3.9 Exercises

Exercise 3.1 (random financing). Consider the fixed-investment model of Section 3.2. We know that if $A \geq \bar{A}$, where

$$I - \bar{A} = p_H \left( R - \frac{B}{\Delta p} \right),$$

it is both optimal and feasible for the borrower to sign a contract in which the project is undertaken for certain. We also noted that for $A < \bar{A}$, the borrower cannot convince investors to undertake the project with probability 1. With $A > 0$, the entrepreneur benefits from shirking. The entrepreneur obtains private benefit $p > 0$ if she shirks. The entrepreneur obtains private benefit $R > 0$ if the entrepreneur works and $p_L = p_H - \Delta p$ ($\Delta p > 0$) if she shirks. The entrepreneur obtains private benefit $B$ if she shirks and 0 otherwise. Assume that $A \geq c_0$ to ensure that the entrepreneur is not in the $-\infty$ range in the absence of financing.

Compute the minimum equity level $\bar{A}$ for which the project is financed by risk-neutral investors when the market rate of interest is 0. Discuss the difference between $p_H = 1$ and $p_H < 1$.

(ii) Generalize the analysis to risk aversion. Let $u(x)$ denote the entrepreneur's utility from consumption with $u' > 0$, $u'' < 0$. Conduct the analysis assuming either limited liability or the absence of limited liability.

Exercise 3.2 (impact of entrepreneurial risk aversion). Consider the fixed-investment model developed in this chapter: an entrepreneur has cash amount $A$ and wants to invest $I > A$ into a project. The project yields $R > 0$ with probability $p$ and 0 with probability $1 - p$. The probability of success is $p_H$ if the entrepreneur works and $p_L = p_H - \Delta p$ ($\Delta p > 0$) if she shirks. The entrepreneur obtains private benefit $B$ if she shirks and 0 otherwise. Assume that

$$I > p_H \left( R - \frac{B}{\Delta p} \right).$$

(Suppose that $p_H R + B < I$; so the project is not financed if the entrepreneur shirks.)

(i) In contrast with the risk-neutrality assumption of this chapter, assume that the entrepreneur has utility for consumption $c$:

$$u(c) = \begin{cases} c & \text{if } c \geq c_0, \\ -\infty & \text{otherwise.} \end{cases}$$

Interpret this result.

Exercise 3.3 (random private benefits). Consider the variable-investment model: an entrepreneur initially has cash $A$. For investment $I$, the project yields $RI$ in the case of success and 0 in the case of failure. The probability of success is equal to $p_H \in (0, 1)$ if the entrepreneur works and $p_L = 0$ if the entrepreneur shirks. The entrepreneur obtains private benefit $BI$ when shirking and 0 when working. The per-unit private benefit $B$ is unknown to all ex ante and is drawn from (common knowledge) uniform distribution $F$:

$$\Pr(B < \hat{B}) = F(\hat{B}) = \hat{B}/R \text{ for } \hat{B} \leq R,$$

with density $f(\hat{B}) = 1/R$. The entrepreneur borrows $I - A$ and pays back $Rl = rI$ in the case of success. The timing is described in Figure 3.10.

(i) For a given contract $(I, r_l)$, what is the threshold $B^*$, i.e., the value of the private per-unit benefit above which the entrepreneur shirks?

(ii) For a given $B^*$ (or equivalently $r_l$, which determines $B^*$), what is the debt capacity? For which value of $B^*$ (or $r_l$) is this debt capacity highest?

(iii) Determine the entrepreneur's expected utility for a given $B^*$. Show that the contract that is optimal for the entrepreneur (subject to the investors breaking even) satisfies

$$\frac{1}{2}p_H R < B^* < p_H R.$$

Interpret this result.
(iv) Suppose now that the private benefit $B$ is observable and verifiable. Determine the optimal contract between the entrepreneur and the investors (note that the reimbursement can now be made contingent on the level of private benefits: $R_i = r_i(B)I$).

**Exercise 3.4 (product-market competition and financing).** Two firms, $i = 1, 2$, compete for a new market. To enter the market, a firm must develop a new technology. It must invest (a fixed amount) $I$.

Each firm is run by an entrepreneur. Entrepreneur $i$ has initial cash $A_i < I$. The entrepreneurs must borrow from investors at expected rate of interest 0. As in the single-firm model, an entrepreneur enjoys private benefit $B$ from shirking and 0 when working. The probability of success is $p_{hi}$ and $p_L = p_{hi} - \Delta p$ when working and shirking.

The return for a firm is

$$ R = \begin{cases} D & \text{if both firms succeed in developing the technology (which results in a duopoly),} \\ M & \text{if only this firm succeeds (and therefore enjoys a monopoly situation),} \\ 0 & \text{if the firm fails,} \end{cases} $$

where $M > D > 0$.

Assume that $p_{hi}(M - B/\Delta p) < I$. We look for a Nash equilibrium in contracts (when an entrepreneur negotiates with investors, both parties correctly anticipate whether the other entrepreneur obtains funding). In a first step, assume that the two firms’ projects or research technologies are independent, so that nothing is learned from the success or failure of the other firm concerning the behavior of the borrower.

(i) Show that there is a cutoff $A$ such that if $A_i < A$, entrepreneur $i$ obtains no funding.

(ii) Show that there is a cutoff $\bar{A}$ such that if $A_i > \bar{A}$ for $i = 1, 2$, both firms receive funding.

(iii) Show that if $A < A_i < \bar{A}$ for $i = 1, 2$, then there exist two (pure-strategy) equilibria.

(iv) The previous questions have shown that when investment projects are independent, product-market competition makes it *more difficult* for an entrepreneur to obtain financing. Let us now show that when projects are correlated, product-market competition may *facilitate financing* by allowing financiers to benchmark the entrepreneur’s performance on that of competing firms.

Let us change the entrepreneur’s preferences slightly:

$$ u(c) = \begin{cases} c & \text{if } c \geq c_0, \\ -\infty & \text{otherwise.} \end{cases} $$

That is, the entrepreneur is infinitely risk averse below $c_0$ (this assumption is stronger than needed, but it simplifies the computations).

Suppose, first, that only one firm can invest. Show that the necessary and sufficient condition for investment to take place is

$$ p_{hi}(M - B/\Delta p) - c_0 \geq I - A. $$

(v) Continuing on from question (iv), suppose now that there are two firms and that their technologies are *perfectly correlated* in that if both invest and both entrepreneurs work, then they both succeed or both fail. (For the technically oriented reader, there exists an underlying state variable $\omega$ distributed uniformly on $[0, 1]$ and common to both firms such that a firm always succeeds if $\omega < p_L$, always fails if $\omega > p_{hi}$, and succeeds if and only if the entrepreneur works when $p_L < \omega < p_{hi}$.)

Show that if

$$ p_{hi}D - c_0 \geq I - A, $$

then it is an equilibrium for both entrepreneurs to receive finance. Conclude that product-market competition may facilitate financing.

**Exercise 3.5 (continuous investment and decreasing returns to scale).** Consider the continuous-investment model, with one modification: investment $I$ yields return $R(I)$ in the case of success, and 0 in the case of failure, where $R' > 0$, $R'' < 0$, $R'(0) > 1/p_{hi}$, $R'(\infty) < 1/p_{hi}$. The rest of the model is unchanged. (The entrepreneur starts with cash $A$. The probability of success is $p_{hi}$ if the entrepreneur behaves and $p_L = p_{hi} - \Delta p$ if she misbehaves. The entrepreneur obtains private benefit $Bl$ if she misbehaves and 0 otherwise. Only the final outcome is observable.) Let $I^*$ denote the level of investment that maximizes total surplus: $p_{hi}R'(I^*) = 1$.

(i) How does investment $I(A)$ vary with assets?

(ii) How does the shadow value $\nu$ of assets (the derivative of the borrower's gross utility with respect to assets) vary with the level of assets?
Exercise 3.6 (renegotiation and debt forgiveness). When computing the multiplier $k$ (given by equation (3.12)), we have assumed that it is optimal to specify a stake for the borrower large enough that the incentive constraint $(IC_b)$ is satisfied. Because condition (3.8) implies that the project has negative NPV in the case of misbehavior, such a specification is clearly optimal when the contract cannot be renegotiated. The purpose of this exercise is to check in a rather mechanical way that the borrower cannot gain by offering a loan agreement in which $(IC_b)$ is not satisfied, and which is potentially renegotiated before the borrower chooses her effort. While there is a more direct way to prove this result, some insights are gleaned from this pedestrian approach. Indeed, the exercise provides conditions under which the lender is willing to forgive debt in order to boost incentives (the analysis will bear some resemblance to that of liquidity shocks in Chapter 5, except that the lender's concession takes the form of debt forgiven rather than cash infusion).

(i) Consider a loan agreement specifying investment $I$ and stake $R_b < BI/\Delta p$ for the borrower. Suppose that the loan agreement can be renegotiated after it is signed and the investment is sunk and before the borrower chooses her effort. Renegotiation takes place if and only if it is mutually advantageous. Show that the loan agreement is renegotiated if and only if

$$\Delta p_R I - \frac{p_H B I}{\Delta p} + p_L R_b \geq 0.$$ 

(ii) Interpret the previous condition. In particular, show that it can be obtained directly from the general theory. Hint: consider a fictitious, “fixed-investment” project with income $(\Delta p) R I$, investment 0, and cash on hand $p_I R_b$.

(iii) Assume for instance that the entrepreneur makes a take-it-or-leave-it offer in the renegotiation (that is, the entrepreneur has the bargaining power). Compute the borrowing capacity when $R_b < BI/\Delta p$ and the loan agreement is renegotiated.

(iv) Use a direct, rational expectations argument to point out in a different way that there is no loss of generality in assuming $R_b \geq BI/\Delta p$ (and therefore no renegotiation).

Exercise 3.7 (strategic leverage). (i) A borrower has assets $A$ and must find financing for an investment $I(\tau) > A$. As usual, the project yields $R$ (success) or 0 (failure). The borrower is protected by limited liability. The probability of success is $p_H + \tau$ or $p_L + \tau$, depending on whether the borrower works or shirks, with $\Delta p = p_H - p_L > 0$. There is no private benefit when working and private benefit $B$ when shirking. The financial market is competitive and the expected rate of return demanded by investors is equal to 0. It is never optimal to give incentives to shirk.

The investment cost $I$ is an increasing and convex function of $\tau$ (it will be further assumed that $p_H R > I(0)$, that in the relevant range $p_H + \tau < 1$, and that $I'(0)$ is “small enough” so as to guarantee an interior solution). Let $\tau^*$, $A^*$, and $\tau^{**}$ be defined by

$$I'(\tau^*) = R,$$

$$[p_H + \tau^*] [R - \frac{B}{\Delta p}] = I(\tau^*) - A^*,$$

$$I'(\tau^{**}) = R - \frac{B}{\Delta p}.$$ 

Can the borrower raise funds? If so, what is the equilibrium level $\tau$ of “quality of investment”?

(ii) Suppose now that there are two firms (that is, two borrowers) competing on this product market. If only firm $i$ succeeds in its project, its income is (as in question (i)), equal to $R$ (and firm $j$'s income is 0). If the two firms succeed (both get hold of “the technology”), they compete à la Bertrand in the product market and get 0 each. For simplicity, assume that the lenders observe only whether the borrower's income is $R$ or 0, rather than whether the borrower has succeeded in developing the technology (showoffs: you can discuss what would happen if the lenders observed “success/failure”).

So, if $q_i \equiv p_i + \tau_i$ denotes the probability that firm $i$ develops the technology (with $p_i = p_H$ or $p_L$), the probability that firm $i$ makes $R$ is $q_i (1 - q_j)$. (This assumes implicitly that projects are independent.)

Consider the following timing. (1) Each borrower simultaneously and secretly arranges financing (if feasible). A borrower's leverage (or quality of investment) is not observed by the other borrower. (2) Bor-
rowers choose whether to work or shirk. (3) Projects succeed or fail.

• Let \( \hat{\tau} \) be defined by
  \[
  I'(\hat{\tau}) = [1 - (p_H + \hat{\tau})]R.
  \]
  Interpret \( \hat{\tau} \).

• Suppose that the two borrowers have the same initial net worth \( \hat{A} \). Find the lower bound \( \hat{A} \) on \( \hat{A} \) such that \((\hat{\tau}, \hat{\tau})\) is the (symmetric) Nash outcome.

• Derive a sufficient condition on \( \hat{A} \) under which it is an equilibrium for a single firm to raise funds.

(iii) Consider the set up of question (ii), except that borrower 1 moves first and publicly chooses \( \tau_1 \). Borrower 2 may then try to raise funds (one will assume either that \( \tau_2 \) is secret or that borrower 1 is rewarded on the basis of her success/failure performance; this is in order to avoid strategic choices by borrower 2 that would try to induce borrower 1 to shirk). Suppose that each has net worth \( \hat{A} \) given by
  \[
  \hat{q} \left( (1 - \hat{q})R - \frac{B}{\Delta p} \right) = I(\hat{q} - p_H) - \hat{A},
  \]
  where \( \hat{q} \) satisfies
  \[
  I'(\hat{q} - p_H) = (1 - \hat{q})R - \frac{B}{\Delta p}.
  \]
  • Interpret \( \hat{q} \).
  • Show that it is optimal for borrower 1 to choose \( \tau_1 > \hat{q} - p_H \).

Exercise 3.8 (equity multiplier and active monitoring). (i) Derive the equity multiplier in the variable-investment model. (Reminder: the investment \( I \in [0, \infty) \) yields income \( RI \) in the case of success and 0 in the case of failure. The borrower’s private benefit from misbehaving is equal to \( RI \). Misbehaving reduces the probability of success from \( p_H \) to \( p_L = p_H - \Delta p \). The borrower has cash \( \Delta p \) and is protected by limited liability. Assume that \( \rho_1 = p_H R > 1 \), \( \rho_0 = p_H (R - B/\Delta p) < 1 \) and \( 1 > p_L R + B \). The investors’ rate of time preference is equal to 0.) Show that the equity multiplier is equal to \( 1/(1 - \rho_0) \).

(ii) Derive the equity multiplier with active monitoring: the entrepreneur can hire a monitor, who, at private cost \( cl \), reduces the entrepreneur’s private benefit from shirking from \( RI \) to \( b(c)I \), where \( b(0) = B, b' < 0 \). The monitor must be given incentives to monitor (denote by \( R_m \) his income in the case of success). The monitor wants to break even, taking into account his private monitoring cost (so, there is “no shortage of monitoring capital”).

• Suppose that the entrepreneur wants to induce level of monitoring \( c \). Write the two incentive constraints to be satisfied by \( R_m \) and \( R_b \) (where \( R_b \) is the borrower’s reward in the case of success).

• What is the equity multiplier?
• Show that the entrepreneur chooses \( c \) so as to maximize
  \[
  \max_c \left\{ \frac{\rho_1 - 1 - c}{1 - \rho_0 + (p_H/\Delta p)B(c) + c - B} \right\}.
  \]

Exercise 3.9 (concave private benefit). Consider the variable-investment model with a concave private benefit. The entrepreneur obtains \( B(I) \) when shirking and 0 when behaving, where \( B(0) = 0 \), \( B' > 0 \), \( B'' < 0 \) (and \( B'(0) \) large, \( \lim_{I \to \infty} B'(I) = B \), where \( p_H (R - B/\Delta p) < 1 \)).

(i) Compute the borrowing capacity.

(ii) How does the shadow price \( v \) of the entrepreneur’s cash on hand vary with \( A \)?

Exercise 3.10 (congruence, pledgeable income, and power of incentive scheme). The credit rationing model developed in this chapter assumes that the entrepreneur’s and investors’ interests are a priori dissonant, and that incentives must be aligned by giving the entrepreneur enough of a stake in the case of success.

Suppose that the entrepreneur and the investors have indeed dissonant preferences with probability \( x \), but have naturally aligned interests with probability \( 1 - x \). Which prevails is unknown to both sides at the financing stage and is discovered (only) by the entrepreneur just before the moral-hazard stage.

More precisely, consider the fixed-investment model of Section 3.2. The investors’ outlay is \( I - A \) and they demand an expected rate of return equal to 0. The entrepreneur is risk neutral and protected by limited liability. With probability \( x \), interests are dissonant: the entrepreneur obtains private benefit \( B \) by misbehaving (the probability of success is \( p_L \)) and 0 by behaving (probability of success \( p_H \)). With probability \( 1 - x \), interests are aligned: the entrepreneur takes her private benefit \( B \) coincides with choosing probability of success \( p_H \).
(i) Consider a “simple incentive scheme” in which
the entrepreneur receives $R_b$ in the case of success
and 0 in the case of failure. $R_b$ thus measures the
“power of the incentive scheme.”

Show that it may be optimal to choose a low-
powered incentive scheme if preferences are rather
congruent ($x$ low) and that the incentive scheme is
necessarily high-powered if preferences are rather
dissonant ($x$ high).

(ii) Show that one cannot improve on simple incen-
tive schemes by presenting the entrepreneur with a
menu of two options (two outcome-contingent incen-
tive schemes) from which she will choose once
she learns whether preferences are congruent or
dissonant.

**Exercise 3.11 (retained-earnings benefit).** An entre-
preneur has at date 1 a project of fixed size with
characteristics $\{I^1, R^1, p^1_1, p^1_2, B^1\}$ (see Section 3.2).
This entrepreneur will at date 2 have a different fixed
size project with characteristics $\{I^2, R^2, p^2_1, p^2_2, B^2\}$, which
will then require new financing. So, we are con-
sidering only a short-term loan for the first project.
Retained earnings from the first project can, how-
ever, be used to defray part of the investment cost
of the second project. Assume that all the charac-
teristics of the second project are known at date 1
except $B^2$, which is distributed on $[B^2, \bar{B}^2]$ accord-
ing to the cumulative distribution $F(B^2)$. Assume for
simplicity that $\bar{B}^2 > \Delta p^2 (p^2_1 R^2 - F(B^2))/p^2_1$. The charac-
teristics of the second project become common
knowledge at the beginning of date 2.

(i) Compute the shadow value of retained earn-
ings. (Hint: what is the entrepreneur’s gross utility
in period 2?)

(ii) Show that it is possible that the first project is
funded even though it would not be funded if the
second project did not exist and even though the
entrepreneur cannot pledge at date 1 income result-
ing from the second project.

**Exercise 3.12 (investor risk aversion and risk pre-
mia).** One of the key developments in the theory of
market finance has been to find methods to price
claims held by investors. Market finance emphasizes
state-contingent pricing, the fact that 1 unit of in-
come does not have a uniform value across states
of nature. This book assumes that investors are risk
neutral, and so it does not matter how the pledge-
able income is spread across states of nature. This
assumption is made only for the sake of computa-
tional simplicity, and can easily be relaxed.

Consider a two-date model of market finance with
a representative consumer/investor. This consumer
has utility of consumption $u(c_0)$ at date 0, the date
at which he lends to the firm, and utility of consump-
tion $u(c(\omega))$ at date 1, date at which he receives
the return from investment. There is macroeconomic
uncertainty in that the representative consumer’s
date-1 consumption depends on the state of nature
$\omega$. The state of nature describes both what happens
in this particular firm and in the rest of the economy
(even though aggregate consumption is independent
of the outcome in this particular firm to the extent
that the firm is atomistic, which we will assume).

Suppose that the entrepreneur works. Let $S$ de-
note the event “the project succeeds” and $F$ the event
“the project fails.” Let

$$q_S = \mathbb{E} \left[ \frac{u'(c(\omega))}{u'(c_0)} \right] \quad \omega \in S$$

and

$$q_F = \mathbb{E} \left[ \frac{u'(c(\omega))}{u'(c_0)} \right] \quad \omega \in F.$$

The firm’s activity is said to covary positively with
the economy (be “procyclical”) if $q_S < q_F$, and nega-
tively (be “countercyclical”) if $q_F < q_S$.

Suppose that

$$p_0 q_S + (1 - p_0) q_F = 1.$$

(i) Interpret this assumption.

(ii) In the fixed-investment model of Section 3.2
(and still assuming that the entrepreneur is risk neu-
tral), derive the necessary and sufficient condition
for the project to receive financing.

(iii) What is the optimal contract between the in-
vestors and the entrepreneur? Does it involve max-
imum punishment ($R_b = 0$) in the case of failure?

How would your answer change if the entrepreneur
were risk averse? (For simplicity, assume that her
only claim is in the firm. She does not hold any of
the market portfolio.)

**Exercise 3.13 (lender market power).** (i) Fixed in-
vestment. An entrepreneur has cash amount $A$ and
wants to invest $I > A$ into a (fixed-size) project. The
project yields \( R > 0 \) with probability \( p \) and 0 with probability \( 1 - p \). The probability of success is \( p_H \) if the entrepreneur works and \( p_L = p_H - \Delta p \) (\( \Delta p > 0 \)) if she shirks. The entrepreneur obtains private benefit \( B \) if she shirks and 0 otherwise. The borrower is protected by limited liability and everyone is risk neutral. The project is worthwhile only if the entrepreneur behaves.

There is a single lender. This lender has access to funds that command an expected rate of return equal to 0 (so the lender would content himself with a 0 rate of return, but he will use his market power to obtain a superior rate of return). Assume

\[
V = p_H R - I > 0
\]

and let \( \bar{A} \) and \( \hat{A} \) be defined by

\[
p_H \left[ R - \frac{B}{\Delta p} \right] = I - \bar{A}
\]

and

\[
p_H \frac{B}{\Delta p} - \hat{A} = 0.
\]

Assume that \( \bar{A} > 0 \) and that the lender makes a take-it-or-leave-it offer to the borrower (i.e., the lender chooses \( R_0 \), the borrower’s compensation in the case of success).

- What contract is optimal for the lender?
- Is the financing decision affected by lender market power (i.e., compared with the case of competitive lenders solved in Section 3.2)?
- Draw the borrower’s net utility (i.e., net of \( A \)) as a function of \( A \) and note that it is nonmonotonic (distinguish four regions: \( (-\infty, \bar{A}), (\bar{A}, \hat{A}), (\hat{A}, I), (I, \infty) \)). Explain.

(ii) Variable investment. Answer the first two bullets in question (i) (lender’s optimal contract and impact of lender market power on the investment decision) in the variable-investment version. In particular, show that lender market power reduces the scale of investment. (Reminder: \( I \) is chosen in \( [0, \infty) \).

The project yields \( RI \) if successful and 0 if it fails. Shirking, which reduces the probability of success from \( p_H \) to \( p_L \), yields private benefit \( BI \). Assume that \( p_H R > 1 > p_L (R - B/\Delta p) \). Hint: show that the two constraints in the lender’s program are binding.)

**Exercise 3.14 (liquidation incentives).** This exercise extends the fixed-investment model of Section 3.2 by adding a signal on the profitability of the project that (a) accrues after effort has been chosen, and (b) is privately observed. (The following model is used as a building block in a broader context by Dessi (2005).)

An entrepreneur has cash \( A \) and wants to invest \( I > A \) into a project. The project yields \( R \) (success) or 0 (failure) at the end. An intermediate signal reveals the probability \( y \) that the project will succeed, with \( y = \hat{y} \) or \( y = \gamma_R \). The probability, \( p \), that \( y = \hat{y} \) depends on the entrepreneur’s effort. If the entrepreneur behaves, then \( p = p_H \) and the entrepreneur receives no private benefit. If the entrepreneur misbehaves, then \( p = p_L \) and the entrepreneur receives private benefit \( B \). Investors and entrepreneur are risk neutral and the latter is protected by limited liability. The competitive rate of return is equal to 0.

Introduce further an option to liquidate after the signal is realized but before the final profit accrues. Liquidation yields \( L \), and \( L \) is entirely pledgeable to investors.

One will assume that

\[
\gamma_R > L > \gamma R,
\]

so that it is efficient to liquidate if and only if the signal is bad; and that

\[
p_H \gamma R + (1 - p_H) L > I
\]

(which will imply that the NPV is positive).

Figure 3.11 summarizes the timing.

