
CONTENTS

PREAMBLE	xiii
LINEAR SYSTEMS I — BASIC CONCEPTS	1
I SYSTEM REPRESENTATION	3
1 STATE-SPACE LINEAR SYSTEMS	5
1.1 State-Space Linear Systems	5
1.2 Block Diagrams	7
1.3 Exercises	11
2 LINEARIZATION	12
2.1 State-Space Nonlinear Systems	12
2.2 Local Linearization Around an Equilibrium Point	12
2.3 Local Linearization Around a Trajectory	15
2.4 Feedback Linearization	16
2.5 Practice Exercises	22
2.6 Exercises	27
3 CAUSALITY, TIME INVARIANCE, AND LINEARITY	31
3.1 Basic Properties of LTV/LTI Systems	31
3.2 Characterization of All Outputs to a Given Input	33
3.3 Impulse Response	34
3.4 Laplace and \mathcal{Z} Transforms (Review)	37
3.5 Transfer Function	38
3.6 Discrete-Time Case	39
3.7 Additional Notes	40
3.8 Exercises	42
4 IMPULSE RESPONSE AND TRANSFER FUNCTION OF STATE-SPACE SYSTEMS	43
4.1 Impulse Response and Transfer Function for LTI Systems	43
4.2 Discrete-Time Case	44
4.3 Elementary Realization Theory	45
4.4 Equivalent State-Space Systems	49
4.5 LTI Systems in MATLAB [®]	50
4.6 Practice Exercises	52
4.7 Exercises	53
5 SOLUTIONS TO LTV SYSTEMS	56
5.1 Solution to Homogeneous Linear Systems	56
5.2 Solution to Nonhomogeneous Linear Systems	58

viii CONTENTS

5.3	Discrete-Time Case	59
5.4	Practice Exercises	61
5.5	Exercises	62
6	SOLUTIONS TO LTI SYSTEMS	64
6.1	Matrix Exponential	64
6.2	Properties of the Matrix Exponential	65
6.3	Computation of Matrix Exponentials Using Laplace Transforms	67
6.4	The Importance of the Characteristic Polynomial	68
6.5	Discrete-Time Case	69
6.6	Symbolic Computations in MATLAB [®]	70
6.7	Practice Exercises	72
6.8	Exercises	74
7	SOLUTIONS TO LTI SYSTEMS: THE JORDAN NORMAL FORM	76
7.1	Jordan Normal Form	76
7.2	Computation of Matrix Powers using the Jordan Normal Form	78
7.3	Computation of Matrix Exponentials using the Jordan Normal Form	80
7.4	Eigenvalues with Multiplicity Larger than 1	81
7.5	Practice Exercise	82
7.6	Exercises	83
II	STABILITY	85
8	INTERNAL OR LYAPUNOV STABILITY	87
8.1	Lyapunov Stability	87
8.2	Vector and Matrix Norms (Review)	88
8.3	Eigenvalue Conditions for Lyapunov Stability	90
8.4	Positive-Definite Matrices (Review)	91
8.5	Lyapunov Stability Theorem	91
8.6	Discrete-Time Case	95
8.7	Stability of Locally Linearized Systems	98
8.8	Stability Tests with MATLAB [®]	103
8.9	Practice Exercises	103
8.10	Exercises	105
9	INPUT-OUTPUT STABILITY	108
9.1	Bounded-Input, Bounded-Output Stability	108
9.2	Time Domain Conditions for BIBO Stability	109
9.3	Frequency Domain Conditions for BIBO Stability	112
9.4	BIBO versus Lyapunov Stability	113
9.5	Discrete-Time Case	114
9.6	Practice Exercises	115
9.7	Exercises	118

1 0	PREVIEW OF OPTIMAL CONTROL	120
10.1	The Linear Quadratic Regulator Problem	120
10.2	Feedback Invariants	121
10.3	Feedback Invariants in Optimal Control	122
10.4	Optimal State Feedback	122
10.5	LQR with MATLAB [®]	124
10.6	Practice Exercise	124
10.7	Exercise	125
III	CONTROLLABILITY AND STATE FEEDBACK	127
1 1	CONTROLLABLE AND REACHABLE SUBSPACES	129
11.1	Controllable and Reachable Subspaces	129
11.2	Physical Examples and System Interconnections	130
11.3	Fundamental Theorem of Linear Equations (Review)	134
11.4	Reachability and Controllability Gramians	135
11.5	Open-Loop Minimum-Energy Control	137
11.6	Controllability Matrix (LTI)	138
11.7	Discrete-Time Case	141
11.8	MATLAB [®] Commands	145
11.9	Practice Exercise	146
11.10	Exercises	147
1 2	CONTROLLABLE SYSTEMS	148
12.1	Controllable Systems	148
12.2	Eigenvector Test for Controllability	150
12.3	Lyapunov Test for Controllability	152
12.4	Feedback Stabilization Based on the Lyapunov Test	155
12.5	Eigenvalue Assignment	156
12.6	Practice Exercises	157
12.7	Exercises	159
1 3	CONTROLLABLE DECOMPOSITIONS	162
13.1	Invariance with Respect to Similarity Transformations	162
13.2	Controllable Decomposition	163
13.3	Block Diagram Interpretation	165
13.4	Transfer Function	166
13.5	MATLAB [®] Commands	166
13.6	Exercise	167
1 4	STABILIZABILITY	168
14.1	Stabilizable System	168
14.2	Eigenvector Test for Stabilizability	169
14.3	Popov-Belevitch-Hautus (PBH) Test for Stabilizability	171
14.4	Lyapunov Test for Stabilizability	171
14.5	Feedback Stabilization Based on the Lyapunov Test	173
14.6	MATLAB [®] Commands	174
14.7	Exercises	174

