical technique, the emphasis is very much on the physics, and I have striven to provide physical context and motivation for every topic in the book. Naturally, greater mastery of the mathematics will lead to greater enjoyment of the physics. I try to model good and efficient methods of calculation, but this is never an end in itself, and I do not hesitate to use the physicist's quick and dirty methods when appropriate. I urge readers, students in particular, to approach the subject in the same spirit.

This book started out many years ago as lecture notes for students in a graduate course. Over the years, to keep my own enthusiasm from waning, I varied the topics in the latter part of the course. A few others that I never got to teach are also included in the book.

As a result, there is more material here than can be covered in a single full-year course. I hope that all instructors will find something to interest them among the more advanced topics and applications, and that the book will serve as a reference for students after the course is complete.

The sequence of topics is as follows. After a very brief survey [1] and a longish math review [2],[1] we proceed to electrostatics [3] and magnetostatics [4] in vacuum, Faraday's law of induction [5], and electromagnetic waves in vacuum [7]. Chapter 6 is on the symmetries of the laws of electromagnetism, and it is here that the concepts of field energy and momentum are introduced. These chapters are part of the core of the subject and belong in all courses.

Chapter 8 deals with interference and diffraction. This material is somewhat nonstandard in an electromagnetism course and may be skipped without loss of continuity. On the other hand, the phenomena of interference and diffraction are the most striking manifestations of the wave aspects of light, and the related topics of coherence and intensity interferometry (the Hanbury-Brown and Twiss effect) are among the more subtle aspects of light that were understood in the twentieth century. They also connect to our understanding of quantum mechanics, and similar ideas are active research topics even today in other areas of physics. A section on the Pancharatnam phase and how the polarization of light affects interference rounds out the chapter.

Next we come to radiation from accelerated charges [9, 10]. Although the field of a moving charge is found in full generality [9], only nonrelativistic sources are considered for the present [10], and relativistic sources are treated later [25]. While this is also core material, the more advanced sections, such as those on higher multipoles, radiation of angular momentum, and radiation reaction, can be omitted in a first reading.

The motion of charges and magnetic moments in external fields is studied next [11]. In addition to charges in uniform and static fields, and the betatron, I have included a moderately complete account of Alfven's guiding center method for charges in inhomogeneous magnetic fields. This method is indispensable for the study of plasmas. I also discuss Larmor precession of magnetic moments, in both static and time-dependent magnetic fields. The concept of adiabatic invariants is developed both for charges and moments in magnetic fields. Again, subsequent chapters do not depend on this development in an essential way and many sections in this chapter may be skipped in a first pass.

Chapter 12 puts electromagnetism within the framework of the action principle. It is thus an essential launching point for the study of *quantum* electrodynamics, and even within the classical theory it is a deep unifying principle. For the study of electromagnetic *phenomena*, however, it is inessential, and less formally minded readers may choose to skip it.

Chapters 3 through 12 constitute a fairly complete treatment (except for relativity, for which see later) of the basic laws and phenomena of electromagnetic fields in vacuum. With these fundamentals over, we turn to electromagnetic fields in matter [13–21]. This

[1] Chapter numbers are in square brackets.

separation of the study of fields in vacuum and matter is a little nonstandard,[2] and it is more common to study the electrostatics of conductors [14] and dielectrics [15] along with electrostatics in vacuum, and electromagnetic waves in matter [20, 21] along with those in vacuum. I believe that no good can come of this fusion. It appears to me to be motivated by the incidental similarity of the mathematics required. However, the interesting physical questions are quite different in the two situations, and the basic electromagnetic equations in vacuum and in macroscopic materials have rather different meaning and ontological status. Thus, studying the propagation of light in glass and vacuum as common instances of the solution of the wave equation obscures the physical content of the dielectric constant, or the Kramers-Kronig relations, or the interplay between electromagnetic energy, mechanical work, and heat dissipation in a medium. A mathematically oriented approach also misses out on the beautiful physics involved in modeling constitutive relations. With specific reference to electrostatics, most students have studied it with conductors and in vacuum as one topic as undergraduates, so they already know that the field is strong near sharp points and edges and other similar facts, and there is little compelling reason to follow the same path again. Nevertheless, instructors who wish to stress the solution of Laplace's equation through methods such as separation of variables and orthogonal functions, can cover chapter 14 earlier.