(i) Suppose first that \( y \) is verifiable. Argue that the entrepreneur should be rewarded solely as a function of the realization of \( y \). What is the pledgeable income? Show that the project is financed if and only if \( A \geq \bar{A} \), where

\[
p_H \left( \gamma R - \frac{B}{\Delta p} \right) + (1 - p_H) L = I - \bar{A}.
\]

(ii) Suppose now that \( y \) is observed only by the entrepreneur. This implies that the entrepreneur must be induced to tell the truth about \( y \). Without loss of generality, consider an incentive scheme in which the entrepreneur receives \( R_0 \) in the case she announces \( y = \hat{y} \) (and therefore the project continues) and the final profit is \( R, L_0 \) if she announces \( y = \gamma \) (and therefore the project is liquidated), and 0 otherwise.
Show that the project is funded if and only if
\[ A \geq X + Y \left( \frac{B}{(\Delta P)(\Delta \gamma)} \right). \]

**Exercise 3.15 (project riskiness and credit rationing).** Consider the basic, fixed-investment model (the investment is \( I \), the entrepreneur borrows \( I - A \); the probability of success is \( p_H \) (no private benefit) or \( p_L = p_H - \Delta p \) (private benefit \( B \)); success (failure) yields verifiable profit \( R \) (respectively 0)). There are two variants, “A” and “B,” of the projects, which differ only with respect to “riskiness”:

\[ p_A^H R_A^H = p_B^H R_B^H, \quad \text{but} \quad p_A^H > p_B^H, \]

so project B is “riskier.” The investment cost is the same for both variants and, furthermore,

\[ p_A^H - p_A^L = p_B^H - p_B^L. \]

Which variant is less prone to credit rationing?

**Exercise 3.16 (scale versus riskiness tradeoff).** Consider an entrepreneur with a project of variable investment \( I \). The entrepreneur has initial wealth \( A \), is risk neutral, and is protected by limited liability. Investors are risk neutral and demand a rate of return equal to 0.

The project comes in two versions:

**Risky.** The project costs \( I \) and ends up (potentially) productive only with probability \( x < 1 \). The timing goes as follows. (a) The scale of investment \( I \) is selected. (b) After the investment has been sunk, news accrues as to the profitability of the project. With probability \( 1 - x \), the project stops and yields 0. With probability \( x \), the project continues (without any need for reinvestment). In the latter case, (c) the entrepreneur chooses an effort; good behavior confers no private benefit on the entrepreneur and yields subsequent probability of success \( p_H \); misbehavior confers private benefit \( B \) and yields probability of success \( p_L \). Finally, (d) the outcome accrues: success yields \( RI \) and failure 0.

**Safe.** The investment cost, \( XI \) with \( X > 1 \), is higher for a given size \( I \). But the project is always productive (“\( x = 1 \”). The moral hazard and outcome stages are as in the case of a risky choice.

We will assume that the contract aims at inducing good behavior. Letting

\[ \rho_1 \equiv p_H R \quad \text{and} \quad \rho_0 \equiv p_H \left( R - \frac{B}{\Delta p} \right), \]

one will further assume that \( x > 1/\rho_1 \) and \( X < \rho_1 \).

Assume that entrepreneur and investors contract on which version will be selected.

(i) Show that the risky version is chosen if and only if

\[ xX \geq 1. \]

(ii) Interpret this condition in terms of a “cost of bringing 1 unit of investment to completion.”

**Exercise 3.17 (competitive product market interactions).** There is a mass of identical entrepreneurs with the variable-investment technology described in Section 3.4. The representative entrepreneur has wealth \( A \), is risk neutral, and is protected by limited liability.

Denote the average investment by \( I \) and the individual investment \( i \) (in equilibrium \( i = I \) by symmetry but we need to distinguish the two in a first step in order to compute the competitive equilibrium). A project produces \( Ri \) units of goods when successful and 0 when it fails. The probability of success is \( p_H \) in
the case of good behavior (the entrepreneur receives no private benefit) and \( p_1 = p_{HI} - \Delta p \) in the case of misbehavior (the entrepreneur then receives private benefit \( BI \)). Assume that it is optimal to induce the entrepreneur to behave.

The market price of output is \( P = P(Q) \), with \( P' < 0 \), where \( Q \) is aggregate production (with \( P(Q) \) tending to 0 as \( Q \) goes to infinity, to ensure that aggregate investment is finite). Finally, the shocks faced by the firms are independent (there is no industry-wide uncertainty) and the risk-neutral investors demand a rate of return equal to 0.

Show that the equilibrium is unique. Compute the equilibrium level of investment. (Hint: distinguish two cases, depending on whether \( A \) is large or small.)

**Exercise 3.18 (maximal incentives principle in the fixed-investment model).** Pursue the analysis of Section 3.4.3, but for the fixed-investment model of Section 3.2: the investment cost \( I \) is given and the income is either \( R^S \) or \( R^F \) (instead of \( R \) or 0), where \( R^S > R^F > 0 \). We assume that

\[
R^F < I - A,
\]

so the project cannot be straightforwardly financed by bringing in net worth \( A \) and pledging the lower income \( R^F \) to lenders. Let

\[
R = R^S - R^F
\]

denote the increase in income from the low to the high level. Show that the debt contract is optimal, but unlike in the variable-investment case it may not be uniquely optimal.

**Exercise 3.19 (balanced-budget investment subsidy and profit tax).** This exercise shows that a balanced-budget public policy that is not based on information that is superior to investors’ does not boost pledgeable income and therefore outside financing capacity (unless there are externalities among firms: see Exercise 3.17). This general point is illustrated in the context of the variable-investment model: an entrepreneur has cash amount \( A \) and wants to invest \( I > A \) into a (variable size) project. The project yields \( RI > 0 \) with probability \( p \) and 0 with probability \( 1 - p \). Reaching a probability of success \( p \) requires the entrepreneur to sink (unobservable) effort cost \( \frac{1}{2} p^2 \) (there is no private benefit in this version). The borrower is risk neutral and is protected by limited liability. Investors are risk neutral and the market rate of interest is 0. Assume that \( \sqrt{2T} < R < 1 \).

(i) Note that, had the borrower no need to borrow \( (A \geq I) \), the borrower’s net utility would be

\[
U_b = V^* = \frac{1}{2} R^2 - I,
\]

independently of \( A \).

(ii) Find the threshold \( \overline{A} \) under which the project is not funded. (Hint: write the pledgeable income as a
function of the entrepreneur’s reward $R_b$ in the case of success. Argue that one can focus attention on the values of $R_b$ that exceed $\frac{1}{2} R$. Do not forget that the NPV must be nonnegative.)

Letting $V(A)$ denote the NPV in the region in which the entrepreneur’s project is financed. Show that the shadow price of net worth, $V''(A)$, satisfies

$$V'(A) > 0,$$

$$V'(I) = 0,$$

$$V''(A) < 0.$$

(iii) Following Cestone and Fumagalli (2005), consider two entrepreneurs, each with net worth $A$. They will each have a project described as above, but with random investment cost. For simplicity, one of them will face investment cost $I_H$ and the other $I_L$, where

$$I_L - A < \frac{3}{8} R^2 < I_H - A,$$

but it is not known in advance who will face which investment cost (each is equally likely to be the lucky entrepreneur). Investment costs, however, will become publicly known before the investments are sunk. Assume that

$$\frac{3}{8} R^2 > I_H,$$

so that the only binding constraint for financing in question (ii) is the investors’ breakeven constraint; and that

$$\frac{1}{2} R^2 > (I_L + I_H) - 2A,$$

and so both projects can be financed by pooling resources. Do the entrepreneurs, behind the veil of ignorance, want to pool their resources and commit to force the lucky firm to cross-subsidize the unlucky one? (Hint: show that under pooling, and if both invest, the net worth is split so that both entrepreneurs have the same stake in success.)

Exercise 3.21 (hedging or gambling on net worth?).

Froot et al. (1993) analyze an entrepreneur’s risk preferences with respect to net worth. In the notation of this book, the situation they consider is summarized in Figure 3.12.

The entrepreneur is risk neutral and protected by limited liability. The investors are risk neutral and demand a rate of return equal to 0.

At date 0, the entrepreneur decides whether to insure against a date-1 income risk

$$r = A_0 + \varepsilon,$$

where $\varepsilon \in [\xi, \xi]$, $E[\varepsilon] = 0$, and $A_0 + \xi \geq 0$.

For simplicity, we allow only a choice between full hedging and no hedging (the theory extends straightforwardly to arbitrary degrees of hedging). Hedging (which wipes out the noise and thereby guarantees that the entrepreneur has cash on hand $A_0$ at date 1) is costless.

After receiving income, the entrepreneur uses her cash to finance investment $I$ and must borrow $I - A$ from investors, with $A = A_0$ in the case of hedging and $A = A_0 + \varepsilon$ in the absence of hedging (provided that $A \leq I$; otherwise there is no need to borrow).

Note that there is no overall liquidity management as there is no contract at date 0 with the financiers as to the future investment.

This exercise investigates a variety of situations under which the entrepreneur may prefer either hedging or “gambling” (here defined as “no hedging”).

(i) Fixed investment, binary effort. Suppose that the investment size is fixed (as in Section 3.2), and
Suppose that the investment size is variable and that the income from investment $R(I)$ is unobservable by investors (fully appropriated by the entrepreneur) and is concave. Suppose that it is always optimal for the entrepreneur to invest her cash on hand.

Show that the entrepreneur hedges.

(v) Liquidity and risk management. Suppose, in contrast with Froot et al.’s analysis, that the entrepreneur can sign a contract with investors at date 0. Show that the entrepreneur’s utility can be maximized by insulating the date-1 volume of investment from the realization of $\epsilon$, i.e., with full hedging, even in situations where gambling was optimal when funding was secured only at date 1.

4.8 Exercises

Exercise 4.1 (maintenance of collateral and asset depletion just before distress). This exercise analyzes the impact of the existence of a privately received signal about distress on credit rationing. Consider the model of Section 4.3.4 with $A' = A$ (so the asset has the same value for the borrower and the lender). The new feature is that the resale value of the asset is $A$ only if the borrower invests in maintenance; otherwise the final value of the asset is 0, regardless of the state of nature. The loan agreement cannot monitor the borrower’s maintenance decision (but the resale value is verifiable). So, there are two dimensions of moral hazard for the borrower. The borrower incurs private disutility $c < A$ from maintaining the asset, and 0 from not maintaining it. Assume that $p_1 B / (\Delta p) \geq c$, and that the entrepreneur is protected by limited liability.

(i) Suppose that the borrower receives no signal about the likelihood of distress (that is, the maintenance decision can be thought of as being simultaneous with that of choosing between probabilities $p_1$ and $p_2$ of success). Show that the analysis of this chapter is unaltered except that the borrower’s utility $U_0$ is reduced by $c$.

(ii) Suppose now that with probability $\xi$ in the case of failure the borrower privately learns that failure will occur with certainty. With probability $(1 - \xi)$ in the case of failure and with probability 1 in the case
of success, no signal accrues. ($\xi = 0$ corresponds to question (i).) The signal, if any, is received after the choice between $p_H$ and $p_L$ but before the maintenance decision. Suppose further that the asset is pledged to the lenders only in the case of failure. Show that, if the entrepreneur is poor and $c$ is “not too large,” constraint (ICb) must now be written

$$\Delta p (R_b + A) \geq B + (\Delta p) \xi c.$$  

Interpret this inequality. Find a necessary and sufficient condition for the project to be funded.

(iii) Keeping the framework of question (ii), when is it better not to pledge the asset at all than to pledge it in the case of failure?

**Exercise 4.2 (diversification across heterogeneous activities).** Consider two variable-investment activities, $\alpha$ and $\beta$, as described in Section 3.4. The probabilities of success $p_H$ (when working) and $p_L$ (when shirking) are the same in both activities. The two activities are independent (as in Section 4.2). The two activities differ in their per-unit returns ($R^\alpha$ and $R^\beta$) and private benefits ($B^\alpha$ and $B^\beta$). Let, for $i \in \{\alpha, \beta\}$,

$$\rho_i^\alpha \equiv p_H R^\alpha > 1 \quad \text{and} \quad \rho_i^\beta = p_H \left( R^\beta - \frac{B^i}{\Delta p} \right) < 1.$$  

For example, $\rho_1^\alpha < \rho_1^\beta$ but $\rho_0^\alpha > \rho_0^\beta$.

(i) Suppose that the entrepreneur agrees with the investors to focus on a single activity. Which activity will they choose?

(ii) Assume now that the firm invests $I^\alpha$ in activity $\alpha$ and $I^\beta$ in activity $\beta$ and that this allocation can be contracted upon with the investors. Write the incentive constraints and breakeven constraint.

Show that it may be that the optimum is to invest more in activity $\beta$ ($I^\beta > I^\alpha$) even though the entrepreneur would focus on activity $\alpha$ if she were forced to focus.

**Exercise 4.3 (full pledging).** In Section 4.3.1, we claimed that it is optimal to pledge the full value of the resale in the case of distress before committing any of the income $R$ obtained in the absence of distress. Prove this formally.

**Exercise 4.4 (“value at risk” and benefits from diversification).** This exercise looks at the impact of portfolio correlation on capital requirements. An entrepreneur has two identical fixed-investment projects. Each involves investment cost $I$. A project is successful (yields $R$) with probability $p$ and fails (yields 0) with probability $1 - p$. The probability of success is endogenous. If the entrepreneur works, the probability of success is $p_H = \frac{1}{2}$ and the entrepreneur receives no private benefit. If the entrepreneur shirks, the probability of success is $p_L = 0$ and the entrepreneur obtains private benefit $B$. The entrepreneur starts with cash $2A$, that is, $A$ per project.

We assume that the probability that one project succeeds conditional on the other project succeeding (and the entrepreneur behaving) is

$$\frac{1}{2} \left( 1 + \alpha \right)$$  

(it is, of course, 0 if the entrepreneur misbehaves on this project). $\alpha \in [-1, 1]$ is an index of correlation between the two projects.

The entrepreneur (who is protected by limited liability) has the following preferences:

$$u(R_b) = \begin{cases} R_b & \text{for } R_b \in [0, \bar{R}], \\ \bar{R} & \text{for } R_b > \bar{R}. \end{cases}$$

(i) Write the two incentive constraints that will guarantee that the entrepreneur works on both projects.

(ii) How is the entrepreneur optimally rewarded for $\bar{R}$ large?

(iii) Find the optimal compensation scheme in the general case. Distinguish between the cases of positive and negative correlation. How is the ability to receive outside funding affected by the coefficient of correlation?

**Exercise 4.5 (liquidity of entrepreneur’s claim).**

(i) Consider the framework of Section 4.4 (without speculative monitoring). In Section 4.4, we assumed that none of the value $\mu r_b$ (with $\mu > 1$) obtained by reinvesting $r_b$ was appropriated by the entrepreneur. Assume instead that $\mu_0 r_b$ is returned to investors, where $\mu_0 < 1$. For consistency, assume that investors observe whether the entrepreneur faces a liquidity shock (this corresponds to case (a) in Section 4.4). And, to avoid having to consider the correlation of activities and the question of diversification (see Section 4.2), assume that $(\mu - \mu_0) r_b$ is a private benefit that automatically accrues to
the entrepreneur and therefore cannot be “cross-pledged.”

There is an equivalence between rewarding success with payment $R_b$ when there was no interim investment opportunity and rewarding success with $(1 - \lambda)R_b$ independently of interim investment opportunity. As in Section 4.4 we assume that the entrepreneur is rewarded with $R_b$ only when there was no interim investment opportunity.

How is the liquidity of the entrepreneur’s claim affected by $\mu_0 > 0$?

(ii) Suppose now that the probability of a “liquidity shock,” i.e., a new investment opportunity, is endogenous. If the entrepreneur does not search, then $\lambda = 0$; if she searches, which involves private cost $\lambda c$ for the entrepreneur, then $\lambda = \tilde{\lambda}$. Rewrite the financing constraint.

Exercise 4.6 (project size increase at an intermediate date). An entrepreneur has initial net worth $A$ and starts at date 0 with a fixed-investment project costing $I$. The project succeeds (yields $R$) or fails (yields 0) with probability $p \in \{p_l, p_H\}$. The entrepreneur obtains private benefit $B$ at date 0 when misbehaving (choosing $p = p_l$) and 0 otherwise. Everyone is risk neutral, investors demand a 0 rate of return, and the entrepreneur is protected by limited liability.

The twist relative to this standard fixed-investment model is that, with probability $\lambda$, the size may be doubled at no additional cost to the investors (i.e., the project duplicated) at date 1. The new investment is identical with the initial one (same date-2 stochastic revenue; same description of moral hazard, except that it takes place at date 1) and is perfectly correlated with it. That is, there are three states of nature: either both projects succeed independently of the entrepreneur’s effort, or both fail independently of effort, or a project for which the entrepreneur behaved succeeds and the other for which she misbehaved fails.

Denote by $R_b$ the entrepreneur’s compensation in the case of success when the reinvestment opportunity does not occur, and by $R_b'$ that when both the initial and the new projects are successful. (The entrepreneur optimally receives 0 if any activity fails.)

Show that the project and its (contingent) duplication receive funding if and only if

$$(1 + \lambda) \left[ p_H (R - \frac{B}{\Delta p}) \right] \geq I - A.$$ 

Exercise 4.7 (group lending and reputational capital). Consider two economic agents, each endowed with a fixed-investment project, as described, say, in Section 3.2. The two projects are independent.

Agent $i$’s utility is

$$R_b^i + a R_b^j,$$

where $R_b^i$ is her income at the end of the period, $R_b^j$ is the other agent’s income, and $0 < a < 1$ is the parameter of altruism. Assume that

$$p_H (R - \frac{B}{(1 + a)\Delta p}) < I - A < p_H R.$$

(i) Can the agents secure financing through individual borrowing? Through group lending?

(ii) Now add a later or “stage-2” game, which will be played after the outcomes of the two projects are realized. This game will be played by the two agents and will not be observed by the “stage-1” lenders. In this social game, which is unrelated to the previous projects, the two agents have two strategies C (cooperate) and D (defect). The monetary (not the utility) payoffs are given by the following payoff matrix:

<table>
<thead>
<tr>
<th></th>
<th>Agent 1</th>
<th>Agent 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td></td>
<td>1, 1</td>
</tr>
<tr>
<td>D</td>
<td>-2, 2</td>
<td></td>
</tr>
</tbody>
</table>

(the first number in an entry is agent 1’s monetary payoff and the second agent 2’s payoff).

Suppose $a = \frac{1}{2}$. What is the equilibrium of this game? What would the equilibrium be if the agents were selfish ($a = 0$)?

(iii) Now, assemble the two stages considered in (i) and (ii) into a single, two-stage dynamic game. Suppose that the agents in stage 1 (the corporate finance stage) are slightly unsure that the other agent is altruistic: agent $i$’s beliefs are that, with probability $1 - \varepsilon$, the other agent ($j$) is altruistic ($a^j = \frac{1}{2}$) and, with probability $\varepsilon$, the other agent is selfish ($a^j = 0$). For simplicity, assume that $\varepsilon$ is small (actually, it is convenient to take the approximation $\varepsilon = 0$ in the computations).
4.8. Exercises

The two agents engage in group lending and receive Rb each if both projects succeed and 0 otherwise. Profits and payments to the lenders are realized at the end of stage 1.

At stage 2, each agent decides whether to participate in the social game described in (ii). If either refuses to participate, each gets 0 at stage 2 (whether she is altruistic or selfish); otherwise, they get the payoffs resulting from equilibrium strategies in the social game.

Let δ denote the discount factor between the two stages. Compute the minimum discount factor that enables the agents to secure funding at stage 1.

Exercise 4.8 (peer monitoring). The peer monitoring model studied in the supplementary section assumes that the projects are independent. Suppose instead that they are (perfectly) correlated. (See Sections 3.2.4 and 4.2. There are three states of nature: favorable (both projects always succeed), unfavorable (both projects always fail), and intermediate (a project succeeds if and only if the entrepreneur behaves), with respective probabilities p_H, p_L, and Δp.)

(i) Replace the limited liability assumption by [no limited liability, but strong risk aversion for R_b < 0 and risk neutrality for R_b ≥ 0]. Show that group lending is useless and that there is no credit rationing.

(ii) Come back to the limited liability assumption and assume that

\[ p_{H}(R - \frac{B}{\Delta p}) < I - A. \]

Assume that \( b + c < B \). Find a condition under which the agents can secure funding.

Exercise 4.9 (borrower-friendly bankruptcy court). Consider the timing described in Figure 4.7.

The project, if financed, yields random and verifiable short-term profit \( r \in [0, \tilde{r}] \) (with a continuous density and \( \text{ex ante mean } E[r] \)). After \( r \) is realized and cashed in, the firm either liquidates (sells its assets), yielding some known liquidation value \( L > 0 \), or continues. Note that (the random) \( r \) and (the deterministic) \( L \) are not subject to moral hazard. If the firm continues, its prospects improve with \( r \) (so \( r \) is "good news" about the future). Namely, the probability of success is \( p_{H}(r) \) if the entrepreneur works between dates 1 and 2 and \( p_{L}(r) \) if the entrepreneur shirks. Assume that \( p_{H} > 0, p_{L} > 0, \) and

\[ p_{H}(r) - p_{L}(r) = \Delta p \]

is independent of \( r \) (so shirking reduces the probability of success by a fixed amount independent of prospects). As usual, one will want to induce the entrepreneur to work if continuation obtains. It is convenient to use the notation

\[ \rho_{1}(r) \equiv p_{H}(r)R \quad \text{and} \quad \rho_{0}(r) \equiv p_{H}(r)\left[R - \frac{B}{\Delta p}\right]. \]

Investors are competitive and demand an expected rate of return equal to 0. Assume

\[ \rho_{1}(r) > L \quad \text{for all } r \quad (1) \]

and

\[ E[r] + L > I - A > E[r + \rho_{0}(r)]. \quad (2) \]

(i) Argue informally that, in the optimal contract for the borrower, the short-term profit and the liquidation value (if the firm is liquidated) ought to be given to investors.
Argue that, in the case of continuation, \( R_0 = B/\Delta p \). (If you are unable to show why, take this fact for granted in the rest of the question.)

Interpret conditions (1) and (2).

(ii) Write the borrower’s optimization program.

Assume (without loss of generality) that the firm continues if and only if \( r > r^* \) for some \( r^* \in (0, r) \). Exhibit the equation defining \( r^* \).

(iii) Argue that this optimal contract can be implemented using, inter alia, a short-term debt contract at level \( d = r^* \). Interpret “liquidation” as a “bankruptcy.”

How does short-term debt vary with the borrower’s initial equity? Explain.

(iv) Suppose that, when the decision to liquidate is taken, the firm must go to a bankruptcy court. The judge mechanically splits the bankruptcy proceeds \( L \) equally between investors and the borrower.

Define \( \hat{r} \) by

\[
\rho_0(\hat{r}) = \frac{1}{2} L.
\]

Assume first that

\[
r^* > \hat{r}
\]

(where \( r^* \) is the value found in question (ii)).

Show that the borrower-friendly court actually prevents the borrower from having access to financing. (Note: a diagram may help.)

(v) Continuing on question (iv), show that when

\[
r^* < \hat{r},
\]

the borrower-friendly court either prevents financing or increases the probability of bankruptcy, and in all cases hurts the borrower and not the lenders.

**Exercise 4.10 (benefits from diversification with variable-investment projects).** An entrepreneur has two variable-investment projects \( i \in \{1, 2\} \). Each is described as in Section 3.4. (For investment level \( I^i \), project \( i \) yields \( RI^i \) in the case of success and 0 in the case of failure). The probability of success is \( p_H \) if the entrepreneur behaves (and thereby gets no private benefit) and \( p_L = p_H - \Delta p \) if she misbehaves (and then obtains private benefit \( BI^i \)). Universal risk neutrality prevails and the entrepreneur is protected by limited liability.) The two projects are independent (not correlated). The entrepreneur starts with total wealth \( A \). Assume

\[
\rho_1 \equiv p_H R > 1 > \rho_0 \equiv p_H \left( R - \frac{B}{\Delta p} \right)
\]

and

\[
\rho_0' = p_H \left( R - \frac{p_H}{p_H + p_L} \frac{B}{\Delta p} \right) < 1.
\]

(i) First, consider project finance (each project is financed on a stand-alone basis). Compute the borrower’s utility. Is there any benefit from having access to two projects rather than one?

(ii) Compute the borrower’s utility under cross-pledging.

**Exercise 4.11 (optimal sale policy).** Consider the timing in Figure 4.8.

The probability of success \( s \) is not known initially and is learned publicly after the investment is sunk. If the assets are not sold, the probability of success is \( s \) if the entrepreneur works and \( s - \Delta p \) if she shirks (in which case she gets private benefit \( B \)). Assume that the (state-contingent) decision to sell the firm to an acquirer can be contracted upon ex ante. It is optimal to keep the entrepreneur (not sell) if and only if \( s > s^* \) for some threshold \( s^* \). (Assume in the following that \( s \) has a wide enough support and that there are no corner solutions. Further assume that, conditional on not liquidating, it is optimal to induce the entrepreneur to exert effort. If you want to show off, you may derive a sufficient condition for this to be the case.) As is usual, everyone is risk neutral, the entrepreneur is protected by limited liability, and the market rate of interest is 0.

(i) Suppose that the entrepreneur’s reward in the case of success (and, of course, continuation) is \( R_0 = B/\Delta p \). Assuming that the financing constraint is binding, write the NPV and the investors’ break-even constraint and show that

\[
s^* = \frac{(1 + \mu)L}{R + \mu(R - B/\Delta p)}
\]

for some \( \mu > 0 \). Explain the economic tradeoff.

