INTRODUCTION

The object of this book is to present the new quantum mechanics in a unified representation which, so far as it is possible and useful, is mathematically rigorous. This new quantum mechanics has in recent years achieved in its essential parts what is presumably a definitive form: the so-called "transformation theory." Therefore the principal emphasis shall be placed on the general and fundamental questions which have arisen in connection with this theory. In particular, the difficult problems of interpretation, many of which are even now not fully resolved, will be investigated in detail. In this connection, the relation of quantum mechanics to statistics and to the classical statistical mechanics is of special importance. However, we shall as a rule omit any discussion of the application of quantum mechanical methods to particular problems, as well as any discussion of special theories derived from the general theory—at least so far as this is possible without endangering the understanding of general relationships. This seems the more advisable since several excellent treatments of these problems are either in print or in process of publication.[1]

On the other hand, a presentation of the mathematical tools necessary for the purposes of this theory will be given, *i.e.*, a theory of Hilbert space and the so-called Hermitian operators. For this end, an accurate introduction to unbounded operators is also necessary, *i.e.*, an extension of the theory beyond its classical limits (developed by D. Hilbert and E. Hellinger, F. Riesz, E. Schmidt, O. Toeplitz). The following may be said regarding the method employed in this mode of treatment: as a rule, calculations should be performed with the operators themselves (which represent physical quantities) and not with the matrices, which, after the introduction of a (special and arbitrary) coordinate system in Hilbert space, result from them. This "coordinate free," *i.e.*, invariant, method, with its strongly geometrical language, possesses noticeable formal advantages.

[1] There are, among others, the following comprehensive treatments: Sommerfeld, Supplement to the 4th edition of *Atombau und Spectrallinien*, Braunschweig, 1928; Weyl, *The Theory of Groups and Quantum Mechanics* (translated by H. P. Robertson), London, 1931; Frenkel, *Wave Mechanics*, Oxford, 1932; Born and Jordan, *Elementare Quantenmechanik*, Berlin, 1930; Dirac, *The Principles of Quantum Mechanics*, 2nd edition, Oxford, 1936.

Dirac, in several papers, as well as in his recently published book,[2] has given a representation of quantum mechanics which is scarcely to be surpassed in brevity and elegance, and which is at the same time of invariant character. It is therefore perhaps fitting to advance a few arguments on behalf of our method, which deviates considerably from that of Dirac.

The method of Dirac, mentioned above (and this is overlooked today in a great part of the quantum mechanical literature, because of the clarity and elegance of the theory) in no way satisfies the requirements of mathematical rigor—not even if these are reduced in a natural and proper fashion to the extent common elsewhere in theoretical physics. For example, the method adheres to the fiction that every self-adjoint operator can be put in diagonal form. In the case of those operators for which this is not actually the case, this requires the introduction of "improper" functions with self-contradictory properties. The insertion of such a mathematical "fiction" is frequently necessary in Dirac's approach, even though the problem at hand is merely one of calculating numerically the result of a clearly defined experiment. There would be no objection here if these concepts, which cannot be incorporated into the present-day framework of analysis, were intrinsically necessary for the physical theory. Thus, as Newtonian mechanics first brought about the development of the infinitesimal calculus, which, in its original form, was undoubtedly not self-consistent, so quantum mechanics might suggest a new structure for our "analysis of infinitely many variables"—*i.e.*, the mathematical technique would have to be changed, and not the physical theory. But this is by no means the case. It should rather be pointed out that the quantum mechanical "Transformation theory" can be established in a manner which is just as clear and unified, but which is also without mathematical objections. It should be emphasized that the correct structure need not consist in a mathematical refinement and explanation of the Dirac method, but rather that it requires a procedure differing from the very beginning, namely, the reliance on the Hilbert theory of operators.

In the analysis of fundamental questions, it will be shown how the statistical formulas of quantum mechanics can be derived from a few qualitative, basic assumptions. Furthermore, there will be a detailed discussion of the problem as to whether it is possible to trace the statistical character of quantum mechanics to an ambiguity (*i.e.*, incompleteness) in our description of nature. Indeed, such an interpretation would be a natural concomitant of the general principle that every probability statement arises from the incompleteness of our knowledge. This explanation "by hidden parameters," as well as another, related to it, which ascribes the "hidden parameter" to the observer and not to the observed system, has been proposed more than once. However, it will appear that this can scarcely succeed in a satisfactory way, or more precisely, such an explanation

[2] See Proc. Roy. Soc. London, **109** (1925) and subsequent issues, especially **113** (1926). Independently of Dirac, P. Jordan, (Z. Physik **40** (1926)) and F. London (Z. Physik **40** (1926)) gave similar foundations for the theory.

is incompatible with certain qualitative fundamental postulates of quantum mechanics.[3]

The relation of quantum statistics to thermodynamics is also considered. A closer investigation shows that the well-known difficulties of classical mechanics, which are related to the "disorder" assumptions necessary for the foundations of thermodynamics, can be eliminated here.[4]

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[3] See **IV** and **VI.3**

[4] See **V**.