In more detail, the treatment of electromagnetism in matter is organized as follows. A short survey chapter [13] introduces the concepts of spatially averaged or macroscopic fields, magnetization and polarization fields, and the vital role of constitutive relations. It is particularly stressed that the latter are largely phenomenological, but no pejorative connotation is attached to this word. It is also stressed that the similarity of form of the macroscopic or material Maxwell equations with those in vacuum is deceptive and sweeps important physics under the rug. This chapter is very important, and readers are urged not to skip any of it.

Chapter 14 covers electrostatics with conductors. My treatment is quite traditional, and it is with some ambivalence that so much detail is given. The prevalence of numerical methods has rather reduced the importance of the classical mathematical techniques, but they provide invaluable insight, and even the numerical solver would be unwise to ignore them. I have also included a section on the work function and contact potentials, if for no other reason than to show the reader that there is subtle physics in this otherwise mathematical territory. The coverage of electrostatics with dielectrics [15] is also traditional. In both chapters, there are several advanced sections that may be skipped without loss of continuity.

Magnetostatics in matter comes next [16]. The central point of physics here is to understand the constitutional differences (constitutive relations, if you will) between para-, dia-, and ferromagnets. Much of the chapter is devoted to ferromagnets, and relevant phenomena, such as domain walls, hysteresis, and demagnetization, ancient topics that are astonishingly current for the design of hard disks and other computer elements. The dangers of uncritical use of equations such as $\mathbf{B} = \mu\mathbf{H}$ are dwelt on. I have

[2] The same approach is taken in the Landau and Lifshitz *Course of Theoretical Physics*, vols. 2 and 8.

tried hard to obey Paul Muzikar's exhortation to explain the difference between **B** and **H** carefully, and I am sure I will hear from him if I have let him down! The chapter also includes a short section on superconductors and the Meissner effect.

The next chapter is on Ohm's law, emf, and electrical circuits [17]. These topics are of great practical importance, but they are often regarded as belonging to the realm of engineering. My focus is on understanding of the meaning of emf, and of the physics of the lumped circuit approximation. I also discuss the distribution of current in extended conductors, and van der Pauw's method of measuring resistivities.

With this, we turn to dynamic phenomena, beginning with the general issue of frequency-dependent response or constitutive relations [18]. This chapter covers the Drude and Drude-Lorentz models for the frequency-dependent conductivity and dielectric functions, which are of great pedagogical value because they fix several key physical concepts in the student's mind. I also discuss Kramers-Kronig relations and electromagnetic energy in material media. None of this material should be skipped.

Quasistatic phenomena [19], such as the skin effect, eddy currents, and maglev, come next, followed by electromagnetic waves in insulators [20] and conductors [21]. The former chapter covers dispersion, and reflection and refraction at interfaces, while the latter includes plasma oscillations, ultraviolet transparency and metallic reflection, waveguides, and resonant cavities.

This ends the discussion of macroscopic electromagnetism. Completeness would demand that I include topics such as spin waves and Walker modes in ferromagnets, light in anisotropic media, some magnetohydrodynamics, and nonlinear optics. But life is short, and the interested reader must seek these elsewhere.

Scattering of light [22] is a vast subject, and the selection of subtopics reflects my personal tastes. I do not discuss scattering of charged particles, collision radiation, virtual quanta, or the energy loss formula, as there are excellent treatments of these topics by other authors, and my own knowledge of them is limited.

The formalism of special relativity [23], its role in electromagnetism [24], and radiation from relativistic particles [25] bring up the rear, but they could in fact be covered any time after chapter 12. I have chosen not to present electromagnetism as an outgrowth of relativity as many other authors do, partly because that is ahistorical, but mainly because I have found that students find it hard to relate the formal relativistic underpinnings to actual electromagnetic phenomena, especially in matter, and the machinery of four-vectors and four-tensors seems unconnected to concepts such as the multipole expansion and mutual inductance. This machinery also requires more sophistication to master, so I have found it helpful to delay introducing it. Nevertheless, relativity is used to provide insight throughout the book, the first-order transformation of **E** and **B** fields is found early on, and much of the treatment of radiation and charged particle motion is fully relativistically correct.