(ii) Endogenize \( R_0(s) \) assuming that effort is to be encouraged and show that indeed \( R_0(s) = B/\Delta p \) for all \( s \). What is the intuition for this “minimum incentive result”?

(iii) Suppose now that \( s \) can take only two values, \( s_1 \) and \( s_2 \), with \( s_2 > s_1 \) and

\[
s_2 \left( R - \frac{B}{\Delta p} \right) > \max(L, I - A).
\]

Introduce a first-stage moral hazard (just after the investment is sunk). The entrepreneur chooses between taking a private benefit \( B_0 \), in which case \( s = s_1 \)
Entrepreneur needs $I - A > 0$ to finance investment of fixed size $I$. Probability of success $s$, drawn from continuous distribution $f(s)$ on $[s, S]$, is publicly observed.

Moral hazard (work yields no private benefit, shirk yields $B$).

Sell assets to acquirers willing to pay $L$.

Asset is sold

for certain, and taking no private benefit, in which case $s = s_2$ for certain. Assume that financing is infeasible if the contract induces the entrepreneur to misbehave at either stage. What is the optimal contract? Is financing feasible? Discuss the issue of contract renegotiation.

Exercise 4.12 (conflict of interest and division of labor). Consider the timing in Figure 4.9.

The entrepreneur (who is protected by limited liability) is assigned two simultaneous tasks (the moral-hazard problem is bidimensional):

- The entrepreneur chooses between probabilities of success $p_H$ (and then receives no private benefit) and $p_L$ (in which case she receives private benefit $B$).
- The entrepreneur is in charge of overseeing that the asset remains attractive to external buyers in the case where the project fails and the asset is thus not used internally. At private cost $c$, the entrepreneur maintains the resale value at level $L$. The resale value is 0 if the entrepreneur does not incur cost $c$. The resale value is observed by the investors if and only if the project fails.

Let $R_b$ denote the entrepreneur's reward if the project is successful (by assumption, this reward is not contingent on the maintenance performance); $\hat{R}_b$ is the entrepreneur's reward if the project fails and the asset is sold at price $L$; last, the entrepreneur (optimally) receives nothing if the project fails and the asset is worth nothing to external buyers.

The entrepreneur and the investors are risk neutral and the market rate of interest is 0. Assume that to enable financing the contract must induce good behavior in the two moral-hazard dimensions.

(i) Write the three incentive compatibility constraints; show that the constraint that the entrepreneur does not want to choose $p_L$ and not maintain the asset is not binding.

(ii) Compute the nonpledgeable income. What is the minimum level of $A$ such that the entrepreneur can obtain financing?

(iii) Suppose now that the maintenance task can be delegated to another agent. The latter is also risk neutral and protected by limited liability. Show that the pledgeable income increases and so financing is eased.
Exercise 4.13 (group lending). Consider the group lending model with altruism in the supplementary section, but assume that the projects are perfectly correlated rather than independent. What is the necessary and sufficient condition for the borrowers to have access to credit?

Exercise 4.14 (diversification and correlation). This exercise studies how necessary and sufficient conditions for the financing of two projects undertaken by the same entrepreneur vary with the projects’ correlation. The two projects are identical, taken on a stand-alone basis. A project involves a fixed investment cost $I$ and yields profit $R$ with probability $p$ and 0 with probability $1-p$, where the probability of success $p$ is chosen by the entrepreneur for each project: $p_H$ (no private benefit) or $p_L = p_H - \Delta p$ (private benefit $B$).

The entrepreneur has wealth $2A$, is risk neutral, and is protected by limited liability. The investors are risk neutral and demand rate of return equal to 0.

In the following questions, assume that, conditional on financing, the entrepreneur receives $R_k$ when $k \in \{0, 1, 2\}$ projects succeed, and that $R_0 = R_1 = 0$ (this involves no loss of generality).

(i) Independent projects. Suppose that the projects are uncorrelated. Show that the entrepreneur can get financing provided that

$$p_H \left[ R - \left( \frac{p_H}{p_H + p_L} \right) \frac{B}{\Delta p} \right] \geq I - A.$$

(ii) Perfectly correlated projects. Suppose that the shocks affecting the two projects are identical. (The following may, or may not, help in understanding the stochastic structure. One can think for a given project of an underlying random variable $\omega$ uniformly distributed in $[0, 1]$. If $\omega < p_L$, the project succeeds regardless of the entrepreneur’s effort. If $\omega > p_H$, the project fails regardless of her effort. If $p_L < \omega < p_H$, the project succeeds if and only if she behaves. In the case of independent projects, $\omega_1$ and $\omega_2$ are independent and identically distributed (i.i.d.). For perfectly correlated projects, $\omega_1 = \omega_2$.)

Show that the two projects can be financed if and only if

$$p_H \left[ R - \frac{B}{\Delta p} \right] \geq I - A.$$

(iii) Imperfectly correlated projects. Suppose that with probability $x$ the projects will be perfectly correlated, and with probability $1-x$ they will be independent (so $x = 0$ in question (i) and $x = 1$ in question (ii)). Derive the financing condition. What value of $x$ would the entrepreneur choose if she were free to pick the extent of correlation between the projects: (a) before the projects are financed, in an observable way; (b) after the projects are financed?

Exercise 4.15 (credit rationing and bias towards less risky projects). This exercise shows that a shortage of cash on hand creates a bias toward less risky projects. The same proposition in the context of a tradeoff between collateral value and profitability. The timing, depicted in Figure 4.10, is similar to that studied in Section 4.3.

The entrepreneur must finance a fixed-size project costing $I$, and has initial net worth $A < I$. If investors consent to funding the project, investors and entrepreneurs agree, as part of the loan agreement, on which variant, $i = s$ (safe) or $r$ (risky) is selected. A public signal accrues at an intermediate stage. With probability $x$ (independent of the project specification), the firm experiences no distress and continues. The production is then subject to moral hazard. The
entrepreneur can behave (yielding no private benefit and probability of success \( p_i^H \)) or misbehave (yielding a private benefit \( R \) and probability of success \( p_i^L \)); success generates profit \( R \). One will assume that

\[
p_i^S - p_i^L = p_i^H - p_i^L \equiv \Delta p > 0.
\]

With probability \( 1 - x \), the firm’s asset must be resold, at price \( L^s \) with \( L^s < p_i^H R \).

We assume that two specifications are equally profitable but the risky project yields a higher long-term profit but a smaller liquidation value (for example, it may correspond to an off-the-beaten-track technology that creates more differentiation from competitors, but also generates little interest in the asset resale market):

\[
L^s > L^r
\]

and

\[
(1 - x)L^s + xp_i^H R = (1 - x)L^r + xp_i^L R.
\]

The entrepreneur is risk neutral and protected by limited liability, and the investors are risk neutral and demand a rate of return equal to 0.

(i) Show that there exists \( A > \bar{A} \) such that for \( A > \bar{A} \), the entrepreneur is indifferent between the two specifications, while for \( A < \bar{A} \), she strictly prefers offering the safe one to investors.

(ii) What happens if the choice of specification is not contractible and is to the discretion of the entrepreneur just after the investment is sunk?

Exercise 4.16 (fire sale externalities and total surplus-enhancing cartelizations). This exercise endogenizes the resale price \( P \) in the redeployability model of Section 4.3.1 (but with variable investment). The timing is recapped in Figure 4.11.

The model is the variable-investment model, with a mass 1 of identical entrepreneurs. The representative entrepreneur and her project of endogenous size \( I \) are as in Section 4.3.1. In particular, with probability \( x \) the project is viable, and with probability \( 1 - x \) the project is unproductive. The assets are then resold to “third parties” at price \( P \). The shocks faced by individual firms (whether productive or not) are independent, and so in equilibrium a fraction \( x \) of firms remain productive, while a volume of assets \( J = (1 - x)I \) (where \( I \) is the representative entrepreneur’s investment) has become unproductive under their current ownership.

The third parties (the buyers) have demand function \( J = \Delta (P) \), inverse demand function \( P = \Delta (J) \), gross surplus function \( S(J) \) with \( S'(J) = P \), net surplus function \( S^p(P) = S(J(P)) - PD(P) \) with \( (S^p)' = -J \). Assume \( P(\omega) = 0 \) and \( 1 > x\rho_0 \).

(i) Compute the representative entrepreneur’s borrowing capacity and NPV.

(ii) Suppose next that the entrepreneurs ex ante form a cartel and jointly agree that they will not sell more than a fraction \( z < 1 \) of their assets when in distress.

Show that investment and NPV increase when asset sales are restricted if and only if the elasticity of demand is greater than 1:

\[
\frac{p}{P} > 1.
\]

Check that this condition is not inconsistent with the stability of the equilibrium (the competitive equilibrium is stable if the mapping from aggregate investment \( I \) to individual investment \( i \) has slope greater than \(-1\)).

(iii) Show that total (buyers’ and firms’) surplus can increase when \( z \) is set below 1.

Exercise 4.17 (loan size and collateral requirements). An entrepreneur with limited wealth \( A \) finances a variable-investment project. A project of size \( I \in \mathbb{R} \) if successful yields \( R(I) \), where \( R(0) = 0 \), \( R' > 0 \), \( R'' < 0 \), \( R'(0) = \infty \), \( R''(\infty) = 0 \). The probability of success is \( p_i^H \) if the entrepreneur behaves (she then receives no private benefit) and \( p_i^L = p_i^H - \Delta p \) if she misbehaves (she then receives private benefit \( BI \)).

The entrepreneur can pledge an arbitrary amount of collateral with cost \( C \geq 0 \) to the entrepreneur and value \( \phi(C) \) for the investors with \( \phi(0) = 0 \), \( \phi' > 0 \), \( \phi'' < 0 \), \( \phi'(0) = 1 \), \( \phi''(\infty) = 0 \).

The entrepreneur is risk neutral and protected by limited liability and the investors are competitive, risk neutral, and demand a rate of return equal to 0.

Assume that the first-best policy does not yield enough pledgeable income. (This first-best policy is \( C^* = 0 \) and \( I^* \) given by \( p_i[R'(I^*')] = 1 \). Thus, the assumption is \( p_i[R(I^*) - BI^*/\Delta p] < I^* - A \).)

Assume that the entrepreneur pledges collateral only in the case of failure (on this, see Section 4.3.5),
and that the investors’ breakeven constraint is binding. Show that as $A$ decreases or the agency cost (as measured by $B$ or, keeping $p_H$ constant, $p_H/\Delta p$) increases, the optimal investment size decreases and the optimal collateral increases.

5.7 Exercises

Exercise 5.1 (long-term contract and loan commitment). Consider the two-project, two-period version of the fixed-investment model of Section 3.2 and a unit discount factor. Assume, say, that the borrower initially has no equity ($A = 0$). Show the following.

(i) If $p_H(p_H R - I) + (p_H R - I - p_H B/\Delta p) \geq 0$, then the optimal long-term contract specifies a loan commitment in which the second-period project is financed at least if the first-period project is successful. Show that if $p_H(p_H R - I) + (p_H R - I - p_H B/\Delta p) > 0$, then the optimal long-term contract specifies that the second-period project is implemented with probability 1 in the case of first-period success, and with probability $\xi \in (0, 1)$ in the case of failure.

(ii) In question (i), look at how $\xi$ varies with various parameters.

(iii) Is the contract “renegotiation proof,” that is, given the first-period outcome, would the parties want to modify the contract to their mutual advantage?

(iv) Investigate whether the long-term contract outcome can be implemented through a sequence of short-term contracts where the first-period contract specifies that the borrower receives $\bar{A} = I - p_H(R - B/\Delta p)$ with probability 1 in the case of success and with probability $\xi$ in the case of failure.

Exercise 5.2 (credit rationing, predation, and liquidity shocks). (i) Consider the fixed-investment model. An entrepreneur has cash $A$ and can invest $I_1 > A$ in a project. The project’s payoff is $R_1$ in the case of success and 0 otherwise. The entrepreneur can work, in which case her private benefit is 0 and the probability of success is $p_H$, or shirk, in which case her private benefit is $B_1$ and the probability of success $p_L$. The project has positive NPV ($p_H R_1 > I_1$), but will not be financed if the contract induces the entrepreneur to shirk. The (expected) rate of return demanded by investors is 0.

What is the threshold value of $A$ such that the project is financed?

In the following, let $\rho_1 = p_H R_1 - I_1 - p_H B_1/\Delta p$.

The next three questions add a prior period, period 0, in which the entrepreneur’s equity $A$ is determined. The discount factor between dates 0 and 1 is equal to 1.

(ii) In this question, the entrepreneur’s date-1 (entire) equity is determined by her date-0 profit. This profit can take one of two values, $a$ or $A$, such that $a < I_1 - \rho_0 < A$.

At date 0, the entrepreneur faces a competitor in the product market. The competitor can “prey” or “not prey.” The entrepreneur’s date-0 profit is $a$ in the case of predation and $A$ in the absence of predation. Preying reduces the competitor’s profit at date 0, but by an amount smaller than the com-
5.7. Exercises

Entrepreneur chooses distribution \( G(L) \) or \( \bar{G}(L) \).

- What is the optimal probability \( x^* \)? (Assume that \( (\Delta q)p_{hl}B_1 \geq (\Delta p)p_{hl}B_0 \).)
- Assuming that \( \rho_0^1 > I_1 \), is the previous contract robust to (a mutually advantageous) renegotiation?

(iii) Forget about the competitor, but keep the assumption that the entrepreneur’s date-0 profit can take the same two values, \( a \) and \( A \). We now introduce a date-0 moral-hazard problem on the entrepreneur’s side.

Assume that the entrepreneur’s date-0 production involves an investment cost \( I_0 \) and that the entrepreneur initially has no cash. The entrepreneur can work or shirk at date 0. Working yields no private benefit and probability of profit \( A \) equal to \( q_H \) (and probability \( 1 - q_H \) of obtaining profit \( a \)). Shirking yields private benefit \( B_0 \) to the entrepreneur, but reduces the probability of profit \( A \) to \( q_L = q_H - \Delta q \) (\( 0 < q_L < q_H < 1 \)). Assume that

\[
I_1 + I_0 - (q_L A + (1 - q_L)a) > \rho_0^1.
\]

- Interpret this condition.

Consider the following class of long-term contracts between the entrepreneur and investors. “The date-1 project is financed with probability 1 if the date-0 profit is \( A \) and with probability \( x < 1 \) if this profit is \( a \). The entrepreneur receives \( R_h = B_1 / \Delta p \) if the date-1 project is financed and succeeds, and 0 otherwise.” Assume that such contracts are not renegotiated.

- What are the optimal specifications \( \{\rho^* (L), \Delta(L)\} \) (where \( \rho^* (L) \) and \( \Delta(L) \) are the state-contingent threshold and extra rent (see Section 5.5.2)) in the absence of the soft budget constraint (that is, the commitment to the contract is credible). Show that

Exercise 5.3 (asset maintenance and the soft budget constraint). Consider the variable-investment framework of Section 5.3.2, except that the date-0 moral hazard affects the per-unit salvage value \( L \).

Date-1 income is now equal to a constant (0, say). Assets are resold at price \( LI \) in the case of date-1 liquidation. The distribution of \( L \) on \([0, L]\) is \( G(L) \), with density \( g(L) \), if the borrower works at date 0, and \( \bar{G}(L) \), with density \( \bar{g}(L) \), if the borrower shirks at date 0. We assume the monotone likelihood ratio property:

\[
\frac{g(L)}{\bar{g}(L)} \text{ is increasing in } L.
\]

The borrower enjoys date-0 private benefit \( B_0 I \) if she shirks, and 0 if she shirks. The timing is summarized in Figure 5.11.

As usual, let \( \rho_1 \equiv p_{hl}R \) and \( \rho_0 = p_{hl}(R - B / \Delta p) \). And let

\[
\ell(L) = \frac{g(L) - \bar{g}(L)}{g(L)}.
\]

(i) Determine the optimal contract \( \{\rho^* (L), \Delta(L)\} \) (where \( \rho^* (L) \) and \( \Delta(L) \) are the state-contingent threshold and extra rent (see Section 5.5.2)) in the absence of the soft budget constraint (that is, the commitment to the contract is credible). Show that
where

and

G(R) grows

project stops and yields nothing. If

cash infusion equal to

fixed-size investment

assets).

Exercise 5.5 (liquidity needs and pricing of liquid assets). Consider the liquidity-needs model with a fixed investment and two possible liquidity shocks. The borrower has cash A and wants to finance a fixed-size investment I > A at date 0. At date 1, a cash infusion equal to p is needed in order for the project to continue. If p is not invested at date 1, the project stops and yields nothing. If p is invested, the borrower chooses between working (no private benefit, probability of success p₁₁) and shirking (private benefit B, probability of success p₁ = p₁₁ - Δp). The project then yields, at date 2, R in the case of success and 0 in the case of failure.

The liquidity shock is equal to ρ₁ with probability (1 - λ) and to ρ₁₁ with probability λ, where

ρ₁ < p₀ < ρ₁₁ < ρ₁,

where ρ₁ ≡ p₁₁R and ρ₀ ≡ p₁₁(R - B/Δp). Assume further that

ρ₀ - ρ₁ > I - A. \hspace{1cm} (1)

There is a single liquid asset, Treasury bonds. A Treasury bond yields 1 unit of income for certain at date 1 (and none at dates 0 and 2). It is sold at date 0 at price q ⩾ 1. (The investors’ rate of time preference is equal to 0.)

(i) Suppose that the firm has the choice between buying enough Treasury bonds to withstand the high liquidity shock and buying none. Show that it chooses to hoard liquidity if

\[(q - 1)(p₁₁ - p₀) \leq (1 - \lambda)(p₀ - p₁) - \lambda(p₁₁ - p₀) - I + A \hspace{1cm} (2)\]

and

\[(q - 1)(p₁₁ - p₀) \leq \lambda(p₁ - p₁₁). \hspace{1cm} (3)\]

(ii) Suppose that the economy is composed of a continuum, with mass 1, of identical firms with characteristics as described above. The liquidity shocks of the firms are perfectly correlated. There are T Treasury bonds in the economy, with \(T < p₁₁ - p₀\). Show that when λ is small, the liquidity premium \((q - 1)\) commanded by Treasury bonds is proportional to the probability of a high liquidity shock. (Hint: show that either (2) or (3) must be binding, and use (1) to conclude that (3) is binding.)

(iii) Suppose that, in the economy considered in the previous subquestion, the government issues at date 0 Treasury bonds, but also a security that yields at date 1 a payoff equal to 1 in the good state (the firms experience liquidity shock ρ₁) and 0 in the bad state (the firms experience liquidity shock ρ₁₁). What is the equilibrium date-0 price \(q'\) of this new asset? (Prices of the Treasury bonds and of this new asset are market clearing prices.)

Exercise 5.6 (continuous entrepreneurial effort; liquidity needs). (i) An entrepreneur with initial cash A and protected by limited liability wants to invest in a fixed-size project with investment cost I > A. After the investment is made, the entrepreneur chooses the probability p of success \(0 \leq p \leq 1\). Consider the...
5.7. Exercises

Entrepreneur and investors contract, and sink \( I_0 \)

Date 0

News about date-2 prospect accrues (publicly).

Date 1

Stop

Moral hazard.

Invest \( I_1 \)

Outcome: success (\( R \)) or failure (0).

Table 5.12

<table>
<thead>
<tr>
<th>Pr (success)</th>
<th>Private benefit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Behaves ( p_H + \tau )</td>
<td>0</td>
</tr>
<tr>
<td>Misbehaves ( p_L + \tau )</td>
<td>( B )</td>
</tr>
</tbody>
</table>

Figure 5.12

1); the disutility of effort is \( g(p) = \frac{1}{2}p^2 \). (The entrepreneur enjoys no private benefit in this model.) In question (i) only, the profit is \( R = 2\sqrt{I - A} \) in the case of success and 0 in the case of failure. (We assume that \( R < 1 \) to avoid considering probabilities of success exceeding 1. \( R \) takes an arbitrary value in question (ii).) As usual, the uninformed investors demand an expected rate of interest equal to 0 and everyone is risk neutral. Let \( R_0 \) denote the entrepreneur’s reward in the case of success.

Solve for the optimal contract \( (R_0) \). Show that

\[
R_0 = \frac{1}{2}R.
\]

(ii) Now introduce an intermediate liquidity shock \( \rho \) (for a now arbitrary level \( A \) of cash on hand). The cumulative distribution of \( \rho \) is \( F(\rho) \) on \([0, \infty)\) and the density \( f(\rho) \). The effort decision is made after the value of \( \rho \) is realized and, of course, conditional on the choice of continuing (incurring reinvestment cost \( \rho \)). Suppose that the entrepreneur’s stake in the case of continuation \( (R_0) \) is independent of \( \rho \). Write the investors’ breakeven condition. Write the optimization program yielding \( (R_0, \rho^*) \), where \( \rho^* \) is the cutoff liquidity shock.

Exercise 5.7 (decreasing returns to scale). Extend the treatment of Section 5.6.1 to the case of decreasing returns to scale: the payoff in the case of continuation and success is \( R(I) \), with \( R(0) = 0, R' > 0, R'' < 0, R'(0) = \infty, \) and \( R'(\infty) = 0 \). The rest is unchanged (the short-term income is \( RI \), the reinvestment need is \( \rho I \), and the private benefit is \( BI \)).

(i) What are the first-order conditions yielding the optimal investment level \( I \) and cutoff \( \rho^* \)?

(ii) Assuming that \( r > \rho^* \), and that \( (R(I)/I) - R'(I) \) is increasing in \( I \) (a condition satisfied, for example, by \( R(I) \) quadratic), derive the impact of the strength of the balance sheet as measured, say, by \( A \) on debt maturity.

Exercise 5.8 (multistage investment with interim accrual of information about prospects). In this chapter we have focused mostly on the case of shocks about the reinvestment need (cost overruns, say). Consider, instead, the case of news about the final profitability. In the two-outcome framework, news can accrue about either the probability of success or the payoff in the case of success. We consider both, in sequence. The investment is a multistage one: let

\[
I = I_0 + I_1,
\]

where \( I_0 \) is the date-0 investment and \( I_1 \) is the date-1 reinvestment. In contrast with \( I_0, I_1 \) is not incurred if the firm decides to stop. The timing is as in Figure 5.12.

As usual, the entrepreneur has initial wealth \( A \), is risk neutral, and protected by limited liability. Investors are risk neutral. The discount rate is equal to 0. If reinvestment cost \( I_1 \) is sunk at date 1, then the firm can continue. Misbehavior reduces the probability of success by \( \Delta p \), but yields private benefit \( B \) to the entrepreneur.

Assume

\[
B < (\Delta p)R.
\]

As announced, we consider two variants.

(c) News about the probability of success. \( R \) is known at date 0, but the probability of success is \( p_H + \tau \) in the case of good behavior and \( p_L + \tau \) in
the case of misbehavior, where \( \tau \) is publicly learned at the beginning of date 1. The random variable \( \tau \) is distributed according to the distribution function \( \bar{F}(\tau) \) with density \( f(\tau) \) on \( [\tau, \tau] = [-p_L, 1 - p_H] \). Let \( \tau^e \) denote the expectation of \( \tau \).

(d) News about the payoff in the case of success. The probabilities of success are known: \( p_H \) and \( p_L \) (normalize: \( \tau = 0 \)). By contrast, the profit \( R \) in the case of success is drawn from distribution \( G(R) \) with density \( g(R) \) on \( (0, \infty) \). (The profit in the case of failure is always equal to 0.)

(i) For each variant, show that there exist two thresholds, \( A_0 \) and \( A_1 \), such that the first best prevails for \( A \geq A_1 \) and financing is secured if and only if \( A \geq A_0 \). Show that the continuation rules take the form of cutoffs, as described in Figure 5.13.

Determine \( \tau_0^*, \tau_1^*, R_0^*, R_1^* \).

(ii) For each variant, assume that \( A = A_0 \). Let \( y \equiv (p_H + \tau)R \) denote the expected income, and \( R^*(y) \) denote the entrepreneur’s rent in the case of continuation. Show that (above the threshold \( y^* \))

- \( 0 < R'(y) < 1 \) in variant (a);
- \( R \) is constant in variant (b).

Exercise 5.9 (the priority game: uncoordinated lending leads to a short-term bias). This chapter, like Chapters 3 and 4, has assumed that the firm’s balance sheet is transparent. In particular, each investor has perfect knowledge of loans made by other lenders and of the firm’s obligations to them.

This exercise argues that uncoordinated lending leads to financing that is too oriented to the short term. In a nutshell, lenders, by cashing out early, exert a negative externality on other lenders. Because this externality is not internalized, the resulting financial structure contains too much short-term debt.