x CONTENTS

IV	OBSERVABILITY AND OUTPUT FEEDBACK	177
15	OBSERVABILITY	179
15.1	Motivation: Output Feedback	179
15.2	Unobservable Subspace	180
15.3	Unconstructible Subspace	182
15.4	Physical Examples	182
15.5	Observability and Constructibility Gramians	184
15.6	Gramian-Based Reconstruction	185
15.7	Discrete-Time Case	187
15.8	Duality for LTI Systems	188
15.9	Observability Tests	190
15.10	MATLAB [®] Commands	193
15.11	Practice Exercises	193
15.12	Exercises	195
16	OUTPUT FEEDBACK	198
16.1	Observable Decomposition	198
16.2	Kalman Decomposition Theorem	200
16.3	Detectability	202
16.4	Detectability Tests	204
16.5	State Estimation	205
16.6	Eigenvalue Assignment by Output Injection	206
16.7	Stabilization through Output Feedback	207
16.8	MATLAB [®] Commands	208
16.9	Exercises	208
17	MINIMAL REALIZATIONS	210
17.1	Minimal Realizations	210
17.2	Markov Parameters	211
17.3	Similarity of Minimal Realizations	213
17.4	Order of a Minimal SISO Realization	215
17.5	MATLAB [®] Commands	217
17.6	Practice Exercises	217
17.7	Exercises	219
	LINEAR SYSTEMS II — ADVANCED MATERIAL	221
V	POLES AND ZEROS OF MIMO SYSTEMS	223
18	SMITH-MCMILLAN FORM	225
18.1	Informal Definition of Poles and Zeros	225
18.2	Polynomial Matrices: Smith Form	226
18.3	Rational Matrices: Smith-McMillan Form	229
18.4	McMillan Degree, Poles, and Zeros	230

18.5	Blocking Property of Transmission Zeros	232
18.6	MATLAB [®] Commands	233
18.7	Exercises	233
19	STATE-SPACE POLES, ZEROS, AND MINIMALITY	235
19.1	Poles of Transfer Functions versus Eigenvalues of State-Space Realizations	235
19.2	Transmission Zeros of Transfer Functions versus Invariant Zeros of State-Space Realizations	236
19.3	Order of Minimal Realizations	239
19.4	Practice Exercises	241
19.5	Exercise	242
20	SYSTEM INVERSES	244
20.1	System Inverse	244
20.2	Existence of an Inverse	245
20.3	Poles and Zeros of an Inverse	246
20.4	Feedback Control of Invertible Stable Systems with Stable Inverses	248
20.5	MATLAB [®] Commands	249
20.6	Exercises	250
VI	LQR/LQG OPTIMAL CONTROL	251
21	LINEAR QUADRATIC REGULATION (LQR)	253
21.1	Deterministic Linear Quadratic Regulation (LQR)	253
21.2	Optimal Regulation	254
21.3	Feedback Invariants	255
21.4	Feedback Invariants in Optimal Control	256
21.5	Optimal State Feedback	256
21.6	LQR in MATLAB [®]	258
21.7	Additional Notes	258
21.8	Exercises	259
22	THE ALGEBRAIC RICCATI EQUATION (ARE)	260
22.1	The Hamiltonian Matrix	260
22.2	Domain of the Riccati Operator	261
22.3	Stable Subspaces	262
22.4	Stable Subspace of the Hamiltonian Matrix	262
22.5	Exercises	266
23	FREQUENCY DOMAIN AND ASYMPTOTIC PROPERTIES OF LQR	268
23.1	Kalman's Equality	268
23.2	Frequency Domain Properties: Single-Input Case	270
23.3	Loop Shaping Using LQR: Single-Input Case	272
23.4	LQR Design Example	275

xii CONTENTS

23.5 Cheap Control Case	278
23.6 MATLAB [®] Commands	281
23.7 Additional Notes	282
23.8 The Loop-Shaping Design Method (Review)	283
23.9 Exercises	288
24 OUTPUT FEEDBACK	289
24.1 Certainty Equivalence	289
24.2 Deterministic Minimum-Energy Estimation (MEE)	290
24.3 Stochastic Linear Quadratic Gaussian (LQG) Estimation	295
24.4 LQR/LQG Output Feedback	295
24.5 Loop Transfer Recovery (LTR)	296
24.6 Optimal Set-Point Control	297
24.7 LQR/LQG with MATLAB [®]	302
24.8 LTR Design Example	303
24.9 Exercises	304
25 LQG/LQR AND THE Q PARAMETERIZATION	305
25.1 Q -Augmented LQG/LQR Controller	305
25.2 Properties	306
25.3 Q Parameterization	309
25.4 Exercise	309
26 Q DESIGN	310
26.1 Control Specifications for Q Design	310
26.2 The Q Design Feasibility Problem	313
26.3 Finite-Dimensional Optimization: Ritz Approximation	314
26.4 Q Design Using MATLAB [®] and CVX	316
26.5 Q Design Example	321
26.6 Exercise	323
BIBLIOGRAPHY	325
INDEX	327