Several appendixes provide mathematical reference material (Bessel and Airy functions, the Wiener-Khinchine theorem, etc.), which need not be covered explicitly. Appendix A on spherical harmonics is an exception, as these are used throughout the book, and an easy familiarity with them is a must. I have also included two appendixes on caustics (the rainbow and the teacup nephroid) and the motion of charged particles in the earth's magnetic field as specialized applications of matter in the main text.

Several colleagues have asked me if my book would include numerical methods. It does, but in a very limited way. My experience has been that good use of numerical methods requires so much attention to basic calculus and linear algebra that it reduces the time students can devote to physical concepts. Further, solving problems numerically requires much trial and error, making the exercise closer to experimental than theoretical physics, and thus ill suited to a traditional lecture-based course. Secondly, the peripheral aspects of computer use—the operating system, the precise computer language, the graphics package—are so varied and riddled with minutiae that they often end up dominating the learning of the numerical methods themselves. I do not advocate the use of canned black-box packages, as the student learns neither much physics nor much numerical analysis from them.

The book includes over three hundred exercises. These appear in each section rather than at the ends of chapters, because in my view the immediacy of the problems makes them more relevant. Some exercises are quite simple and designed to develop technical skills, while others may require significant extension of concepts in the text. The latter are often accompanied by short solutions or hints. I have tried to make sure that all exercises have a point and to explain that point, often by adding explicit commentary, and that none are just make-work for the sake of having a large number. A few exercises *are* purely mathematical or formal, but these are generally used to develop results used elsewhere in the book.

The book does not require in-depth understanding of quantum mechanics or thermodynamics, and though quantal and thermodynamic concepts are used or mentioned in a matter-of-fact way when needed, they are not critical to the central development, and an acquaintance with them at the undergraduate level will suffice.

I have benefited from encouragement and discussions with so many colleagues and students over the years that I am sure I cannot remember them all. I am indebted, among others, to Greg Anderson, Mike Bedzyk, Carol Braun, Paul Cadden-Zimansky, Pulak Dutta, Shyamsunder Erramilli, André de Gouvêa, Nick Giordano, C. Jayaprakash, John Ketterson, Mike Peskin, Mohit Randeria, Wayne Saslow, Surendra Singh, Mike Stone, and Tony Zee. I am materially indebted, in addition, to David Mermin and Saul Teukolsky for lectures at Cornell University, which I relied on in writing appendix A on spherical harmonics and chapters 23 and 24 on relativity. I am likewise indebted to Jens Koch for his invaluable help with some of the figures. I am particularly indebted to Onuttam Narayan and Paul Sievert for reading large sections of the text, and their valuable feedback and criticisms. None of these people are, of course, responsible for any errors or omissions in the book.

I also wish to thank Karen Fortgang and Ingrid Gnerlich at Princeton University Press for their gentle prodding and guidance through the production process.

I owe a different kind of debt to the authors of the texts from which I learned this subject. These include the delightful *Electricity and Magnetism* by Purcell; *The Feynman Lectures* (all three volumes) by Feynman, Leighton, and Sands; *Classical Electrodynamics*

by Jackson; Vols. 2 and 8 of the *Course of Theoretical Physics* by Landau and Lifshitz; and *Classical Electrodynamics* by Schwinger, DeRaad, Milton, and Tsai. I have consulted these books so often that I am sure I have internalized and come to regard as my own the ideas and the very language that they use, and I hope I have not failed to give them credit on any occasion. To all these authors, I am deeply grateful.

I cannot adequately acknowledge my wife, Neerja, for her unwavering encouragement and support and uncomplaining acceptance of the domestic burdens created by my absorption with this project.

Lastly, I thank you, the readers, in advance. Your comments, suggestions, and notifications of mistakes are welcome and will be gratefully received.

Anupam Garg
Wilmette, Illinois
December 2010