We consider a three-period model: \( t = 0, 1, 2 \). The entrepreneur has no cash (\( A = 0 \)), is risk neutral, and is protected by limited liability. At date 0, a fixed investment \( I \) is made. The project yields a known return \( r > 0 \) at date 1, and an uncertain return \( (R \text{ or } 0) \) at date 2. Because the point is quite general and does not require credit constraints, we assume away moral hazard; or, equivalently, the private benefit from misbehaving is 0. The probability of a date 2 success is

\[ p + \tau(I_1), \]

where \( I_1 \) is the date-1 deepening investment, equal to \( r \) minus the level of short-term debt repaid to lenders and the date-1 payment to the entrepreneur (the firm does not return to the capital market at date 1), and \( \tau \) is an increasing and concave function (with \( \tau'(0) = \infty \)). Assume that \( \tau'(r)R < 1 \).

We assume that the entrepreneur cannot engage in “fraud,” that is, cannot fail to honor the short-term debt and, if the project succeeds at date 2, the long-term debt. By contrast, obligations to lenders, and in particular \( I_1 \), cannot be verified as the firm’s balance sheet is opaque.

(i) Derive the first-best investment \( I_1^* \). Show how this allocation can be implemented by a mixture of short- and long-term debt (note that in this model without moral hazard the structure of compensation for the entrepreneur exhibits a degree of indeterminacy).

(ii) Assume that \( r - I_1^* < I \) (creditors must hold long-term debt). Suppose next that financing is not transparent. Start from the first-best solution, with a large number (a continuum of mass 1) of lenders, with the representative lender owning short-term
Entrepreneur has wealth \( A \) and fixed-size investment project \( I \).

\[ \rho \text{ is realized.} \]

Reinvesting \( \rho \) raises probabilities of success to \( p + \tau \).

\[ \text{Outcome } (R \text{ or } 0). \]

Figure 5.14

Entrepreneur has wealth \( A \) and income \( Pr > 0 \).

\( \text{Fixed-investment project costing } I > A \).

Reinvestment need \( \rho \) (drawn from \( F(.) \)).

Success (profit \( PR \)) with probability \( p \), failure (profit 0) with probability \( 1 - p \).

Figure 5.15

Claim \( r_1 \) and contingent long-term claim \( R_1 \) on the firm.

Show that the entrepreneur has an incentive to secretly collude with any lender to increase the latter’s short-term claim in exchange for a smaller long-term claim.

Given the constraint that financing is provided by many lenders and that the latter do not observe each other’s contracts, is the indeterminacy mentioned in question (i) resolved?

**Exercise 5.10 (liquidity and deepening investment).** (i) Consider the fixed-investment model. The entrepreneur has cash \( A \) and can invest \( I > A \) in a project. The project’s return in the case of success (respectively, failure) is \( R \) (respectively, 0). The probability of success is \( p_H \) if the entrepreneur behaves (she then gets no private benefit) and \( p_L = p_H - \Delta p \) if she misbehaves (in which case she gets private benefit \( B \)). In this subquestion and in the subsequent extension, one will assume that the project is viable only if the incentive scheme induces the entrepreneur to behave. The entrepreneur and the capital market are risk neutral; the entrepreneur is protected by limited liability; and the market rate of interest is equal to 0.

Let

\[ \rho_1 \equiv p_H R \quad \text{and} \quad \rho_0 \equiv p_H (R - B/\Delta p), \]

and assume \( \rho_1 > I > \rho_0 \).

What is the necessary and sufficient condition for the project to be financed?

(ii) Now add an intermediate stage, in which there is an option to make a deepening investment. This investment increases the probability of success to \( p_H + \tau \) (in the case of good behavior) and \( p_L + \tau \) (in the case of misbehavior).

If the deepening investment is not made, the probabilities of success remain \( p_H \) and \( p_L \), respectively. This deepening investment costs \( \rho \), where \( \rho \) is unknown ex ante and distributed according to distribution \( F(\rho) \) and density \( f(\rho) \) on \([0, \infty)\). The timing is summarized in Figure 5.14.

Let \( \mu \equiv \tau / p_H \), \( \hat{\rho}_1 \equiv \mu \rho_1 \), and \( \hat{\rho}_0 \equiv \mu \rho_0 \).

Write the incentive compatibility constraint and (for a given cutoff \( \rho^* \)) the investors’ breakeven condition.

(iii) What is the optimal cutoff \( \rho^* \)? (Hint: consider three cases, depending on whether

\[ \rho_0 [1 + \mu F(\hat{\rho}_k)] \leq I - A + \int_0^{\hat{\rho}_k} \rho f(\rho) \, d\rho, \]

with \( k = 0, 1 \).

(iv) Should the firm content itself with returning to the capital market at date 1 in order to finance the deepening investment (if any)?

**Exercise 5.11 (should debt contracts be indexed to output prices?).** This exercise returns to optimal corporate risk management when profits are positively serially correlated (see Section 5.4.2). The
source of serial correlation is now a permanent shift in the market price of output, as summarized in Figure 5.15. The model is the fixed-investment model, except that the date-1 and date-2 incomes depend on an exogenous market price $P$, with mean $\bar{P}$, that is realized at date 1. The realizations of $P$ and $\rho$ are independent.

The rest of the model is otherwise the same as in Section 5.2. Following the steps of Section 5.4.2:

(i) Determine the optimal reinvestment policy $\rho^\ast(P)$.

(ii) Show that, accounting for seasoned offerings, the optimal debt is fully indexed debt:

\[ d(P) = Pr - \ell_0, \]

where $\ell_0$ is a positive constant.

### 6.10 Exercises

**Exercise 6.1 (privately known private benefit and market breakdown).** Section 6.2 illustrated the possibility of market breakdown without the possibility of signaling. This exercise supplies another illustration. Let us consider the fixed-investment model of Section 3.2 and assume that only the borrower knows the private benefit associated with misbehavior. When the borrower has private information about this parameter, lenders are concerned that this private benefit might be high and induce the borrower to misbehave. In the parlance of information economics, the “bad types” are the types of borrower with high private benefit. We study the case of two possible levels of private benefit (see Exercise 6.2 for an extension to a continuum of possible types). The borrower wants to finance a fixed-size project costing $I$, and, for simplicity, has no equity ($A = 0$). The project yields $R$ (success) or 0 (failure). The probability of success is $p_{H}$ or $p_{L}$, depending on whether the borrower works or shirks, with $\Delta p = p_{H} - p_{L} > 0$. There is no private benefit when working. The private benefit $B$ enjoyed by the borrower when shirking is either $B_{L} > 0$ or $B_{H} > B_{L}$. The borrower will be labeled a “good borrower” when $B = B_{H}$ and a “bad borrower” when $B = B_{L}$. At the date of contracting, the borrower knows the level of her private benefit, while the capital market puts (common knowledge) probabilities $\alpha$ that the borrower is a good borrower and $1 - \alpha$ that she is a bad borrower. All other parameters are common knowledge between the borrower and the lenders.

To make things interesting, let us assume that under asymmetric information, the lenders are uncertain about whether the project should be funded:

\[ p_H \left( R - \frac{B_H}{\Delta p} \right) < I < p_H \left( R - \frac{B_L}{\Delta p} \right). \tag{1} \]

Assume that investors cannot break even even if the borrower shirks:

\[ p_{H}R < I. \tag{2} \]

(i) Note that the investor cannot finance only good borrowers. Assume that the entrepreneur receives no reward in the case of failure (this is indeed optimal); consider the effect of rewards $R_{b}$ in the case of success that are (a) smaller than $B_{H}/\Delta p$, (b) larger than $B_{H}/\Delta p$, (c) between these two values.

(ii) Show that there exists $\alpha^\ast$, $0 < \alpha^\ast < 1$, such that

- no financing occurs if $\alpha < \alpha^\ast$,
- financing is an equilibrium if $\alpha \geq \alpha^\ast$.

(iii) Describe the “cross-subsidies” between types that occur when borrowing is feasible.

**Exercise 6.2 (more on pooling in credit markets).** Consider the model of Exercise 6.1, in which the borrower has private information about her benefit of misbehaving, except that the borrower’s type is drawn from a continuous distribution instead of a binary one. We will also assume that there is a monopoly lender, who makes a credit offer to the borrower. The borrower has no equity ($A = 0$).

Only the borrower knows the private benefit $B$ of misbehaving. The lender only knows that this private benefit is drawn from an ex ante cumulative distribution $H(B)$ on an interval $[0, \bar{B}]$ (so, $H(0) = 0$, $H(\bar{B}) = 1$). (Alternatively, one can imagine that lenders face a population of borrowers with characteristic $B$ distributed according to distribution $H$, and are unable to tell different types of borrower apart in their credit analysis.) The lender knows all other parameters. For a loan agreement specifying share $R_{b}$ for the borrower in the case of success, and 0 in the case of failure, show that the lender’s expected
6.10. Exercises

profit is

\[ U_1 = H((\Delta p) R_0) p_H(R - R_b) + [1 - H((\Delta p) R_0)] p_L(R - R_b) - I. \]

Show that

- the proportion of “high-quality borrowers” (that is, of borrowers who behave) is endogenous and increases with \( R_0 \);
- adverse selection reduces the quality of lending (if lending occurs, which as we will see cannot be taken for granted);
- there is an externality among different types of borrower, in that the low-quality types (\( B \) large) force the lender to charge an interest rate that generates strictly positive profit on high-quality types (those with small \( B \));
- the credit market may “break down,” that is, it may be the case that no credit is extended at all even though the borrower may be creditworthy (that is, have a low private benefit). To illustrate this, suppose that \( p_L = 0 \) and \( H \) is uniform (\( H(B) = B/\hat{B} \)). Show that if

\[
\frac{p_H^2 R^2}{\hat{B}} < I
\]

(which is the case for \( \hat{B} \) large enough), no loan agreement can enable the lender to recoup on average his investment.

**Exercise 6.3 (reputational capital).** Consider the fixed-investment model. All parameters are common knowledge between the borrower and the investors, except the private benefit which is known only to the borrower. The private benefit is equal to \( B \) with probability \( 1 - \alpha \) and to \( b \) with probability \( \alpha \), where \( B > b > 0 \).

(i) Consider first the one-period adverse-selection problem. Suppose that the borrower has assets \( A > 0 \) such that

\[
p_H\left(R - \frac{b}{\Delta p}\right) > I - A > \max\left(p_H\left(R - \frac{B}{\Delta p}\right), p_L R\right).
\]

Show that the project receives funding if and only if

\[
(p_H - (1 - \alpha)\Delta p)\left(R - \frac{b}{\Delta p}\right) \geq I - A.
\]

(ii) Suppose now that there are two periods \((t = 1, 2)\). The second period is described as in question (i), except that the belief \( \hat{\alpha} \) at date 2 is the posterior belief updated from the prior belief \( \alpha \), and that the borrower has cash \( A \) only if she has been successful at date 1 (and has 0 and is not funded if she has been unsuccessful). So, suppose that the first-period project is funded and that the borrower receives at the end of date 1 a reward \( A \) when successful and 0 when unsuccessful. The first-period funding is project finance and does not specify any funding for the second project. Suppose for notational simplicity that the private benefit is the same \((B \ or b)\) in period 1 and in period 2. Let \( \Delta p_1 \) denote the increase in the probability of success when diligent in period 1. Assume that

\[
b < (\Delta p_1) A < B
\]

\[
< (\Delta p_1) \left[p_H\left(R - \frac{I - A}{p_H - (1 - \alpha_S)\Delta p}\right) + B\right]
\]

and

\[
(p_H - (1 - \alpha)\Delta p)\left(R - \frac{b}{\Delta p}\right) < I - A
\]

\[
< (p_H - (1 - \alpha_S)\Delta p)\left(R - \frac{b}{\Delta p}\right),
\]

where \( 1 - \alpha_S \equiv (1 - \alpha) p_L / ((1 - \alpha) p_L + \alpha p_H) \).

A “pooling equilibrium” is an equilibrium in which the borrower’s first-period effort is independent of her private benefit. A "separating equilibrium" is (here) an equilibrium in which the \( b \)-type works and the \( B \)-type shirks in period 1. A "semiseparating" equilibrium is (here) an equilibrium in which in period 1 the \( b \)-type works and the \( B \)-type randomizes between working and shirking.

- Show that there exists no pooling and no separating equilibrium.
- Compute the semiseparating equilibrium. Does this model formalize the notion of reputational capital?

**Exercise 6.4 (equilibrium uniqueness in the suboptimal risk-sharing model).** In the suboptimal risk-sharing model of Application 8, prove the claim made in the text that the low-information-intensity optimum depicted by \{S,B\} in Figure 6.3 is interim efficient if and only if the belief that the borrower
is a good borrower lies below some threshold $\alpha^*$, $0 < \alpha^* < 1$. (Verify the weak-monotonic-profit condition in the supplementary section, and show that $\alpha^*$ is in the interior of the interval $[0, 1]$.)

**Exercise 6.5 (asymmetric information about the value of assets in place and the negative stock price reaction to equity offerings with a continuum of types).** Consider the privately-known-prospects model of Application 2 in Section 6.2.2, but with a continuum of types. The entrepreneur already owns a project, which with probability $p$ yields profit $R$ and probability $1 - p$ profit 0. The probability $p$ is private information of the borrower. From the point of view of the investors, $p$ is drawn from cumulative distribution $F(p)$ with continuous density $f(p) > 0$ on some interval $[p_0, \tilde{p}]$. Assume that the distribution has monotone hazard rates:

$$\frac{f(p)}{F(p)} \text{ is decreasing in } p$$

and

$$\frac{f(p)}{1 - F(p)} \text{ is increasing in } p.$$  

(This assumption, which is satisfied by most usual distributions, is known to imply that the truncated means $m^{-}(p)$ and $m^{+}(p)$ have slope less than 1:

$$0 < (m^{-}(p))' = \frac{d}{dp}[E(\hat{p} | \hat{p} < p)] \leq 1$$

and

$$0 < (m^{+}(p))' = \frac{d}{dp}[E(\hat{p} | \hat{p} > p)] \leq 1$$

(see, for example, An 1998).

The model is otherwise as in Section 6.2.2. A seasoned offering may be motivated by a profitable deepening investment: at cost $I$, the probability of success can be raised by an amount $\tau$ such that

$$\tau R > I$$

(of course, we need to assume that $\tilde{p} + \tau \leq 1$). The entrepreneur has no cash on hand, is risk neutral, and is protected by limited liability. The investors are risk neutral and demand a rate of return equal to 0.

(i) Show that in any equilibrium, only types $p \leq p^*$, for some cutoff $p^*$, raise funds and finance the deepening investment.

(ii) Show that $p^* > p$ and that if $p^* < \tilde{p}$, then

$$\frac{\tau R}{I} = \frac{p^* + \tau}{m^{-}(p^*) + \tau}.$$  

Show that if the benefits from investment are “not too large,” in that

$$\frac{\tau R}{I} < \frac{\tilde{p} + \tau}{E[p] + \tau},$$

then indeed $p^* < \tilde{p}$.

Show that if there are multiple equilibria, the one with the highest cutoff $p^*$ Pareto-dominates (is better for all types than) the other equilibria.

(iii) Is there a negative stock price reaction upon announcement of an equity issue?

(iv) Focusing on an interior Pareto-dominant equilibrium, show that, when $\tau$ increases, the volume of equity issues increases.

**Exercise 6.6 (adverse selection and ratings).** A borrower has assets $A$ and must find financing for a fixed investment $I > A$. As usual, the project yields $R$ (success) or 0 (failure). The borrower is protected by limited liability. The probability of success is $p_H$ or $p_L$, depending on whether the borrower works or shirks, with $\Delta p = p_H - p_L > 0$. There is no private benefit when working. The private benefit enjoyed by the borrower when shirking is either $b$ (with probability $\alpha$) or $B$ (with probability $1 - \alpha$). At the date of contracting, the borrower knows her private benefit, but the market (which is risk neutral and charges a 0 average rate of interest) does not know it. Assume that $p_H R + B < I$ (the project is always inefficient if the borrower shirks) and that

$$p_H \left( R - \frac{B}{\Delta p} \right) < I - A < p_H \left( R - \frac{b}{\Delta p} \right)$$

and

$$[\alpha p_H + (1 - \alpha) p_L] \left( R - \frac{b}{\Delta p} \right) < I - A.$$  

(i) Interpret conditions (1) and (2) and show that there is no lending in equilibrium.

(ii) Suppose now that the borrower can at cost $r(x) = rx$ (which is paid from the cash endowment $A$) purchase a signal with quality $x \in [0, 1]$. (This quality can be interpreted as the reputation or the number of rating agencies that the borrower contracts with.) With probability $x$, the signal reveals
6.10. Exercises

Borrower chooses quality of signal $x$ (this quality is observed by the capital market).

Borrower’s type revealed with probability $x$.

Nothing revealed with probability $1-x$.

Borrower goes to the capital market.

Figure 6.5

the borrower’s type ($b$ or $B$) perfectly; with probability $1-x$, the signal reveals nothing. The financial market observes both the quality $x$ of the signal chosen by the borrower and the outcome of the signal (full or no information). The borrower then offers a contract that gives the borrower $R_b$ and the lenders $R - R_b$ in the case of success (so, a contract is the choice of an $R_b \in [0,R]$). The timing is summarized in Figure 6.5.

Look for a pure strategy, *separating* equilibrium, that is, an equilibrium in which the two types pick different signal qualities.

- Argue that the bad borrower (borrower $B$) does not purchase a signal in a separating equilibrium.
- Argue that the good borrower (borrower $b$) borrows under the same conditions regardless of the signal’s realization, in a separating equilibrium.
- Show that the good borrower chooses signal quality $x \in (0,1)$ given by

$$A = x(A-rx) + (1-x) \left[ \frac{R - I + A + rx}{pI} + B \right].$$

- Show that this separating equilibrium exists only if $r$ is “not too large.”

**Exercise 6.7 (endogenous communication among lenders).** Padilla and Pagano (1997) and others have observed that information sharing about creditworthiness is widespread among lenders (banks, suppliers, etc.). For example, Dun & Bradstreet Information Services, one of the leading rating agencies, collects information from thousands of banks. Similarly, over 600,000 suppliers communicate information about delays and defaults by their customers; and credit bureaux centralize information about the consumer credit markets.

Padilla and Pagano (see also Pagano and Jappelli (1993) and the references therein) argue that information sharing has both costs and benefits for the banks. By sharing information, they reduce their differentiation and compete more with each other. But this competition protects their borrowers’ investment and therefore enhances opportunities for lending. In a sense, the “tax rate” (the markup that banks can charge borrowers) decreases but the “tax base” (the creditworthiness of borrowers) expands. This exercise builds on the Padilla–Pagano model.

There are two periods ($t = 1,2$). The discount factor between the two periods is $\delta$. A risk-neutral borrower protected by limited liability has no cash on hand ($A = 0$). Each period, the borrower has a project with investment cost $I$. The project delivers at the end of the period $R$ or 0. There is no moral hazard. The probability of success is $p$ if the entrepreneur is talented (which has probability $\alpha$), and $q$ if she is not (which has probability $1-\alpha$). We will assume that the market rate of interest in the economy is 0, that the lenders are risk neutral, and that only the good type is creditworthy:

$$pR > I > qR.$$ 

The date-1 and date-2 projects (if financed) are correlated and yield the same profit (they both succeed or both fail).

There are $n$ towns. Each town has one bank and one borrower. The “local bank” has local expertise and thereby learns the local borrower’s type; the other banks, the “foreign banks,” learn nothing (and therefore have beliefs $\alpha$ that the entrepreneur is talented) at date 1. At date 2, the foreign banks learn

- only whether the borrower was financed at date 1, if there is no information sharing among banks;
- whether the borrower was financed at date 1 and whether she repaid (i.e., whether she was successful), if there is information sharing about riskiness.

In other words, information sharing is feasible on hard data (repayments), but not on soft data (assessment of ability).

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3. One will assume that if the signal reveals the borrower’s type, the investors put probability 1 on this type, even when they put weight 0 on the corresponding type after observing the quality of the signal.
Padilla and Pagano add two twists to the model. First, banks decide ex ante whether they will communicate information about default and they make this decision public. Second, the borrower’s type may be endogenous (in which it refers more to an investment in the projects or industry than in “pure talent”): at increasing and convex cost \( C(\alpha) \{ C' > 0, C'' > 0, C(0) = 0, C'(0) = 0, C'(1) = \infty \} \), the borrower develops a \( p \) project with probability \( \alpha \) and a \( q \) project with probability \( 1 - \alpha \). \( C \) can be viewed as an investment cost and represents a nonmonetary cost borne by the borrower.

Contracts between banks and borrowers are short-term contracts. These contracts just specify a payment \( R_b \) for the borrower in the case of success during the period (and 0 in the case of failure). Furthermore, in each period, banks simultaneously make take-it-or-leave-it offers to borrowers. And at date 2, the incumbent bank (the bank that has lent at date 1) makes its offer after the other banks.

The timing is summarized in Figure 6.6. 

(i) Suppose first that the probability \( \alpha \) of being a \( p \)-type is exogenous (there is no borrower investment), that \( [\alpha p + (1 - \alpha) q] R - I + \delta(\alpha p + (1 - \alpha) q)(R - I) < 0 \) , and that \( q R - I + \delta q(R - I) < 0 \). Show that the banks prefer not to share information.

(ii) Next, suppose that the borrower chooses \( \alpha \). Assuming that the two assumptions made in (i) still hold in the relevant range of \( \alpha \) (for example, \( \alpha \in [0, \alpha] \) , where \( \alpha \) satisfies the conditions), show that the banks choose to share information.

Exercise 6.8 (pecking order with variable investment). Consider the privately-known-prospects model with risk neutrality and variable investment. For investment \( I \), the realized income is either \( R^2 I \) (in the case of success) or \( R^3 I \) (in the case of failure), where \( R^3 > R^2 \geq 0 \). A good borrower has probability \( p_H \) of success when working and \( p_I \) when shirking; similarly, a bad borrower has probability \( q_H \) of success when working and \( q_I \) when shirking, where \( p_H - p_I = \Delta p = q_H - q_I \), for simplicity. The entrepreneur’s private benefit is 0 when working and \( BI \) when shirking. The entrepreneur is risk neutral and protected by limited liability; the investors are risk neutral and demand a rate of return equal to 0.

(i) Let \( U_B^{SI} \) denote the bad borrower’s gross utility under symmetric information.\(^4\) Consider the problem of maximizing the good borrower’s utility subject to the investors’ breaking even on that borrower, to the mimicking constraint that the good borrower’s terms not be preferred by the bad borrower to her symmetric-information terms, and to the no-shirking constraint. Let \( \{R^B, R^S\} \) denote the (nonnegative) rewards of the good borrower in the cases of success and failure. Write the separating program.

(ii) Show that \( R^B = 0 \).

(iii) (Only if you have read the supplementary section.) Show that the separating outcome is the only perfect Bayesian equilibrium of the issuance game if and only if \( \alpha \leq \alpha^* \) for some threshold \( \alpha^* \).

Exercise 6.9 (herd behavior). It is often argued that the managers of industrial companies, banks, or mutual funds are prone to herd.\(^5\) They engage in similar investments with sometimes little evidence that their strategy is the most profitable. An economic agent may indeed select a popular strategy against her own information that another strategy may be more profitable. A number of contributions have demonstrated that herding behavior may actually be

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4. This utility was derived in Section 3.4.2. It is equal to \[
\frac{1}{1 - \frac{q_B R - 1}{1 - q_B (R - \delta \Delta p)}} A
\]
if \( q_B R \geq 1 \), and to \( A \) otherwise.

5. One of the first empirical papers on herding behavior is Lakonishok et al. (1992). The large empirical literature on the topic includes Chevalier and Ellison (1999).
individually rational even though it is often collectively inefficient. The literature on herding behavior starts with the seminal contributions of Banerjee (1992), Bikhchandani et al. (1992), Scharfstein and Stein (1990), and Welch (1992); see Bikhchandani and Sharma (2001) for a survey of applications of this literature to financial markets.

There are several variants of the following basic argument. Consider first a sequence of agents $i = 1, 2, \ldots$ choosing sequentially between strategies $A$ and $B$. Agents receive their own signals; they observe previous decisions but not the others’ signals. Suppose that agents 1 and 2 have, on the basis of their own information, selected $A$. Agent 3, observing the first two choices, may well then select strategy $A$ even if her own signal favors the choice of $B$. Agent 4, not knowing agent 3’s motivation to choose $A$, may then also choose strategy $A$ even if his own signal points toward the choice of $B$. And so forth. It may therefore be the case that all agents choose $A$, even though the cumulative evidence, if it were shared, would indicate that $B$ is the best choice.

The literature also analyzes herd behavior in situations in which agents have principals (that is, they are not full residual claimants for the consequences of their choices). In particular, such agents may adopt herd behaviors because of reputational concerns (see Chapter 7). Suppose, for instance, that a manager’s job is rather secure; herding with the managers of other firms is then likely to be attractive to the manager: if the strategy fails, the manager has the excuse that other managers also got it wrong (“it was hard to predict”). Choosing an unpopular strategy, even if one’s information points in that direction, is risky, as there will be no excuse if it fails. The literature on herd behavior has also investigated the use of benchmarking by principals in explicit incentives (compensation contracts) rather than in implicit ones (career concerns).

Let us build an example of herding behavior in the context of the privately-known-prospects model of Section 6.2. There are two entrepreneurs, $i = 1, 2$, operating in different markets, but whose optimal strategy is correlated. There are two periods, $t = 1, 2$. Entrepreneur $i$ can raise funds only at date $t = i$ (so they secure funding sequentially). A project yields $R$ when it succeeds and 0 when it fails. The entrepreneurs are risk neutral and protected by limited liability; the investors are risk neutral and demand a rate of return equal to 0. The entrepreneurs have no net worth or cash initially.

The two entrepreneurs each have to choose between strategy $A$ and $B$. Strategies differ in their probability of success. A borrowing contract with investors specifies both the managerial compensation $R_0$ in the case of success (and 0 in the case of failure) and the strategy that the entrepreneur will select. Crucially, entrepreneur 2 and her potential investors observe the date-1 financing contract for entrepreneur 1. Entrepreneurs, but not investors, learn the state of nature.

Consider the following stochastic structure.

*Unfavorable environment (probability $1 - \alpha$).* The probabilities of success are, with equal probabilities, $(q, 0)$ for one project and $(0, q)$ for the other, where the first element is entrepreneur 1’s probability of success and the second entrepreneur 2’s. So entrepreneurs necessarily choose different projects if they apply for funding.

*Favorable environment (probability $\alpha$).* With probability $\theta$, the best project is the same for both and has probability of success $p$; the worst project for both has probability of success $r$, where

$$p > \max\{q, r\}.$$ 

With probability $1 - \theta$, the two entrepreneurs’ best strategies differ: the probabilities of success are $(p, r)$ and $(r, p)$, respectively, for entrepreneur 1’s and entrepreneur 2’s best strategy (which are $A$ or $B$ with equal probabilities). Thus $\theta$ is the probability of correlation of the best strategies in a favorable environment; this probability is equal to 0 in the unfavorable environment.

Let $m \equiv \alpha p + (1 - \alpha) q$ and assume that

$$qR > I.$$ 

Show that funding and herding (with probability $\alpha(1 - \theta)$, entrepreneur 2 chooses entrepreneur 1’s best strategy even though it does not maximize her probability of success) is an equilibrium behavior as

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6. One will assume therefore that the first entrepreneur cannot condition her financing contract on the later choice of strategy by the second entrepreneur.
long as
\[ r \left[ R - \frac{I}{\partial p} (1 - \partial) + q \right] \geq p \left[ R - \frac{I}{q} \right]. \]

Note that entrepreneur 2 is on average worse off than in an hypothetical situation in which investors did not observe the strategy of entrepreneur 1 (or that in which the optimal strategies were uncorrelated).

**Exercise 6.10 (maturity structure).** At date 0 the entrepreneur has cash on hand \( A \) and needs to finance an investment of fixed size \( I \). At date 1, a deterministic income \( r \) accrues; a liquidity shock must be met in order for the firm to continue. Liquidation yields nothing. The probability of success in the case of continuation depends on a date-1 effort: for a good borrower, this probability is \( p_H \) or \( p_L \) depending on whether she behaves (no private benefit) or misbehaves (private benefit \( B \)); similarly, for a bad borrower, it is \( q_H \) or \( q_L \). We assume that
\[ p_H - p_L = q_H - q_L = \Delta p, \]
and so the incentive compatibility constraint in the case of continuation is the same for both types of borrower:
\[ (p_H - p_L) R_b = (q_H - q_L) R_b = (\Delta p) R_b \geq B, \]
where \( R_b \) is the borrower’s reward in the case of success.

The borrower knows at the date of contracting whether she is a “\( p \)-type” or a “\( q \)-type.” Let
\[ \rho_0^G \equiv p_0 \left( R - \frac{B}{\Delta p} \right) \quad \text{and} \quad \rho_0^B \equiv q_0 \left( R - \frac{B}{\Delta p} \right) \]
de note the date-1 pledgeable incomes for the good and bad types.

The liquidity shock is deterministic and equal to \( \rho \). Information is asymmetric at date 0, but the capital market learns the borrower’s type perfectly at date 1, before the liquidity shock has to be met. Assume that
\[ \rho_0^G > \rho > \rho_0^B. \]

Suppose further that under symmetric information only the good borrower is creditworthy (provided that she is incentivized to behave).

Assume that \( r < I - A < r + [\rho_0^G - \rho] \). Show that the good borrower can costlessly signal her type.

7.4 Exercises

**Exercise 7.1 (competition and vertical integration).**
This exercise is inspired by Cestone and White (2003).

(i) A cashless entrepreneur \((A = 0)\) considers a research project requiring a fixed investment \( I \). When financed, the project succeeds with probability \( p_H = 1 \) (for certain) if she works, and with probability \( p_L = 1 - \Delta p \) if she shirks, in which case she receives private benefit \( B \). Regardless of the outcome, there is a verifiable salvage value \( R^f \geq 0 \) (equipment, real estate) at the end. For the moment, there is no other firm in the market and so success brings an additional income \( R = M \) (monopoly profit) on top of the salvage value. Assume that
\[ R^f + \left( M - \frac{B}{\Delta p} \right) \geq I. \quad (1) \]

The investment cost \( I \) includes a fixed cost \( K \leq I \) borne by a supplier who must develop an enabling technology. There is ex ante a competitive supply of such suppliers, who for simplicity have enough cash to finance the entrepreneur’s remaining investment cost, \( I - K \), besides their own cost \( K \). So we can formalize the supplier as a “competitive capital market” for the moment.

In exchange for his contribution (supplying the technology and providing complementary financing \( I - K \) to the entrepreneur), the selected supplier receives a debt claim (the equivalent of a fixed price) and an equity stake in the entrepreneurial firm.

A debt claim is a payment \( R^i_f \) to the supplier/lender from the safe income \( R_f^s \):
\[ 0 \leq R^s_f \leq R^f. \]

An equity claim is a share \( \theta_l \in [0, 1] \) of the firm’s profit beyond \( R^f \) (here, a claim on \( M \)).

- Can the project be financed?
- Characterize the set of feasible contracts \((R^i_f, \theta_l)\).
  (There is some indeterminacy, except when the inequality in (1) is an equality. Discuss informally extra elements that could be added to the model to make a debt contract strictly optimal.)

(ii) Suppose now that, after having developed the enabling technology for the entrepreneur, the supplier can, at no extra cost (that is, without incurring
K again), offer the technology to a rival who is in every respect identical to the entrepreneur. If he does so, and the two downstream projects are successful, then the per-firm duopoly profit is $D$ (on top of the salvage value $R^F$), where

$$2D < M$$

(competition destroys profit). Assume that

$$R^F + \left(D - \frac{B}{\Delta p}\right) \geq I - K > R^F. \quad (2)$$

- Note that the entrepreneur always wants to sign an exclusivity contract with the selected supplier (hint: look at the industry profit when the rival receives the enabling technology).
- In the absence of exclusivity provision (say, for antitrust reasons), look at whether the entrepreneur can obtain de facto exclusivity by choosing the debt/equity mix of the supplier properly. Assume for simplicity that $(\Delta p)(1-\theta)D \geq B$. This will hold true in an optimal contract.

**Exercise 7.2 (benefits from financial muscle in a competitive environment).** This exercise extends to liquidity choices the Aghion–Dewatripont–Rey idea that pledgeable income considerations may make financial structures and corporate governance strategic complements in a competitive environment.

(i) Consider a single firm. At date 0, the entrepreneur borrows $I - A$ in order to finance a fixed-size project costing $I$. At date 1, the firm may need to reinvest an amount $\rho$ with probability $\lambda$. With probability $1 - \lambda$, no reinvestment is required. In the case of continuation the entrepreneur may behave (probability of success $p_H$, no private benefit) or misbehave (probability of success $p_L = p_H - \Delta p$, private benefit $B$). Let

$$\rho_1(R) \equiv p_H R \quad \text{and} \quad \rho_0(R) \equiv p_H \left(R - \frac{B}{\Delta p}\right),$$

where $R$ is the profit in the case of success at date 2 (the profit is equal to 0 in the case of failure).

The firm is said to have “financial muscle” if $\rho > \rho_0(R)$ and the firm chooses to withstand the liquidity shock if it occurs.
- Explain the phrase “financial muscle.”
- Does the firm want to have financial muscle when $\rho > \rho_0(R)$? (Hint: consider three regions for the term $(1 - \lambda)\rho_0(R) - (I - A)$: $(-\infty, 0)$, $(0, \lambda[\rho - \rho_0(R)])$, and $(\lambda[\rho - \rho_0(R)], +\infty)$.)

(ii) Suppose now that the firm (now named the incumbent) faces a potential entrant in the innovation market. The entrant is identical to the incumbent in all respects (parameters $A$, $I$, $p_H$, $p_L$, $B$ and profits (see below)) except that the entrant will never face a liquidity shock if he invests (the entrant is therefore endowed with a better technology). Let $R = M$ denote the monopoly profit made by a firm when it succeeds and the other firm either has not invested in the first place or has invested but not withstood its liquidity shock; let

$$R = C = p_H D + (1 - p_H) M$$

(where $D < M$ is the duopoly profit) denote its expected profit when it succeeds and the other firm has invested and withstood its liquidity shock (if any). Assume that

$$\rho > \rho_1(M), \quad (1 - \lambda)\rho_0(C) + \lambda p_0(M) > I - A > \rho_0(C),$$

$$(1 - \lambda)\rho_1(C) + \lambda p_1(M) > I. \quad (3)$$

- Suppose, first, that the two firms choose their financial structures (liquidity) simultaneously at date 0. Show that the entrant invests and the incumbent does not.
- Suppose, second, that, at date 0, the incumbent chooses her financial structure before the entrant. And assume, furthermore, that

$$p_0(M) - \lambda \rho > I - A. \quad (4)$$

Show that the incumbent invests, while the (more efficient) entrant does not.

**Exercise 7.3 (dealing with asset substitution).** Consider the fixed-investment model with a probability that the investment must be resold (redeployed) at an intermediate date because, say, it is learned that there is no demand for the product. The timing is summarized in Figure 7.16.

An entrepreneur has cash $A$ and wants to invest a fixed amount $I > A$ into a project. The shortfall must be raised in a competitive capital market. The project yields $R$ with probability $p$ and 0 with probability $1 - p$, provided that there is a demand for the product (which has probability $x$) and is revealed at
the intermediate stage; the final profit is always 0 if there is no demand, and so it is then optimal to liquidate at the intermediate stage. Investors and entrepreneur are risk neutral, the latter is protected by limited liability, and the market rate of interest is 0.

(i) In a first step, ignore the possibility of asset substitution. The liquidation value is $L = L_0$, and the probability of success is $p_H$ if the entrepreneur works and $p_L = p_H - \Delta p$ if she shirks (in which case she obtains a private benefit $B$). Assume that the NPV of the project is positive if the entrepreneur works, and negative if she shirks.

Assume that $A \geq \bar{A}$, where

$$(1 - x)L_0 + xp_H \left( R - \frac{B}{\Delta p} \right) = I - \bar{A} \quad (1)$$

(and that $L_0 \leq p_H (R - B / \Delta p)$).

- Interpret (1).
- Compute the entrepreneur's expected utility.
- What is the class of optimal contracts (or, at least, characterize the optimal contract for $A = \bar{A}$)?

(ii) Suppose now that, before the state of demand is realized, but after the investment is sunk, the entrepreneur can engage in asset substitution. She can reallocate funds between asset maintenance (value of $L$) and future profit (as characterized by the probability of success, say).

More precisely, suppose that the entrepreneur chooses $L$ and

- the probability of success is $p_H + \tau(L)$ if the entrepreneur behaves and $p_L + \tau(L)$ if she misbehaves;
- the function $\tau$ is decreasing and strictly concave;
- $\tau(L_0) = 0$ and $\tau'(L_0)R = -\frac{1 - x}{x}$; \quad (2)

- the entrepreneur secretly chooses $L$ (multitasking).

Consider contracts in which

- liquidation occurs if and only if there is no demand (hence, with probability $x$);
- the entrepreneur receives $r_b(L)$ if the assets are liquidated, and $R_b$ if they are not and the project is successful (and 0 if the project fails).

Interpret (2). Compute the minimum level of $A$ such that the threat of (excessive) asset substitution is innocuous. Interpret the associated optimal contract. (Hint: what is the optimal asset maintenance (liquidation value)? Note that, in order to induce the entrepreneur to choose this value, in the case of liquidation you may pay $r_b(L) = r_b$ if $L$ is at the optimal level and 0 otherwise.)

Exercise 7.4 (competition and preemption). Consider the “profit-destruction model (with independent processes)” of Section 7.1.1.

As in Fudenberg and Tirole (1985), time is continuous, although both investment $I$ and the research process and outcome are instantaneous (this is in order to simplify expressions). The actual R&D can be performed only at (or after) some fixed date $t_0$. The instantaneous rate of interest is $r$. The monopoly and duopoly profits, $M$ and $D$, and the private benefit $B$ then denote present discounted values (at interest rate $r$) from $t_0$ on. The entrepreneur's cash is worth $e^{r(t_0-t)}A$ at date $t$ and so it grows with interest rate $r$ and is worth $A$ at date $t_0$.

Assume that

$$p_H \left( M - \frac{B}{\Delta p} \right) \geq I - A$$

$$\geq p_H \left[ (1 - p_H)M + p_H D - \frac{B}{\Delta p} \right].$$
This condition states that if investment were constrained to occur at \( t_0 \), there would be scope for funding exactly one entrepreneur (see Section 7.1.1).

The twist is that the investment \( I \) can be sunk at any date \( t \leq t_0 \) (implying an excess expenditure of \( [e^{r(t-t_0)} - 1]I \) from the point of view of date \( t_0 \) since the investment is useless until date \( t_0 \)). The investment is then publicly observed.

Analyze this preemption game, distinguishing two cases depending on whether

\[
P_H M \gtrless p_H \left( M - \frac{B}{\Delta p} \right) + A.
\]

**Exercise 7.5 (benchmarking).** This exercise generalizes the benchmarking analysis of Section 7.1.1.

The assumptions are the same as in that section, except for the descriptions of risk aversion and correlation. Two firms, \( i = 1, 2 \), must develop, at cost \( I \), a new technology in order to be able to serve the market. Individual profits are \( M \) for the successful firm if only one succeeds, \( D \) if both succeed, and 0 otherwise. The probability of success is \( p_H \) in the case of good behavior and \( p_L \) in the case of misbehavior (yielding private benefit \( B \)). Each entrepreneur starts with cash \( A \).

The entrepreneurs exhibit the following form of risk aversion: their utility from income \( w \) is

\[
w \quad \text{for } w \geq 0, \\
(1 + \theta)w \quad \text{for } w < 0,
\]

where \( \theta > 0 \) is both a parameter of risk aversion and a measure of deadweight loss of punishment (similar to that of costly collateral pledging (see Chapters 4 and 6)).

With probability \( \rho \), the realization of the random variable determining success/failure (see Section 7.1.1) is the same for both firms. With probability \( 1 - \rho \), the realizations are independent for the two firms. (So Section 7.1.1 considered the polar cases \( \rho = 0 \) and \( \rho = 1 \).) No one ever learns whether realizations are correlated or not.

(i) Find conditions under which both entrepreneurs' receiving funding (and exerting effort) is an equilibrium. Describe the optimal incentive schemes.

Hints:

(a) Let \( w = a_k \geq 0 \) denote the reward of a successful entrepreneur when \( k \) \((= 1, 2)\) is the number of successful firms. Let \( w = -b_k < 0 \) denote the reward (really, a punishment) of an unsuccessful entrepreneur when the number of unsuccessful firms is \( k \) \((= 1, 2)\).

(b) Each entrepreneur maximizes her NPV subject to (IR1) (the investors' breakeven condition) and (ICb) (the entrepreneur's incentive constraint).

(c) Show that there is no loss of generality in assuming that

\[
a_2 = b_2 = 0.
\]

(d) Use a diagram in the \((a_1, b_1)\)-space.

(ii) What happens when \( \theta \) goes to 0 or \( \infty \)? When \( \rho \) goes to 0 or 1?

**Exercise 7.6 (Brander–Lewis with two states of demand).** Analyze the Brander–Lewis Cournot model with two states of demand, \( \tilde{\theta} \) and \( \theta \), with \( \Delta \theta = \tilde{\theta} - \theta > 0 \), and

\[
\theta = \begin{cases} 
\tilde{\theta} & \text{with probability } \alpha, \\
\theta & \text{with probability } 1 - \alpha.
\end{cases}
\]

The demand function is \( p = \theta - Q \).

Let \( \theta^e = \alpha \tilde{\theta} + (1 - \alpha) \theta \) denote the mean. Assume that \( \frac{1}{\rho} (\theta^e)^2 > I \).

(i) Compute the equilibrium when the two firms issue no debt.\(^7\)

Show that both firms invest.

(ii) Next, follow Brander and Lewis in assuming that firm 1 chooses its financial structure first and picks a debt level \( D_1 \) high enough so that when the intercept is \( \tilde{\theta} \), firm 1 goes bankrupt.

Note that entrepreneur 1 then ignores the bad state. Show that the new equilibrium (assuming that firm 2 enters and remains an all-equity firm) is

\[
q_1 = \frac{1}{\rho} (\theta^e + 2 (1 - \alpha) \Delta \theta)
\]

and

\[
q_2 = \frac{1}{\rho} (\theta^e - (1 - \alpha) \Delta \theta).
\]

(iii) Assume that firm 1 accommodates entry and that firm 2 cannot issue debt. What is the optimal level of debt \( D_1 \) issued by entrepreneur 1?

**Exercise 7.7 (optimal contracts in the Bolton–Scharfstein model).** Redo the Bolton and Scharf-
stein analysis of Section 7.1.2, allowing for fully general contracts: the entrepreneur receives \( r_s^0 \) in the case of date-0 success but no refinancing, \( R^{SS}_0 \) in the cases of date-0 and date-1 success, and \( R^{FS}_0 \) in the cases of date-1 success and date-0 failure (with \( R^{SS}_0, R^{FS}_0 \geq B/\Delta p \)). (Under risk neutrality, there is no point rewarding failures unless it serves to deter predation. Hence, the exception \( R^{FS}_0 \) ) Generalize the conditions (PD) and (IC) and show that \( r_s^0 = 0 \) and that \( R^{SS}_0 \geq R^{FS}_0 \geq B/\Delta p \).

**Exercise 7.8 (playing the soft-budget-constraint game vis-à-vis a customer).** Consider a supplier–customer relationship with the timing as in Figure 7.17.

For simplicity, the customer is described as a self-financing entrepreneur (hence, without external investors). By contrast, the supplier is an entrepreneur who must borrow from the capital market. Thus, the context is that of the standard risk-neutral, fixed-investment model except for one twist: the payoff in the case of success, \( R \), is determined endogenously as part of a later bargaining process with the user of the input. The customer receives gross benefit \( v \) from using the (successfully developed) input and 0 otherwise. The entrepreneur/supplier would therefore like to extract as much of \( v \) as possible from the customer.

Assume that

\[
p_H \left( v - \frac{B}{\Delta p} \right) \geq \max(I - A, p_L v)
\]

and

\[
p_L v + B < I
\]

(and so, if all parties are rational, the investment will not take place if it subsequently induces the entrepreneur to misbehave). One will further assume that the input has no outside value (it is wasted if not used by the customer) and that the date-0 contract between the entrepreneur and the lenders is perfectly observed by the customer.

(i) **Long-term, nonrenegotiable debt.** Suppose, first, that the date-0 contract between the entrepreneur and her investors specifies an amount \( R_l \) of senior debt to be repaid to investors at date 2. This senior debt is purchased by investors who are unable to renegotiate their contract at any date.

Show that, when optimizing over the debt level \( R_l \), the entrepreneur cannot obtain ex ante utility exceeding

\[
U_b = (\Delta p) v - I.
\]

(Hint: work by backward induction. What happens at date 2 if no contract has yet been signed with the customer and the project has been successful? Moving back to date 1, distinguish two cases depending on whether \( p_H (v - R_l - B/\Delta p) \))

(ii) **Short-term, nonrenegotiable debt.** Second, assume that the entrepreneur issues an amount of short-term debt \( r_l \) and no long-term debt. This short-term debt is due at date 1 and thus the firm is liquidated if the debt is not reimbursed (again, we assume that the debt is purchased by dispersed investors who are unable to renegotiate the initial
contract). Because the firm has no date-1 revenue, the customer, if he wants the supplier to continue operating, must offer to cover the debt payment \( r_1 \), besides offering a transfer price \( R \) in case of a successful development of the input. Show that the entrepreneur can obtain expected utility

\[
U_b = p_H v - I.
\]

(Hints: show that the customer offers \( R = B/\Delta p \). Note that the entrepreneur consumes \( (A + r_1) - I \) at date 0.)

**Exercise 7.9 (optimality of golden parachutes).** Return to the manipulation model of Section 7.2.1, with the possibility of informed manipulation. Confirm the heuristic analysis of that section through a careful analysis, allowing for general contracts (the reward \( R_2 \) is contingent on the revealed information and may \textit{a priori} exceed \( B/\Delta p \); a fixed payment can be made in both states and only under revealed poor prospects: \( L_1^L \) and \( L_2^L \leq L \); allow \( q_H R \) to be larger or smaller than \( L \)).

- A manager with high current ability succeeds with probability \( r \), while one with low current ability succeeds with probability \( q < r \).
- The entrepreneur’s date-1 ability is high with probability \( \alpha \) and low with probability \( 1 - \alpha \) (no one knows this ability). The correlation of ability between dates 1 and 2 is equal to \( \rho \in [0, 1] \). That is, the entrepreneur's ability remains the same at date 2 with probability \( \rho \). To simplify computations, assume that the manager’s ability does not change between dates 2 and 3 (this assumption is not restrictive; we could simply require that the date-3 ability be positively correlated with the date-2 ability).
- At date 1, the entrepreneur privately observes the date-1 profit. If the entrepreneur has been successful (\( y_1 = R_1 \)), she can defer income recognition. The reported profit is then \( \hat{y}_1 = 0 \). These savings increase the probability that \( y_2 = R_2 \) by a uniform amount \( \tau \leq 1 - r \) (independent of type), presumably at a cost in terms of NPV \((R_1 > \tau R_2)\).\(^8\)
- Investors at the end of date 2 have the opportunity to replace the entrepreneur with an alternative manager who has probability \( \hat{\alpha} \) of being a high-ability manager. (There is no commitment with regards to this replacement decision.) This decision is preceded by a careful audit that prevents the entrepreneur from manipulating earnings (\( \hat{y}_2 = y_2 \)). One can have in mind a yearly report or a careful audit preceding an opportunity to replace management by a new managerial team.

8. We could also allow the entrepreneur to inflate date-1 earnings from 1 to \( R_1 \) at the cost of a reduction \( \tau \) in the probability of success at date 2 (\( R_1 < \tau R_2 \)). But if \( \tau \) is "not too large," there is no such incentive.
Find conditions under which a “pooling equilibrium,” in which the entrepreneur keeps a low profile \((\hat{y}_1 = 0)\) when successful \((y_1 = R_1)\), prevails.

8.5 Exercises

Exercise 8.1 (early performance measurement boosts borrowing capacity in the variable-investment model). Follow the analysis of Section 8.2.2 (publicly observable signal) and allow that the investment size is variable as in Section 3.4. Derive the entrepreneur’s borrowing capacity and utility.

Exercise 8.2 (collusion between the designated monitor and the entrepreneur). Consider the fixed-investment model of Section 8.2.3 (designated monitor), but assume that the entrepreneur can, at no direct cost to her, tunnel firm resources to the monitor through, say, an advantageous supply or consulting contract that reduces the project’s NPV. Namely, she can transfer an amount \(T(\tau)\) to the monitor at the cost of reducing the probability of success by \(\tau\) (from \(\nu_j\) to \(\nu_j - \tau\), where \(\nu_j\) is the probability of success conditional on signal \(j\)). Assume that \(T(0) = 0\), \(T' > 0\), \(T'(0) = R\) (a small transfer involves almost no deadweight loss), and \(T'' < 0\). (Note that \(T'(\tau) < \tau R\) for \(T'(\tau) > 0\) and so tunneling is inefficient.)

By contrast, transfers from the monitor to the entrepreneur are easily detected by investors. Similarly, the entrepreneur cannot offer to share her reward without being detected.

We look at ex post collusion: the entrepreneur and the monitor both observe the signal \(j \in \{L, H\}\) and the entrepreneur offers some level of \(\tau\) against a specified option exercise behavior by the monitor.

As in the rest of this chapter, we assume that the entrepreneur is incentivized to behave. She obtains \(\hat{R}_0\) if the monitor exercises his option and 0 otherwise. The monitor buys \(s\) shares at strike price \(p_H R\) each if he exercises his call options.

Show that the contract studied in Section 8.2.3 is immune to tunneling if and only if \(s\) exceeds some threshold.

9.6 Exercises

Exercise 9.1 (low-quality public debt versus bank debt). Consider the model of Section 9.2.1, except that the project has a positive NPV even if the entrepreneur misbehaves.

As usual, the entrepreneur is risk neutral and protected by limited liability. She has assets \(A\) and must finance an investment of fixed size \(I > A\). The project yields \(R\) in the case of success and 0 in the case of failure. The probability of success is \(p_H\) if the entrepreneur behaves (no private benefit) and \(p_L\) if she misbehaves (private benefit \(B\)). Investors are risk neutral and demand a 0 rate of return.

Instead of assuming that the project has positive NPV only in the case of good behavior, suppose that \(p_H R > p_L R + B > I\). Suppose further that there is a competitive supply of monitors and abundant monitoring capital. At private cost \(c\), a monitor can reduce the entrepreneur’s private benefit of misbehavior from \(B\) to \(b\). Assume that

\[
p_H \frac{B - b}{\Delta p} > c > (\Delta p)R - p_H \frac{b}{\Delta p}
\]

and

\[
(\Delta p)R > c + B.
\]

Show that there exist thresholds \(A_1 < A_2 < A_3\) such that

- if \(A > A_3\), the firm issues high-quality public debt (public debt that has a high probability of being repaid);
- if \(A_3 > A > A_2\), the firm borrows from a monitor (and from uninformed investors);
- if \(A_2 > A > A_1\), the firm issues junk bonds (public debt that has a low probability of being repaid);
- if \(A_1 > A\), the firm does not invest.

Exercise 9.2 (start-up and venture capitalist exit strategy). There are three periods, \(t = 0, 1, 2\). The rate of interest in the economy is equal to 0, and everyone is risk neutral. A start-up entrepreneur with initial cash \(A\) and protected by limited liability wants to invest in a fixed-size project. The cost of investment, incurred at date 0, is \(I > A\). The project yields,
at date 2, $R > 0$ with probability $p$ and 0 with probability $1 - p$. The probability of success is $p = p_H$ if the entrepreneur works and $p = p_L = p_H - \Delta p$ ($\Delta p > 0$) if the entrepreneur shirks. The entrepreneur’s effort decision is made at date 0. Left unmonitored, the entrepreneur obtains private benefit $B$ if she shirks and 0 otherwise. If monitored (at date 0), the private benefit from shirking is reduced to $b < B$.

There is a competitive industry of venture capitalists (monitors). A venture capitalist (general partner) has no fund to invest at date 0 and incurs private cost $c_A > 0$ when monitoring the start-up and 0 otherwise (the subscript “A” refers to “active monitoring”). The twist is that the venture capitalist wants his money back at date 1, before the final return, which is realized at date 2 (technically, the venture capitalist has preferences $c_0 + c_1$, while the entrepreneur and the uninformed investors have preferences $c_0 + c_1 + c_2$, where $c_1$ is the date-$t$ consumption). Assume that

$$ I - p_H (R - \frac{B}{\Delta p}) > A > I - p_H (R - \frac{b + c_A}{\Delta p}) . $$

(i) Assume first that the financial market learns (for free) at date 1 whether the project will be successful or fail at date 2. Note that we are then in the standard two-period model, in which the outcome can be verified at date 1 (one can, for example, organize an IPO at date 1, at which the shares in the venture are sold at a price equal to their date-2 dividend).

Show that the entrepreneur cannot be financed without hiring a venture capitalist. Write the two incentive constraints in the presence of a venture capitalist and show that financing is feasible. Show that the entrepreneur’s utility is $p_H R - I - \left[p_H c_A / \Delta p \right]$.

(ii) Assume now that at date 1 a speculator (yet unknown at date 0) will be able to learn the (date-2) realization of the venture’s profit by incurring private cost $c_P$, where the subscript “P” refers to “passive monitoring.”

At date 0, the venture capitalist is given $s$ shares. The date-0 contract with the venture capitalist specifies that these $s$ shares will be put for sale at date 1 in a “nondiscriminatory auction” with reservation price $P$. That is, shares are sold to the highest bidder at a price equal to the highest of the unsuccessful bids, but no lower than $P$. If left unsold, the venture capitalist’s shares are handed over for free to the date-0 uninformed investors (the limited partners) in the venture.

(a) Find conditions under which it is an equilibrium for the speculator (provided he has monitored and received good news) to bid $R$ for shares, and for uninformed arbitrageurs to bid 0 (or less than $P$).

(b) Write the condition on $(s, P)$ under which the speculator is indifferent between monitoring and not monitoring. Writing the venture capitalist’s incentive constraint, show that $P$ satisfies

$$ \frac{R - P}{P} = \frac{c_P \Delta p}{c_A p_H} . $$

How should the venture capital contract be structured if these conditions are not satisfied?

**Exercise 9.3 (diversification of intermediaries).**

Consider two identical entrepreneurs. Both are risk neutral, are protected by limited liability, have a project of fixed size $I$, and must borrow $I - A$ in order to finance their project. Each project, if undertaken, yields $R$ with probability $p$ and 0 with probability $1 - p$. The probability of success is $p_H$ if the entrepreneur behaves (receives no private benefit) and $p_L$ if she misbehaves (receives private benefit $B$). The two projects are statistically independent. The rate of interest in the economy is 0.

There is also a competitive supply of monitors, call them venture capitalists. Venture capitalists have no cash. Monitoring a firm involves a nonmonetary cost $c$ for the venture capitalist. The entrepreneur’s private benefit from misbehaving is then reduced from $B$ to $b < B$. Assume that

$$ I - A > \max \left\{ p_H \left( R - \frac{B}{\Delta p} \right), p_L \left( R - \frac{b + c}{\Delta p} \right) \right\} . $$

(i) Show that the entrepreneurs cannot obtain financing without uniting forces (on a stand-alone basis, with or without monitoring).

(ii) Consider now the following structure: the two firms are monitored by the same venture capitalist. By analogy with Diamond’s diversification reasoning (see Chapter 4), argue that the venture capitalist is paid a reward $(R_m)$ only if the two firms succeed. Show that if

$$ p_H \left( R - \frac{b + c p_H}{(p_H + p_L) \Delta p} \right) > I - A , $$

then financing can be arranged.
Exercise 9.4 (the advising monitor model with capital scarcity). Work out the model of Section 9.2.5, but assume that monitors have no capital ($I_m = 0$).

Find conditions under which the enlisting of a monitor facilitates financing, or conversely requires a stronger balance sheet.

Exercise 9.5 (random inspections). This exercise investigates a different way of formalizing monitoring. Rather than limiting the set of options available to the entrepreneur, the monitor ex post inspects, and, when finding evidence of misbehavior, takes a corrective action.

The timing is described in Figure 9.4.

The model is the standard one, with risk-neutral entrepreneur and investors. The entrepreneur is protected by limited liability and the investors demand a rate of return equal to 0.

At private cost $c$, the monitor can learn the choice of effort. If the entrepreneur has behaved, the firm is on the right track (as long as the entrepreneur stays on to finish the project), and there is no action to take. By contrast, if the entrepreneur misbehaves, the best policy is to kick her out, in which case she will enjoy neither her private benefit $B$ nor any reward in the case of success. The remedial action (which includes firing the entrepreneur) raises the probability of success to $p_L + \nu$, where $\nu > 0$ and $p_L + \nu < p_H$.

In questions (i) and (ii), one will assume that the entrepreneur and the monitor are rewarded solely as a function of the final outcome (they get $R_b$ and $R_m$ in the case of success, and 0 in the case of failure).

Assume that $\nu R_m > c$ and $(\Delta p) R_b < B$, and that the monitor has no cash (so $I_m = 0$).

(i) Show that in equilibrium the entrepreneur and the monitor play mixed strategies: the entrepreneur misbehaves with probability $x \in (0, 1)$, and the monitor fails to monitor with probability $y \in (0, 1)$.

(ii) Write the entrepreneur's utility and the uninformed investors' income as functions of $R_m$ and $R_b$.

What is the optimal financing arrangement?

(iii) In view of Chapter 8, is the performance-based contract studied in (i) and (ii) optimal?

Exercise 9.6 (monitor's junior claim). A risk-neutral entrepreneur protected by limited liability has a fixed-size project that yields $R_S$ in the case of success and $R_F \in (0, R_S)$ in the case of failure. Her cash on hand $A$ is smaller than the investment cost $I$.

As in Section 9.2, there are three versions of the project: good (probability of success $p_H$, no private benefit), bad (probability of success $p_L$, private benefit $b$), Bad (probability of success $p_L$, private benefit $B$). A risk-neutral monitor can at private cost $c$ rule out the Bad version. Monitoring capital is scarce; actually consider the polar case in which the monitor has no cash on hand (and is protected by limited liability).

As usual, uninformed investors are risk neutral and demand a rate of return equal to 0; one will also assume that funding can be secured only if the entrepreneur is monitored and is induced to choose the good version.

As in Section 9.2, there are three versions of the project: good (probability of success $p_H$, no private benefit), bad (probability of success $p_L$, private benefit $b$), Bad (probability of success $p_L$, private benefit $B$). A risk-neutral monitor can at private cost $c$ rule out the Bad version. Monitoring capital is scarce; actually consider the polar case in which the monitor has no cash on hand (and is protected by limited liability).

As usual, uninformed investors are risk neutral and demand a rate of return equal to 0; one will also assume that funding can be secured only if the entrepreneur is monitored and is induced to choose the good version.

Compute $R_m^S$ and $R_m^F$, the monitor's compensations in the cases of success and failure, respectively.
Show that

\[ R^F_{in} = 0. \]

**Exercise 9.7 (intertemporal recoupment).** An entrepreneur has a sequence of two projects to be undertaken at \( t = 1, 2 \), respectively. There is no discounting between the two periods. The only link between the two projects is that the second project can be undertaken only if the first has been. Each project is as described in Section 9.2, and has three versions: good (probability of success \( p_H \), no private benefit), bad (probability of success \( p_L \), private benefit \( b \)), Bad (probability of success \( p_L \), private benefit \( B \)). A risk-neutral monitor can at private cost \( c \) rule out the Bad version.

There is no scarcity of monitoring capital, in the sense that a monitor is willing to participate as long as his rate of return (which includes his monitoring cost) exceeds 0. As usual, uninformed investors are risk neutral and demand a rate of return equal to 0; one will also assume that funding can be secured only if the entrepreneur is monitored and is induced to choose the good version.

A project yields \( R \) in the case of success and 0 in the case of failure.

Assume that the entrepreneur has no cash on hand (\( A = 0 \)) and that the investment costs for the two projects, \( I_1 \) and \( I_2 \), satisfy

\[ I_1 + c > p_H \left( R - \frac{b}{\Delta p} \right) > I_2 + c \]

(the second project can for example be viewed as a continuation project, involving a lower investment cost),

\[ I_1 + I_2 + 2c < 2p_H \left( R - \frac{b}{\Delta p} \right), \]

and

\[ p_H R - I_1 - c > 0. \]

Consider two situations depending on whether there is competition among potential monitors:

**Concentrated lending market.** There is a single potential monitor. This monitor furthermore has full bargaining power, i.e., makes a take-it-or-leave-it contract offer (or offers) to the borrower.

**Competitive lending market.** There are multiple potential monitors, who compete for the borrower’s business.

\[ 10.6. \text{Exercises} \]

(i) **Long-term contracts.** First, assume that a contract covers the two periods; characterize the outcome under concentrated and competitive lending, and show that in either case the borrower receives funding for both investments.

(ii) **Short-term contracts.** Suppose now that the only contracts that a monitor can sign are one-period (spot) lending contracts, in which the monitor is compensated through a claim on the current profit only. Show that the borrower secures funding only in a concentrated market.

10.7 Exercises

**Exercise 10.1 (security design as a disciplining device).** Go through the analysis in Section 10.4.2 more formally. The date-1 income is \( r \) with probability \( p_H \) (if the entrepreneur exerts a high effort at date 1) or \( p_L \) (if the entrepreneur exerts a low effort at date 1), and 0 otherwise. The entrepreneur enjoys date-1 private benefit \( B_0 \) when shirking and 0 otherwise. Let \( R^* \) be defined by

\[ I - A - p_H^1 r - (1 - p_H^1)L = p_H^1[p_H(R - R^*)], \]

and assume that

\[ R^* > \frac{B}{\Delta p}, \]

\[ p_H(R - R^*) > L, \]

and

\[ (p_H^1 - p_H^1)(p_H R^*) > B_0. \]

(i) Interpret those conditions.

(ii) Describe an optimal incentive scheme and security design.

(iii) Suppose that \( R^* > B/\Delta p \). Argue that a short-term bonus (a payment in the case of date-1 profit \( r \)) is suboptimal. Argue more generally that there is no benefit in having such a payment.

**Exercise 10.2 (allocation of control and liquidation policy).** This exercise considers the allocation of a control right over liquidation. As described in Figure 10.9, the framework has three dates: date 0 (financing and investment), date 1 (choice of liquidation), and date 2 (payoff in the case of continuation).
tion). There is moral hazard in the case of continuation. As usual, there is universal risk neutrality, the entrepreneur is protected by limited liability, and the investors demand a rate of return equal to 0.

One will assume that the variables \((p_1, p_{1H}, R, B)\) in the case of continuation are known ex ante. As usual, misbehaving (choosing probability \(p_1\)) yields a private benefit \(B > 0\) to the entrepreneur. Let
\[
\rho_0 \equiv p_{1H}\left(R - \frac{B}{\Delta p}\right)
\]
and
\[
\rho_1 \equiv p_{1H}R.
\]
In contrast, the liquidation proceeds \(L\) and the fallback option \(U_b^0\) for the entrepreneur may be ex ante random, even though they become common knowledge at date 1 before the liquidation decision. Lastly, \(L\) is fully pledgeable to investors while none of \(U_b^0\) is.

(i) Solve for the optimal complete (state-contingent) contract, assuming that a court is able to directly verify \(\omega \equiv (L, U_b^0)\) (and the profit in the case of success) and to enforce the contract specifying the probability of continuation \(x(\omega) \in [0, 1]\) and the allocation of \(L\) and \(R\) between the investors and the entrepreneur.

(ii) Assume from now on that,
\[
\rho_0 \equiv p_{1H}(R - \frac{B}{\Delta p})
\]
and
\[
\rho_1 \equiv p_{1H}R.
\]
That is, in the absence of a “golden parachute” given to the entrepreneur in the case of liquidation, the entrepreneur always prefers to continue. Compare the sets \(\Omega^{FB}\) and \(\Omega^{SB}\) of states of nature in which continuation is optimal in the absence and presence of financing constraint. How does \(\Omega^{SB}\) vary with the entrepreneur’s net worth \(A^*/(\Delta p)\) (A diagram will help.)

(iii) From now on, assume that the court observes neither \(L\) nor \(U_b^0\). Only the entrepreneur and the investors do. The remaining questions look at how far one can go toward the implementation of the optimal full-observability contract described in (i) using a simple allocation of the control right concerning liquidation.

One will focus on the case in which \(\Omega^{SB}\) (see question (ii)) is strictly included in \(\Omega^{FB}\), and so inefficient liquidation is required.

Suppose first that the entrepreneur has the control right and that renegotiation occurs once \(\omega\) is realized. Argue that
\[
\Omega^{EN} = \Omega^{FB},
\]
where \(\Omega^{EN}\) is the set of states of nature over which continuation occurs under entrepreneur control.

Conclude that the project is then not financed.

(iv) Investor control. Perform the analysis of question (iii) in the case of investor control in the absence of a golden parachute (the initial contract does not provide for any compensation for the entrepreneur in the case of liquidation). Suppose that the entrepreneur does not keep any savings. Show that
\[
\Omega^{IN} \subset \Omega^{SB},
\]
where \(\Omega^{IN}\) is the set of states of nature over which continuation occurs under investor control. Is the project financed?

(v) Investor control with golden parachute. Argue that a positive golden parachute \((r_b > 0\) given to the entrepreneur in the case of liquidation) is optimal when investors have control.

**Exercise 10.3 (large minority blockholding).** Consider the active monitor model (see Chapter 9). The firm yields \(R\) in the case of success and 0 in the case of failure. The entrepreneur, large shareholder, and small shareholders have shares \(s_1, s_2,\) and \(s_3\), respectively, where \(s_1 + s_2 + s_3 = 1\). (To complete the model’s description, one can, as in Chapter 9, assume that \(s_1R \geq b/\Delta p\) and \(s_2R \geq c/\Delta p\), using the notation of this chapter.) The small shareholders have formal control (one share bears one voting right) and \(s_3 > \frac{1}{2}\).

The project can be modified in a countable number of ways \((k = 0, 1, \ldots)\). Option 0 consists in “not modifying the project” (this option is known to everyone). Options 1 through \(\infty\) do modify the project; all but two of them have disastrous consequences for all parties (so taking a modification at random is dominated by the status quo option 0). The two relevant modifications are such that one increases the probability of success by \(\tau > 0\) and the other reduces it by \(\mu > 0\). One involves a private cost \(\gamma > 0\) or a private benefit \(-\gamma\) for the entrepreneur, with \((\tau + \mu)s_1R < \gamma\), and the other no such cost. Lastly, an action may involve a private benefit \(\xi\) for the large blockholder (or one of his subsidiary). There are three states of nature, as shown in Table 10.2. In each state of nature, the left-hand payoffs correspond to
10.7. Exercises

Exercise 10.4 (monitoring by a large investor). Section 10.6 assumed that the entrepreneur does not have enough pledgeable income to recommend the investor-value-enhancing action in the case of dissonance, but has enough pledgeable income to induce (through the choice of the large investor’s share) the level of monitoring that maximizes the NPV and still receive funding.

Suppose instead that pledgeable income is low so that the level of pledgeable income is not sufficient to attract funding when the NPV-maximizing monitoring level is induced. Go through the steps of case (b) ("fully informed entrepreneur, large investor") assuming that there is no scarcity of monitoring capital (on this, see Section 9.2), and show that the monitoring level \( x \) is given by

\[
p_H \left[ R - \frac{B}{\Delta p} \right] + (1 - \xi) x \tau R = I - A + c_m(x)
\]

and

\[
c_m'(x) > (1 - \xi) (\tau R - \gamma).
\]

Exercise 10.5 (when investor control makes financing more difficult to secure). The general thrust of control rights theory is that investors are reassured, and so are more willing to lend, if they have control rights over the firm. The purpose of this exercise is to build a counterexample in which investor control is self-defeating and jeopardizes financing.

(i) An entrepreneur has cash \( A \) and wants to invest \( I > A \) into a (fixed-size) project. The project yields \( R > 0 \) with probability \( p \) and 0 with probability \( 1 - p \). The probability of success is \( p_H \) if the entrepreneur behaves and \( p_L = p_H - \Delta p (\Delta p > 0) \) if the entrepreneur misbehaves. The entrepreneur receives private benefit \( B > 0 \) in the latter case, and 0 in the former case. All parties are risk neutral, the entrepreneur is protected by limited liability, and the rate of interest in the economy is 0.

---

**Figure 10.9**

[Diagram showing Financing (investors contribute \( I - A \)), Continuation, and Verifiable liquidation value \( L \). Entrepreneur obtains \( U^B_L \geq 0 \) in alternative job.]

**Table 10.2** Probabilities: \( \beta \) (state 1); \( (1 - \beta) \kappa \) (state 2); \( (1 - \beta)(1 - \kappa) \) (state 3).

<table>
<thead>
<tr>
<th>Impact on probability of success</th>
<th>State 1</th>
<th>State 2</th>
<th>State 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Private cost for entrepreneur</td>
<td>( \tau - \mu )</td>
<td>( \tau - \mu )</td>
<td>( \tau - \mu )</td>
</tr>
<tr>
<td>Private benefit for large blockholder</td>
<td>0 - ( \gamma )</td>
<td>0 - ( \gamma )</td>
<td>0 - ( \gamma )</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>0</td>
<td>0 ( \xi )</td>
</tr>
</tbody>
</table>
What is the necessary and sufficient condition for the entrepreneur to be able to obtain financing from investors?

(ii) Now add a control right. This control right can raise the expected revenue in the case of misbehavior, but does nothing in the case of good behavior; namely, the holder of the control right can select an action ("damage control") that raises the probability of success from $p_L$ to $p_L + \nu$ ($\nu > 0$) in the case of misbehavior, but keeps $p_H$ constant. This interim action imposes a cost $\gamma > 0$ on the entrepreneur. (If the action is not selected, the probabilities of success are as in question (i), and there is no private cost $\gamma$.) The choice of action is simultaneous (say) with the entrepreneur’s choice of effort.

First assume "entrepreneur control" (the entrepreneur is given the right to select this action or not). Write the two incentive constraints for the entrepreneur to behave. Show that, compared with question (i), the pledgeable income remains the same if $\nu B / (\Delta p) \leq \gamma$, and is decreased otherwise.

(iii) Next consider “investor control.” Assume that when indifferent, the investors select the dominant strategy, i.e., the damage-control action (alternatively, one can assume that the action raises $p_H$ as well, to $p_H + \epsilon$, where $\epsilon$ is arbitrarily small). Show that the financing condition is now

$$p_H \left[ R - \frac{B}{\Delta p} - \nu \right] \geq I - A.$$

Conclude that investor control, besides reducing NPV, may also make it more difficult for the entrepreneur to secure financing.

Exercise 10.6 (complementarity or substitutability between control and incentives). This exercise pursues the agenda set in Exercise 10.5 by considering various forms of complementarity and substitutability between the exercise of control rights and managerial incentives. It therefore relaxes the assumption of separability between the two.

(i) An entrepreneur has cash $A$ and wants to invest $I > A$ into a (fixed-size) project. The project yields $R > 0$ with probability $p$ and 0 with probability $1 - p$. The probability of success is $p_H$ if the entrepreneur behaves and $p_L = p_H - \Delta p$ ($\Delta p > 0$) if the entrepreneur misbehaves. The entrepreneur receives private benefit $B > 0$ in the latter case, and 0 in the former case. All parties are risk neutral, the entrepreneur is protected by limited liability, and the rate of interest in the economy is 0.

What is the necessary and sufficient condition for the entrepreneur to be able to obtain financing from investors?

(ii) Now consider the possibility that a profit-enhancing action be chosen. For reasons of simplicity (but not for the sake of realism!), assume that this action is chosen simultaneously with effort. This action raises the probability of success to

- $p_H + \tau_H$ if the entrepreneur behaves, and
- $p_L + \tau_L$ if the entrepreneur misbehaves.

The action is indeed profit enhancing ($\tau_L, \tau_H > 0$) and is

- complementary with effort if $\Delta \tau \equiv \tau_H - \tau_L > 0$,
- substitutable with effort if $\Delta \tau < 0$.

The action further inflicts a disutility $\gamma$ on the manager, where

$$\max(\tau_L, \tau_H) \cdot R < \gamma.$$

Lastly, assume that the high effort must be induced in order for financing to occur.

Write the pledgeable income under investor control and entrepreneur control. When does investor control increase the pledgeable income (and therefore facilitate financing)?

Exercise 10.7 (extent of control). A simple variation on the basic model of Section 10.2.1 involves a choice between limited investor control and extended investor control, rather than between entrepreneur control and investor control. Suppose, in the model of Section 10.2.1, that entrepreneur control is out of the picture (after you finish the exercise, you may want to think about a sufficient condition for this to be case), but that there are two degrees of investor control:

Limited. The action taken then increases the probability of success by $\tau_A > 0$ and inflicts cost $y_A > 0$ on insiders.

Extended (investors have control over a wide set of actions). The selected action then increases the probability of success by $\tau_B > \tau_A$ and inflicts cost $y_B > y_A$ on insiders.

Assume that

$$\tau_A R - y_A > \tau_B R - y_B.$$
Find conditions under which limited or extended investor control prevails.

Exercise 10.8 (uncertain managerial horizon and control rights). This exercise considers the allocation of control between investors and management when the entrepreneur has an uncertain horizon.

We consider the fixed-investment model. The investment cost is \( I \) and the entrepreneur has only \( A < I \). The entrepreneur is risk neutral and protected by limited liability; the investors are risk neutral and demand rate of return equal to 0. The profit is equal to \( R \) in the case of success and is 0 in the case of failure. In the absence of profit-enhancing action, the probability of success is \( p \); when the profit-enhancing action is taken this probability becomes \( p + \tau \), where \( \tau > 0 \), but the action imposes a non-monetary cost on insiders, \( \gamma \), where

\[
\gamma > \tau R.
\]

As usual, \( p = p_H \) if the entrepreneur behaves (no private benefit) and \( p = p_L \) if she misbehaves (private benefit \( B \)).

The twist relative to Chapter 10 is that the entrepreneur may not be able to run the project to completion: with probability \( \lambda \), she must quit the firm for exogenous reasons. She learns this after the investment is sunk, but before the moral-hazard stage. If the entrepreneur quits (which will have probability \( \lambda \)), a new and cashless manager will be brought in. This manager is also risk neutral and protected by limited liability and has the same private benefit, probabilities of success, and payoff in the case of success as the entrepreneur.

Figure 10.10 summarizes the timing.

Let \( x \) and \( y \) in \([0, 1]\) denote the probabilities that investors receive control when the entrepreneur and the replacement manager are in charge, respectively. And assume that

\[
(p_H + \tau) \frac{B}{\Delta p} \geq y
\]

(interpret this assumption), and that

\[
\rho_1 \equiv p_H R > I > \rho_0^+ \equiv (p_H + \tau) \left( R - \frac{B}{\Delta p} \right).
\]

(i) Assuming that incentives must be provided for good behavior (by either the entrepreneur or the replacement manager), write down the following.

- The entrepreneur’s utility. (Hint: this utility is slightly different from the project’s social value. Why?)
- The pledgeable income and the breakeven condition.

(ii) Argue that \( y = 1 \). Find the conditions under which the project is undertaken. (Warning. Two conditions must be fulfilled: investors must be willing to finance it, and the entrepreneur must be willing to go ahead with it.)

Exercise 10.9 (continuum of control rights). This exercise extends the analysis of Section 10.2.2 to a continuum of control rights. As in Section 10.2.2, consider a risk-neutral entrepreneur protected by limited liability. The entrepreneur has cash on hand \( A \) and wants to finance a project with cost \( I > A \). The project yields \( R \) if it succeeds and 0 if it fails. Investors are risk neutral and demand a rate of return equal to 0. There is a continuum of control rights, where the decision attached to a control right can be thought of as a modification relative to the initial project and is characterized by the pair \((t, g)\):
$t \geq 0$ is the increase in the probability of success and $g \geq 0$ is the private cost borne by the entrepreneur if the decision is taken (the modification is made). Let $F(t,g)$ denote the continuous joint distribution over the space of control rights and $E_F[\cdot]$ the expectations with respect to distribution $F$.

The probability of success is

$$p + \tau \equiv p + E_F[t x(t,g)],$$

where $x(t,g) = 1$ if the decision $(t,g)$ is taken and 0 otherwise. Similarly, let

$$y \equiv E_F[g x(t,g)].$$

Moral hazard is modeled in the usual way: $p = p_H$ if the entrepreneur behaves (no private benefit) and $p = p_L$ if the entrepreneur misbehaves (and receives private benefit $B$). Assume that the project can be funded only if the entrepreneur is provided with the incentive to behave.

1. Solve for the optimal policy $x(\cdot,\cdot)$, assuming that the investors’ breakeven constraint is binding (which it is for $A$ small enough or $I$ large enough).
2. Show that, as $A$ decreases, $\tau$ and $y$ increase.
3. Discuss the implementation of the optimal $x(\cdot,\cdot)$ function.
4. Consider the degenerate case in which $g$ is the same for all control rights ($g > 0$). Show that

$$\frac{d^2 y}{d\tau^2} > 0.$$
and
\[ c(t) < R(t) \quad \text{for } t > t^* \text{ for some } t^* \in (0, 1). \]

(iii) Show that the “first-best outcome” described above is not incentive compatible, in the sense that depositors may want to withdraw early and reinvest in the technology themselves.

Exercise 12.2 (Allen and Gale (1998) on fundamentals-based panics). Consider the Diamond–Dybvig model developed in Section 12.2 and add randomness in the payoff of the long-term asset. Consumers are Diamond–Dybvig consumers: they invest 1 at date 0, and learn at date 2 whether they are impatient (their utility is \( u(c_1) \)) or patient (their utility is \( u(c_2) \)). The probability of being impatient is \( \lambda \).

The liquid or short-term technology yields one-for-one in each period: \( r_1 = r_2 = 1 \). The illiquid, long-term technology yields a random \( R \) (the same for all illiquid investments). The cumulative distribution is \( F(R) \) and the density \( f(R) \) on \([0, \infty)\). Liquidating the long-term asset yields nothing (\( l = 0 \)). One assumes
\[ E(R) > 1. \]

The realization of \( R \) is publicly observed at date 1.

(i) Compute the socially optimal insurance contract \( \{c_1(R), c_2(R)\} \), ignoring incentive compatibility (the ability of patient types to disguise as impatient ones). Note that this contract is incentive compatible.

(ii) Consider now a deposit contract. Consumers are promised, if they withdraw at date 1, a fixed payment \( c_1 \), or a share of \( i_1 \) if total withdrawal demand exceeds \( i_1 \). The date-2 income is shared among depositors who did not withdraw at date 1. Long-term assets are never liquidated. One will denote by \( x(R) \in [0, 1] \) the fraction of patient consumers who “join the run” (declare they are impatient, and store the money they have withdrawn from the bank).

Show that a judicious choice of \( c_1 \) succeeds in implementing the social optimum described in (i).

Exercise 12.3 (depositors’ game with a public signal). Consider the depositors’ game of Section 12.6.2, except that the depositors receive the same signal:
\[ y = R + \sigma \eta. \]

Determine the range of signals over which there exist multiple equilibria.

Exercise 12.4 (random withdrawal rate). Consider a three-date Diamond–Dybvig economy \((t = 0, 1, 2)\). Consumers are \textit{ex ante} identical; they save 1 at date 0. At date 1, consumers learn their preferences. A fraction \( \lambda \) has utility \( u(c_1) \) and a fraction \((1 - \lambda)\) has utility \( u(c_2) \).

At date 0, the consumers put their savings in a bank. They later cannot withdraw and invest in financial markets, so the Jacklin critique does not apply. That is, incentive compatibility issues are ignored in this exercise (a patient depositor cannot masquerade as an impatient one). The bank invests the per-depositor savings into short- and long-term projects: \( i_1 + i_2 = 1 \). The long-term technology yields \((\text{per unit of investment}) R > 1\) at date 2, but only \( l < 1 \) if liquidated at date 1. The short-term technology yields 1 (so \( r_1 = r_2 = 1 \)).

(i) Show that the optimal allocation \((c_1, c_2)\) satisfies
\[ u'(c_1) = Ru'(c_2). \]

- Suppose that \( u(c) = c^{1-\gamma}/(1-\gamma) \) with \( \gamma > 1 \). How do \( i_1 \) and \( i_2 \) vary with \( \gamma \)?

(ii) Suppose now that there is macroeconomic uncertainty, in that \( \lambda \) is unknown: \( \lambda = \lambda_1 \) with probability \( \beta \) and \( \lambda = \lambda_2 \) with probability \( 1 - \beta \), where \( 0 < \lambda_1 < \lambda_2 < 1 \). Set up the optimal program (let \( y_0 \) and \( z_0 \) denote the fraction of short-term investment that is not rolled over, and the fraction of long-term investment that is liquidated, respectively, in state of nature \( \omega \in \{L, H\} \)). What does the solution look like for \( l = 0 \) and \( l \) close to 1? (Showoffs: characterize the solution for a general \( l \))

13.6 Exercises

Exercise 13.1 (improved governance). There are two dates, \( t = 0, 1 \), and a continuum of mass 1 of firms. Firms are identical except for the initial wealth \( A \) initially owned by their entrepreneur. \( A \) is distributed according to continuous cumulative distribution \( G(A) \) with density \( g(A) \) on \([0, I] \).
Each entrepreneur has a fixed-size project, and must invest $I$, and therefore borrow $I - A$, at date 0 in order to undertake it. Those entrepreneurs who do not invest themselves, invest their wealth in other firms. The savings function of nonentrepreneurs (consumers) is an increasing function $S(r)$, where $r$ is the interest rate, with $S(r) = 0$ for $r < 0$ (so total savings equal $S(r)$ plus the wealth of unfinanced entrepreneurs). Entrepreneurs have utility $c_0 + c_1$ from consumptions $c_0$ and $c_1$.

A project, if financed, yields $R > 0$ at date 1 with probability $p$ and 0 with probability $1 - p$. The probability of success is $p_1$ if the entrepreneur works and $p_1 = p_1 - \Delta p$ if she shirks. The entrepreneur obtains private benefit $B$ by shirking and 0 otherwise. Assume $p_1 R > I > p_1 (R - B/\Delta p)$, that financing cannot occur if the entrepreneur is provided with incentives to misbehave, and that the equilibrium interest rate is strictly positive.

(i) What is the pledgeable income? Write the financing condition.

(ii) Give the expression determining the market rate of interest. How does this interest rate change when improved investor protection lowers $B$?

**Exercise 13.2 (dynamics of income inequality).**

(This exercise builds on the analysis of Section 13.4 and on Matsuyama (2000).)

(i) Consider the “warm-glow” model: generations are indexed by $t = 0, 1, \ldots, \infty$. Each generation lives for one period; each individual has exactly one heir. A generation-$t$ individual has utility from consumption $c_t$ and bequest $L_t$ equal to

$$
\left( \frac{c_t}{1 - a} \right)^{1-a} \left( \frac{L_t}{a} \right)^a
$$

with $0 < a < 1$.

What is the individual’s utility from income $y_t$?

(ii) Consider the entrepreneurship model of Section 13.4, with two twists:

- variable-size investment (instead of a fixed-size one),
- intraperiod rate of interest $r$ (so investors demand $(1 + r)$ times their outlay, in expectation); $r$ is assumed constant for simplicity.

One will assume that $p_1 = 1$ and that each generation $t$ is born with endowment $\hat{A}$ (to which is added bequest $L_{t-1}$, so $A_t = \hat{A} + L_{t-1}$). See Figure 13.12.

A successful project delivers $RI \geq (1 + r)I$, an unsuccessful one 0. The private benefit from misbehaving, $BI$, is also proportional to investment.

Let

$$
\rho_1 = R \quad \text{and} \quad \rho_0 = R - \frac{B}{\Delta p}.
$$

Assume that

$$
a(\rho_1 - \rho_0) < 1 - \frac{\rho_0}{1 + r}.
$$

Show that each dynasty’s long-term wealth converges to

$$
A_\infty = \frac{\hat{A}}{1 - a(\rho_1 - \rho_0)/(1 - \rho_0/(1 + r))},
$$

regardless of its initial total wealth $A_0$ (that is, $\hat{A}$ plus the bequest from generation $-1$, if any).

(iii) Now assume that there is a minimal investment scale $I > 0$ below which nothing can be produced. For $I > I$, the technology is as above (constant returns to scale, profit $RI$ in the case of success, private benefit $BI$ in the case of misbehavior, etc.).

Compute the threshold $A_0^*$ under which the dynasty remains one of lenders (at rate $r$) and never makes it to entrepreneurship.

What is the limit wealth $A_{\infty}^*$ of these poor dynasties? (The limit wealth of dynasties starting with $A_0 \geq A_0^*$ is still $A_\infty$.)

(iv) Finally, close the model by assuming that investors are domestic investors and by describing the equilibrium in the loan market. Focus on steady states. Show that multiple steady states may coexist:

- one in which everyone (investors, entrepreneurs) has the same wealth and $\rho_1 = 1 + r$,
- others, with unequal wealth distribution, in which $\rho_1 > 1 + r$, a fraction $\kappa$ of the population is poor (lends), and a fraction $1 - \kappa$ is rich (borrows to undertake projects).

**Exercise 13.3 (impact of market conditions with and without credit rationing).**

This analysis pursues that of Section 13.5.1. There, we compared the sensitivity of investment with the output price (or installed-base investment) in the presence or absence of credit rationing, focusing on either the fixed-investment variant or the constant-returns-to-scale variant. We now assume decreasing returns to scale.
The representative entrepreneur (there is a unit mass of such entrepreneurs) has initial wealth \( A \), is risk neutral and protected by limited liability, and invests \( I + K \), where \( I \) is the scale of investment and \( K \) a fixed cost that is unrelated to scale. We assume that \( K \geq A \), and so investors are unable to finance by themselves even a small investment.

An entrepreneur is successful with probability \( p \) and fails with probability \( 1 - p \). We assume that the shocks faced by the entrepreneurs are independent.

This hypothesis is consistent with the assumption made below that the output price is deterministic. When successful, the entrepreneur produces \( R(I) \) units of a good (with \( R(0) = 0, R' > 0, R'' < 0, R'(0) = \infty, R'(\infty) = 0 \)); an entrepreneur who fails produces nothing. For concreteness, let \( R(I) = I^\alpha \), with \( 0 < \alpha < 1 \).

As usual, the probability of success is endogenous: \( p \in \{p_L, p_H\} \). Misbehavior, \( p = p_L \) (respectively, good behavior, \( p = p_H \)), brings about private benefit \( BI \) (respectively, no private benefit). To prevent moral hazard, the entrepreneur must receive reward \( R_0 \) in the case of success, such that

\[
(\Delta p)R_0 \geq BI.
\]

The product sells at price \( P \) per unit. Presumably, investors are risk neutral and demand rate of return 0.

Suppose that the fixed cost \( K \) is “not too large” (so that the entrepreneur wants to invest in the absence of credit rationing), and that

\[
\frac{p_H B}{\Delta p} < \frac{1}{\alpha}.
\]

(i) Derive the first- and second-best investment levels as functions of \( P \). Show that they coincide for \( P \geq P_0 \) for some \( P_0 \).

(ii) Using a diagram, argue that there exists a region of output prices in which the second-best investment is more responsive than the first-best investment to the output price.

(iii) How would you analyze the impact of the existence of an installed-base level of investment \( I_0 \)?

14.4 Exercises

Exercise 14.1 (investment externalities in an industry with decreasing returns to scale). Suppose that the entrepreneur’s limited attention, say, induces decreasing returns to scale. Income in the case of success is \( R(I) \), where \( R' > 0, R'' < 0, R'(0) = \infty, R'(\infty) = 0 \). Redo the analysis of the Schleifer–Vishny model with this modification, and determine the sign of the investment externality.

Exercise 14.2 (alternative distributions of bargaining power in the Shleifer–Vishny model). Perform the analysis of Section 14.2.2 for an arbitrary unit price \( P \in [0, \rho_0] \) of resale of a distressed firm’s assets to a productive one. (Assume that bargaining oc-
Exercise 14.3 (liquidity management and acquisitions). Consider the model of Section 14.2.5 when the retooling cost is random. Suppose that this retooling cost is drawn from cumulative distribution function $F(\rho)$ on $[0, \infty)$, with density $f(\rho)$ and monotonic hazard rate ($f(\rho)/F(\rho)$ is decreasing). The level of the retooling cost is privately observed by the potential acquirer (the safe firm). The timing is as described in Figure 14.2.

Assume that the safe firm’s entrepreneur and investors ex ante secretly agree on an investment level $I$ and a credit line $L$. This credit line can be used if needed for the acquisition by the entrepreneur and completed by the liquidity, $\rho_0I$, that can be raised through a seasoned offering that dilutes the initial investors. (Fixing a credit line $L$ of this sort is indeed an optimal policy.)

One will assume that the seller always has the bargaining power ($z = 1$ in the notation of Section 14.2.5) and therefore sets price $P$. Lastly, let $\rho^*$ denote the equilibrium threshold for the retooling cost (that is, assets in equilibrium are acquired and retooled if and only if $\rho \leq \rho^*$).

(i) Write the entrepreneur’s optimal liquidity management (to this end, follow the steps described in Chapter 5). Show that given (anticipated) equilibrium price $P$, the threshold $\rho^*$ satisfies the “indifference between make and buy” equation:

$$P + \rho^* = 1.$$

(ii) Write the objective function of the risky firm when in distress. Compute the equilibrium price $P$.

Note that $P < 1$. What happens to $P$ if for some reason the anticipated level $L$ increases?

(iii) Suppose that the cumulative distribution function $F(\rho)$ converges to a spike at $\rho^*$. Show that

$$P + \rho^* = 1,$$

and that $F(\rho^*)$ converges to 1.

Exercise 14.4 (inefficiently low volume of asset re-allocations). This exercise applies the logic of corporate risk management developed in Chapter 5 to show that, even with frictionless resale markets, there will be an inefficiently low volume of transactions in the secondary market.

There are three dates, $t = 0, 1, 2$, and at least two firms $i = 1, 2$.

Firm 1, the firm of interest, is managed by a risk-neutral entrepreneur, who owns initial wealth $A$ at date 0 and is protected by limited liability. This firm invests at a variable investment level $I \in [0, \infty)$. The per-unit profitability of investment is random and learned at date 1. The investment yields $RI$ with

9. While still satisfying the monotone hazard rate property.
probability \( p + \tau \) and 0 with probability \( 1 - (p + \tau) \). The random variable \( \tau \) is drawn from a continuous distribution. The variable \( p \) is equal to \( p_H \) if the entrepreneur behaves (no private benefit) and \( p_L \) if the entrepreneur misbehaves (private benefit \( BI \)). Let

\[
\rho_1 = (p_H + \tau) R
\]

and

\[
\rho_0 = (p_H + \tau) \left( R - \frac{B}{\Delta p} \right) \equiv \rho_1 - \Delta \rho
\]

denote the random continuation per-unit NPV and pledgeable income when the entrepreneur behaves and the realization of profitability is \( \tau \). The distribution on \( \tau \) induces a cumulative distribution function \( F(\rho_0) \) on \([\rho^*_0, \rho_0]\).

At date 1, the firm may either continue or resell assets \( J \) to firm 2 (or to a competitive market). Firm 2 has a known level \( \hat{\rho}_0 \) of per-unit pledgeable income per unit of investment (its NPV per unit of investment is in general larger than this).

Firms 1 and 2 do not contract with each other at date 0. Rather, investors in firm 1 make a take-it-or-leave-it offer to firm 2 at date 1 if firm 1’s initial contract specifies that assets ought to be reallocated.

Assume for simplicity that the contract between firm 1’s investors and the entrepreneur can be contingent on the realization of \( \rho_0 \).

Show that at the optimal contract assets are resold whenever \( \rho_0 < \rho^*_0 \), where

\[
\rho^*_0 < \hat{\rho}_0,
\]

and so the volume of asset reallocations is inefficiently low.

### 15.5 Exercises

**Exercise 15.1** (downsizing and aggregate liquidity). Consider the variable-investment model with decreasing returns to scale and a liquidity shock. There is a unit mass of identical entrepreneurs. The timing for a given entrepreneur is in Figure 15.9.

At date 1, an amount \( J, 0 < J < I \), is rescued. In the absence of a liquidity shock (event has probability \( 1 - \lambda \)), of course \( J = I \). But in the face of a liquidity shock (which has probability \( \lambda \)), the investment is downsized to \( J \leq I \) (the cost of continuation is then \( \rho J \)). The shock is verifiable. Let \( R(J) \) denote the profit in the case of success.

The moral-hazard stage is described as it usually is: the probability of success is \( p_H \) if the entrepreneur works and \( p_L = p_H - \Delta \rho \) if she shirks. The entrepreneur obtains private benefit \( BI \) by misbehaving and 0 otherwise. Investors and entrepreneur are risk neutral, and the latter is protected by limited liability.

Economic agents do not discount the future (which does not imply that rates of interest are always 0!).

From now on, use \( J \) for the amount that is salvaged when there is a liquidity shock (as we noted, the corresponding amount is \( I \) in the absence of shock).

Assume that \( R(0) = 0, R' > 0, R'' < 0, R'(0) = \infty, R'(\infty) = 0 \).

(i) Assume that there is plenty of liquidity in the economy, so that the firms have access to a store of value (by paying \( q = 1 \) at date 0, they receive 1 at date 1).

From figure i, show that downsizing occurs in the case of a liquidity shock,

\[
J^* < I^*,
\]

if and only if

\[
\rho > \frac{1}{1 - \lambda}.
\]

(Hints: (1) write the incentive constraints (the sharing rule can be adjusted to the realization of the shock) and infer the pledgeable income; (2) maximize the entrepreneur’s utility (employ the usual trick) subject to the investors’ breakeven condition, ignoring the constraint \( J \leq I \); let \( \mu \) denote the shadow price of the constraint; (3) derive the stated result.)

(ii) Suppose that the liquidity shocks are perfectly correlated.

- What is the minimal number \( L^* \) of outside stores of value (delivering 1 unit of good each at date 1) needed to support the allocation described in (i)?
- Argue that if \( L < L^* \), then \( q > 1 \) and \( J < I \) a fortiori if \( \rho > 1/(1 - \lambda) \).

Derive the equations giving the liquidity premium \( (q - 1) \) under these assumptions.

(iii) Suppose now that the liquidity shocks are independent across firms.
Entrepreneur has wealth $A$, borrows $I - A$.

Firm is intact (no reinvestment needed) with probability $1 - \lambda$, and distressed (reinvestment $\rho$ per salvaged unit) with probability $\lambda$.

Moral hazard.

Outcome: success (profit $R(J)$) with probability $p$, failure (profit 0) with probability $1 - p$.

Figure 15.9

Representative entrepreneur invests $I$? If so, she borrows $I - A$.

Firms’ idiosyncratic productivities $y$ revealed (drawn from $G(\cdot)$).

Depending on its productivity realization, each firm decides whether to continue (and spend $J$).

Realization of income ($y$ or 0).

Figure 15.10

- Argue that (provided that the entrepreneurs borrow at date 0) there is enough liquidity to support the allocation derived in (i).
- Suppose that each entrepreneur holds the stock index. When will this provide enough liquidity? How can one prevent this potential waste of liquidity?

**Exercise 15.2 (news about prospects and aggregate liquidity).** Consider an economy with a continuum of identical risk-neutral entrepreneurs. The representative entrepreneur has a fixed-size investment project costing $I$, and limited personal wealth $A < I$. The project, if undertaken, will deliver a random but verifiable income $y \in [0, 1]$, with cumulative distribution function $G(y)$ and density $g(y)$, provided that a reinvestment $J$ is made after $y$ is learned, but before $y$ is produced. The project yields nothing if it is interrupted.

Moreover, in the case of “continuation” (that is, if $J$ is sunk), and regardless of the value of $y$, the entrepreneur may behave, in which case income is $y$ for certain, or misbehave, in which case income is $y$ with probability $p_L$ and 0 with probability $1 - p_L$. The entrepreneur, who is protected by limited liability, obtains private benefit $B$ when misbehaving (and no private benefit otherwise). Let

$$R \equiv \frac{B}{1 - p_L}$$

(one will assume that $B$ is small enough that, in the relevant range, it is worth inducing the entrepreneur to behave in the case of continuation).

The timing is summarized in Figure 15.9.

The rate of interest in the economy is equal to 0.

(i) Compute the NPV and the investors’ net income as functions of the threshold $y^*$ for continuation.

(ii) Let $y^*_0 \equiv J$ and $y^*_1 \equiv J + R$.

Define $A^*_0$ and $A^*_1$ by

$$I - A^*_k \equiv \int_{y^*_k}^{1} y \, dG(y) - \left[1 - G(y^*_k)\right][J + R],$$

for $k \in \{0, 1\}$.

- Argue that, for $A > A^*_1$, the entrepreneur must arrange at date 0 for her firm’s date-1 liquidity.

(iii) Is there enough inside liquidity if productivities are drawn independently from the distribution $G(\cdot)$? Why?

(iv) Suppose, in contrast, that there is a macroeconomic shock $\theta$ that is revealed at the beginning of date 1. (One will denote by $E_0[\cdot]$ the date-0 expectations over the random variable $\theta$.) Let $y^*(\theta)$ denote the state-contingent threshold.

- Argue that, for $A > A^*_1$, the entrepreneur must arrange at date 0 for her firm’s date-1 liquidity.

- Write the date-0 financing constraint.
16.6. Exercises

- Show that the optimal threshold when liquidity is abundant is actually state independent: there exists \( y^* \) such that
  \[
  y^*(\theta) = y^* \quad \text{for all } \theta.
  \]
- Show that the second-best allocation can be implemented when there are at least
  \[
  \min_{\{\theta\}} \int_{y^*}^{1} (y - J - R) \, dG(y | \theta)
  \]
niters of outside liquidity delivering 1 unit of good for certain at date 1.
- What would happen if there were few such stores of value?

**Exercise 15.3 (imperfectly correlated shocks).** This exercise extends the analysis of Section 15.3 to allow for imperfect correlation among the shocks faced by the firms. As in Section 15.3.1, there is a mass 1 of *ex ante* identical entrepreneurs. Each entrepreneur has a constant-returns-to-scale project. An investment of size \( I \) at date 0 yields \( \rho_1 I \) at date 2, of which \( \rho_0 I \) is pledgeable, provided that the liquidity shock \( \rho I \) is met at date 1. \( \rho \) is equal to \( \rho_L \) with probability \( 1 - \lambda \) and \( \rho_H \) with probability \( \lambda \), with \( \rho_L < \rho_0 < \rho_H < \rho_1 \) and \( (1 - \lambda)(\rho_H - \rho_0) \) < 1. As usual, entrepreneurs and investors are risk neutral, and the latter demand a rate of return equal to 0 (see Figure 15.10).

The new feature is that shocks are imperfectly correlated: for a fraction \( 1 - \theta \) of entrepreneurs, shocks are drawn independently (\( \theta = 0 \) in Section 15.2.1). A fraction \( \theta \) of entrepreneurs face the same shock, \( \rho_L \) with probability \( 1 - \lambda \) and \( \rho_H \) with probability \( \lambda \) (\( \theta = 1 \) in Section 15.3.1).

There is no outside store of value, and the long-term projects are the only investment projects available to the corporate sector.

Show that the private sector is self-sufficient (i.e., the efficient allocation can be implemented using the inside liquidity created by the long-term projects) if and only if \( \theta \leq \theta^* \), where
  \[
  (1 - \theta^*)(I - A) = \theta^*(\rho_H - \rho_0)I,
  \]
where \( I \) is independent of \( \theta \).

**Exercise 15.4 (complementarity between liquid and illiquid assets).** Go through the analysis of Section 15.3.1 assuming that entrepreneurs do not want to invest in projects that are discontinued in the adverse state of nature:
  \[
  (1 - \lambda)\rho_1 < 1 + (1 - \lambda)\rho_L.
  \]
Show that an increase in the supply \( L^S \) of liquid assets increases the investment \( I \) in illiquid ones.

16.7 Exercises

The first exercise is inspired by a paper by Gertler and Rogoff (1990).

**Exercise 16.1 (borrowing abroad).** Consider a small country with a mass 1 of identical entrepreneurs. There is a single (tradable) good. The representative entrepreneur has initial wealth \( A \) and a variable-investment constant-returns-to-scale project. A project of size \( I \in [0, \infty) \) at date 1 yields at date 2 verifiable revenue \( RI \) with probability \( p \) and 0 with probability \( 1 - p \). The probability \( p \) is not subject to moral hazard. There is moral hazard, though: instead of investing \( I \) in the firm, the entrepreneur can invest it abroad and get private return \( \mu I \), where \( \mu < 1 \). The investors are unable to seize the return from this alternative investment. Everyone is risk neutral, has discount factor 1 (i.e., has utility equal to the undiscounted sum of consumptions at
dates 1 and 2), and the entrepreneur is protected by limited liability.

One will assume that
\[ p_R > 1 > p_R - \mu. \]

(i) Compute the representative entrepreneur’s borrowing capacity and utility. Show that the outcome is the same as in a situation in which the entrepreneur cannot divert funds and invest them abroad, but can enjoy a private benefit per unit of investment \( B = \mu \). Explain why.

(ii) Adopt the convention that the payment to investors, \( R_l \), is a debt payment. Suppose that the entrepreneurs’ projects are independent and that the government imposes a per-unit-of-income tax on successful projects and offers a guarantee/compensation \( \sigma \) on private debt (so \( \tau R_I \) is the tax on successful projects and \( \sigma R_l \) is the investors’ payoff in the case of bankruptcy). Show that the borrowing capacity and entrepreneur utility are the same as in (i).

In contrast, compute the impact on entrepreneurs when the government starts at date 1 with an inherited public debt outstanding to foreign lenders equal to \( D (\leq A) \) per entrepreneur and must finance it through an income tax on successful projects.

(iii) Coming back to question (i), suppose that the government can through its governance institutions or other policies affect the return \( \mu \) on investments abroad. There are two levels \( \mu_L < \mu_H \) (where both levels satisfy the conditions in (ii)). The choice between the two levels involves no cost (but affects behavior!). The government’s objective function is to maximize the representative entrepreneur’s welfare.

Assuming that all borrowing is foreign borrowing, what is the representative entrepreneur’s utility when

(a) the government can commit to \( \mu \) before foreign investors invest;
(b) the government chooses \( \mu \) after they have invested (but before the entrepreneurs select their action)?

(iv) Suppose now that the output \( R_I \) (in the case of success) is in terms of a nontradable good (but the endowment \( A \) and the investment \( I \) are in tradable goods). Another sector of the economy (the “export sector”) will receive \( R \) in tradable goods at date 2. All domestic agents have utility from date-2 consumptions \( c \) and \( c^∗ \) of nontradable and tradable goods equal to \( c + c^∗ \) (so the two goods are perfect substitutes for domestic residents, while foreigners consume only the tradable good). Define the date-2 exchange rate \( e \geq 1 \) as the price of tradables in terms of nontradables. Compute the borrowing capacity and the exchange rate. (One will, for example, assume that funds fraudulently invested abroad cannot be reimported and must be consumed abroad. So they yield \( \mu \) rather than \( e\mu \).)

Exercise 16.2 (time-consistent government policy). Consider a unit mass of identical entrepreneurs with variable-investment projects. The timing is summarized in Figure 16.11.

The cost of the policy for the country is \( \gamma(\tau)I \) (where \( y'(0) = 0, y'(<\tau) > 0, \) for \( \tau > 0, y'(1 - p_H) = \infty, y''(0) > 0 \)).

All investors are domestic investors (there are no foreign lenders and, when choosing \( \tau \), the government maximizes social welfare, equal to entrepreneurs’ welfare plus investors’ welfare).

Assume that
\[ (p_H + \tau)R > 1 > (p_H + \tau)(R - \frac{B}{\Delta p}) \]
in the relevant range of values of \( \tau \), and that it is never optimal to induce entrepreneurs to misbehave. Everyone is risk neutral, and the entrepreneurs are protected by limited liability.

(i) Show that, when expecting policy \( \tau \), entrepreneurs invest

\[
I(\tau) = \frac{A}{1 - (p_H + \tau)(R - B/\Delta p)}.
\]

- What is the equilibrium value \( \tau^* \)?
(ii) What value would the government choose if it selected \( \tau \) before entrepreneurs borrow?
(iii) Informally explain how your answer to (i) would change if investors were foreign investors and the government discounted their welfare relative to that of domestic residents.

Exercise 16.3 (political economy of exchange rate policies). Consider a country that has liberalized its capital account. There are two goods: a tradable good (the only one consumed by foreigners) and a nontradable good.

- The only investors are foreign investors, with preferences over date-0 and date-1 consumptions

\[
c_1^t + c_1^n,
\]

where an asterisk refers to the tradable good.
- The country is populated by a unit mass of domestic entrepreneurs endowed with a constant-returns-to-scale technology. The representative entrepreneur (1) invests \( I \) units of tradables in equipment (where \( I \) is endogenous), (2) produces \( RI \) units of tradables in the case of success, and 0 in the case of failure, and \( SI \) units of nontradables for certain. We assume that firms’ outcomes are independent (there is no macroeconomic shock).

The model is a variation on the standard variable-investment model:

- Each entrepreneur is initially endowed with \( A \) units of tradables (her only wealth), borrows \( I - A \).
- There is moral hazard. The probability of success in the tradable-good activity is \( p_H \) if the entrepreneur behaves, and \( p_L \) otherwise. The entrepreneur receives private benefit \( BI \) in tradables by misbehaving and 0 otherwise.
- An entrepreneur’s utility is \( c_1 + u(c_1^t) + v(g^*) \), where \( c_1 \) is the consumption of nontradables, \( c_1^t \) the consumption of the tradable good, and \( g^* \) the level of public good supplied by the government. \( u \) and \( v \) are concave.

We add a government. The government has international reserves \( R^* \), of which it consumes \( g^* \) to produce a public good. The rest, \( R^* - g^* \), is dumped on the currency market at the end. So, \( e \), the price of tradables in terms of nontradables, is given \textit{ex post} by

\[
p_HRI + R^* + g^* = c_1^t(e) + d^* + \frac{d}{e},
\]

where \( I \) is the representative entrepreneur's investment, \( d^* \) is the entrepreneurs' average reimbursed debt in tradables and \( d \) is the average reimbursed debt in nontradables. The government cares only about the welfare of entrepreneurs, i.e., does not internalize that of the foreigners.

The timing is summarized in Table 16.1, where “t” and “nt” stand for “tradables” and “nontradables,” respectively.

Consider financing contracts in which

- investors receive \( R^t \) in tradables in the case of success, and 0 in the case of failure;
- investors have nominal claims \( R^n_t \) and \( R^n_i \) in nontradables in the cases of success and failure, respectively.

(i) Relate \((d, d^*)\) and \((R^t, R^n_t, R^n_i)\).
(ii) Fixing an expected exchange rate \( e \), determine the investment \( I \) of the representative entrepreneur in this constant-returns-to-scale model assuming that \( \rho_0 = p_H (R - B/\Delta p) < 1 - (S/e) \) in the relevant range.

Show that \( R I = SI \) and that
\[
I = \frac{A}{1 - ([S/e] + \rho_0)}.
\]

(iii) Compare the exchange rate and the welfare of entrepreneurs when the government chooses \( g^* \) after the private sector borrows abroad ("non-commitment") and when the government can commit to \( g^* \) before entrepreneurs borrow abroad ("commitment").

Assume that the exchange rate depreciates as government expenditures \( g^* \) grow. Show that \( v'(g^*) > e \) under commitment (underspending) and \( v'(g^*) < e \) under noncommitment (overspending).

(iv) Show that there is an externality among borrowers when the government cannot commit.

**Exercise 16.4 (time consistency and the soft budget constraint).** A firm is run by a risk-neutral entrepreneur with wealth \( A \), and has a fixed-size project with investment cost \( I \). The project, if undertaken at date 0, will deliver a verifiable income, \( y \in \{y_L, y_H\} \) in the case of success and 0 in the case of failure, at date 2, provided that one worker is employed in the firm. The project yields nothing if it is interrupted (the worker is laid off). \( y = y_H \) with probability \( \rho \) and \( y = y_L \) with probability \( 1 - \rho \).

Moreover, in the case of \("\) continuation\) and regardless of the value of \( y \), the entrepreneur may behave (the income is \( y \) for certain, the entrepreneur receives no private benefit) or misbehave (the income is \( y \) with probability \( p_L \) and 0 with probability \( 1 - p_L \), the entrepreneur receives private benefit \( B \). The entrepreneur is protected by limited liability. Let
\[
R \equiv \frac{B}{1 - p_L}.
\]

(One will assume that \( B \) is small enough that it is worth inducing the entrepreneur to behave in the case of continuation.) The (risk-neutral) worker is paid \( w \) in the case of continuation and 0 otherwise. He obtains unemployment benefit paid by the state \( w_u < w \) when laid off. We take \( w \) and \( w_u \) as given. (Note: they could be endogenized through some efficiency wage and incentive-to-search stories, but take these as exogenous for this exercise.)

Assume that the interest rate in the economy is 0 and that
\[
w < y_L < w + R
\]
and
\[
I - A \leq \rho (y_H - w + R) + (1 - \rho) (y_L - w - R).
\]

(i) Write the firm’s NPV depending on whether the firm continues (\( x = 1 \)) or stops (\( x = 0 \)) when productivity is low (\( y = y_L \)). Show that \( x^* = 1 \). Assuming a perfectly functioning capital market at date 1, what is the amount of liquidity that is needed to complement capital market refinancing?

(ii) Introduce a government that can at date 1 bring a subsidy \( s \geq 0 \) to the firm (it is a pure subsidy: the government takes no ownership stake in exchange). The shadow cost of public funds is \( \lambda \), and so the cost of subsidy \( s \) for the taxpayers is \( (1 + \lambda) s \). The government maximizes total welfare (entrepreneur, investors, worker, taxpayers). Assuming that
\[
\lambda [(w - w_u) + R] \leq (1 + \lambda) y_L
\]
and that the government selects its subsidy at date 1 (having observed the realization of $y$), what is the liquidity $L$ chosen by entrepreneur and investors at date 0?

How would the government (contingent) choice of $s$ be affected if the government could commit to $s$ at date 0, before the investors and the entrepreneur write their contract?
PART VII

Answers to Selected Exercises, and Review Problems
Answers to Selected Exercises

Exercise 3.1 (random financing). (i) The investors’ breakeven condition is

\[ xI - A \leq xp_H(R - R_b) \].

Because the NPV is negative if the entrepreneur has an incentive to shirk, \( R_b \) must satisfy

\[ (\Delta p)R_b \geq B \].

The investors’ breakeven condition (which will be satisfied with equality under a competitive capital market) is then

\[ B \geq x[p_H(R - \frac{B}{\Delta p}) - I] \geq -A \]

or

\[ x\bar{A} \leq A \].

(ii) The NPV is equal to

\[ U_b = x(p_H R - I) \]

and so maximizing \( U_b \) is tantamount to maximizing \( x \). Hence,

\[ x^* = \frac{A}{\bar{A}} \].

The probability that the project is undertaken grows from 0 to 1 as the borrower’s net worth grows from 0 to \( \bar{A} \).

Exercise 3.2 (impact of entrepreneurial risk aversion). (i) When \( p_H < 1 \), the entrepreneur must receive at least \( c_0 \) in the case of failure, because the probability of failure is positive even in the case of good behavior. Because of risk neutrality above \( c_0 \), it is optimal to give the entrepreneur exactly \( c_0 \) in the case of failure. Let \( R_b \) denote the reward in the case of success.

The incentive constraint is

\[ (\Delta p)(R_b - c_0) \geq B \].  \hfill (IC)

The pledgeable income is

\[ p_H R - (1 - p_H)c_0 - p_H \min R_b = p_H \left( R - \frac{B}{\Delta p} \right) - c_0 \].

To allow financing, this pledgeable income must exceed \( I - A \). Hence, \( \bar{A} = I + c_0 - p_H(R - B/\Delta p) \).

When \( p_H = 1 \), the pledgeable income is then \( p_H R \) (if \( c_0 > 0 \), deviations can be punished harshly by giving the entrepreneur, say, 0 in the case of failure).

(ii) Let \( R_b^S \) and \( R_b^F \) denote the rewards in the cases of success and failure, respectively. The incentive constraint is

\[ (\Delta p)[u(R_b^S) - u(R_b^F)] \geq B \].

The optimal contract solves

\[ \max U_b = p_H u(R_b^S) + (1 - p_H)u(R_b^F) \]

s.t.

\[ p_H R - p_H R_b^S - (1 - p_H)R_b^F \geq I - A, \]

\[ (\Delta p)[u(R_b^S) - u(R_b^F)] \geq B, \]

and (if limited liability is imposed)

\[ R_b^F \geq 0. \]

It must also be the case that the solution to this program exceeds the utility, \( u(A) \), obtained by the entrepreneur if the project is not financed. The entrepreneur’s incentive compatibility constraint is binding; otherwise, the solution to this program would give full insurance to the entrepreneur, which would violate the incentive compatibility condition. We refer to Holmström\(^1\) and Shavell\(^2\) for general considerations on this moral-hazard problem.

Exercise 3.3 (random private benefits). (i) \( B^* = p_H(R - \bar{\eta}) \).

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The investors’ expected income is
given by
\[ p_{II}^2 \frac{r(R - r)}{R} I = \frac{B^*(p_{II}R - B^*)}{R} I. \]
The borrowing capacity is such that this expected income is equal to the investors’ initial investment, \( I = A \). Thus
\[ I = kA, \]
where
\[ k = \frac{1}{1 - B^*(p_{II}R - B^*)/R}. \]

The borrowing capacity is maximized for
\[ B^* = \frac{1}{2} p_{II} R, \]
or, equivalently,
\[ r_1 = \frac{1}{2} R. \]

(iii) Using the fact that investors break even, the entrepreneur’s expected utility is
\[ \left( p_{II}\partial B^* + \frac{R}{2} \right) \Delta = \frac{p_{II} B^* + \frac{1}{2} R - (B^*)^2/2R}{1 - B^*(p_{II}R - B^*)/R} A. \]
At the optimum,
\[ \frac{1}{2} p_{II} R < B^* < p_{II} R. \]

Recall that \( B^* = p_{II} R \) maximizes the return per unit of investment as it eliminates shirking, while \( B^* = \frac{1}{2} p_{II} R \) maximizes borrowing capacity.

(iv) When \( B \) is verifiable, the entrepreneur’s expected utility is still
\[ \left( p_{II}\partial B^* + \frac{R}{2} - \frac{(B^*)^2}{2R} \right) I. \]
For a given \( B^* \), the contract should specify
\[ r_1(B) \begin{cases} = R - B/p_{II} & \text{if } B < B^* & (\text{recall that } p_{II} = 0), \\ > R - B/p_{II} & \text{if } B > B^*. \end{cases} \]
The maximal investment is then
\[ I = \frac{A}{1 - p_{II} B^* + (B^*)^2/2R}. \]
Borrowing capacity is maximized at \( B^* = p_{II} R \). Because this threshold also maximizes per-unit expected income, it is clearly optimal overall.

**Exercise 3.4 (Product-market competition and financing).** (i) Because the two projects are statistically independent, there is no point making an entrepreneur’s reward contingent on the outcome of the other firm’s performance. (Technically, this result is a special case of the “sufficient statistics” results of Holmström and Shavell. This result states that an agent’s reward should be contingent only on variables that the agent can control—a sufficient statistic for the vector of observable variables relative to effort—and not on extraneous noise.) So, let \( R_{b}^S \) and \( R_{b}^F \) denote an entrepreneur’s reward in the cases of success and failure. As usual,
\[ (\Delta p)(R_{b}^S - R_{b}^F) \geq B \quad \text{and} \quad R_{b}^F = 0. \]

Let \( x \in [0, 1] \) denote the probability that the rival firm invests. Then the expected income is
\[ p_{II}[xp_{II}D + (1 - xp_{II})M]. \]
The pledgeable income is equal to this expression minus \( p_{II}B/\Delta p \).

At best, the other firm is not financed, and \( R = M \) in the case of success. The threshold \( A \) is given by
\[ I - A = p_{II} \left( M - \frac{B}{\Delta p} \right). \]

(ii) At worst, the rival firm is funded. So, the expected return in the case of success is
\[ p_{II}D + (1 - p_{II})M. \]

So,
\[ I - A = p_{II} \left( p_{II}D + (1 - p_{II})M - \frac{B}{\Delta p} \right). \]

(iii) One of the firms gets funding while the other does not (obvious). There also exists a third, mixed-strategy equilibrium, in which each firm gets funded with positive probability.

(iv) If only one firm receives financing, then \( R_{b}^F = c_0 \)
(as long as \( p_{II} < 1 \), so that there is always a probability of failing even when the entrepreneur works), and
\[ R_{b}^S = c_0 + \frac{B}{\Delta p}, \]
which yields the minimum net worth given in the statement of the question.

(v) Suppose now that both entrepreneurs receive financing. Consider the following reward scheme for

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the entrepreneur:

\[ R_b < c_0 \quad \text{if the firm fails and} \]
\[ R_b = c_0 \quad \text{otherwise}. \]

There is no longer moral hazard: as long as the other entrepreneur works, shirking yields probability \( \Delta p \) that the other entrepreneur succeeds while this entrepreneur fails (recall that the two technologies are perfectly correlated), resulting in a large (infinite) punishment. If

\[ D > M - \frac{B}{\Delta p}, \]

then product-market competition facilitates financing! Correlation enables benchmarking provided that both firms secure financing.

**Exercise 3.5 (continuous investment and decreasing returns to scale).** (i) The incentive constraint is, as in the model of Section 3.4,

\[(\Delta p)R_b \geq BI. \quad (IC)\]

The pledgeable income is

\[ p_m \left[ R(I) - \min_{[I_c]} R_b \right] = p_m \left[ R(I) - \frac{BI}{\Delta p} \right]. \]

Thus the entrepreneur selects \( I \) to solve

\[
\max NPV = \max U_b = p_m R(I) - I \\
\text{s.t.} \quad p_m \left[ R(I) - \frac{BI}{\Delta p} \right] \geq I - A. \quad (BB)
\]

Clearly, if \( I = I^* \) satisfies (BB) (\( A \) is high), then it solves this program. The shadow price of the budget constraint is then \( \mu = 0 \).

So suppose \( A \) is small enough that (BB) is not satisfied at \( I = I^* \). Then \( I \) is determined by (BB) (since the objective function is concave). In that region, by the envelope theorem

\[
\frac{dU_b}{dA} = v = \left[ p_m R'(I) - 1 \right] \frac{dI}{dA} \]

\[ = \frac{p_m R'/\Delta p}{(p_m R' - 1) - 1}. \]

So \( v \) decreases with \( A \).

**Exercise 3.6 (renegotiation and debt forgiveness).** (i) Suppose that \( R_b < BI/(\Delta p) \).

In the absence of renegotiation, the entrepreneur will shirk and obtain utility

\[ BI + p_1 R_b, \]

and the lender’s expected revenue is

\[ p_l (RI - R_b). \]

Renegotiation must be mutually advantageous. So a necessary condition for renegotiation is that total surplus increases. A renegotiation toward a stake \( \hat{R}_b < BI/(\Delta p) \) does not affect surplus and thus is a mere redistribution of wealth between the investors and the entrepreneur. So renegotiation, if it happens, must yield stake \( \hat{R}_b \geq BI/(\Delta p) \) for the entrepreneur. It constitutes a Pareto-improvement if the following two conditions are satisfied:

\[ p_m \hat{R}_b \geq BI + p_1 R_b \]

and

\[ p_m (RI - \hat{R}_b) \geq p_l (RI - R_b). \]

The second inequality, together with the incentive constraint, implies that

\[ (\Delta p)RI - p_m \frac{BI}{\Delta p} + p_1 R_b \geq 0. \]

Conversely, if this condition is satisfied, then the two parties can find an \( \hat{R}_b \) that makes them both better off.

Note that the standard assumptions

\[ p_m \left[ RI - \frac{BI}{\Delta p} \right] \geq I - A \]

and

\[ I \geq p_1 RI + BI \]

imply that

\[ (\Delta p)RI - p_m \frac{BI}{\Delta p} + A - BI \geq 0. \]

So, if \( A > BI \) and \( R_b \) is small enough, the condition for renegotiation may not be satisfied.

(ii) The “project” consists in creating incentives for the entrepreneur. It creates NPV equal to \((\Delta p)RI\), does not involve any new investment, and the entrepreneur can bring an amount of money \( A \equiv p_1 R_b \) that is the forgone expected income.
For this fictitious project, the pledgeable income is
\[(\Delta p)RI - p_HB_I \Delta p\]
and the investors’ outlay is
\[0 - \hat{A}.
Hence, it is “financed” if and only if
\[(\Delta p)RI - p_HB_I \Delta p \geq -p_LR_0.
Exercise 3.7 (strategic leverage). (i) The NPV, if the project is funded, is
\[(p_H + \tau)R - I(\tau).
So, if \(A \geq A^*, \tau = \tau^*\).
• For \(A < A^*, \) the pledgeable income can be increased by reducing \(\tau\) below \(\tau^*:\)
\[
\frac{d}{d\tau} \left[(p_H + \tau)\left(R - \frac{B}{\Delta p}\right) - I(\tau) - A\right] = R - \frac{B}{\Delta p} - I'(\tau).
\]
Let \(\tau^{**}\) be defined by
\[I'(\tau^{**}) = R - \frac{B}{\Delta p}.
The pledgeable income decreases with \(\tau\) for \(\tau \geq \tau^{**}\). The borrower can raise funds if and only if \(A > A^{**}\), with
\[(p_H + \tau^{**})\left(R - \frac{B}{\Delta p}\right) = I(\tau^{**}) - A^{**}.
The quality of investment increases with \(A\) (for \(A > A^{**}\)) and is flat beyond \(A^*\). For \(A \in [A^{**}, A^*]\),
\[\frac{p_H + \tau(A)}{\Delta p} - \frac{B}{\Delta p} = I(\tau(A)) - A.
For \(A \geq A^*\), \(\tau(A) = \tau^*\).
(ii) Define \(\hat{\tau}\) by
\[I'(\hat{\tau}) = [1 - (p_H + \hat{\tau})]R.
(\(\hat{\tau}\) maximizes a firm’s NPV given that the other firm’s choice is \(\tilde{\tau}\).) Borrower \(i\)’s incentive compatibility constraint is \((\Delta p) (1 - q_j) R_0 \geq B\), where \(R_0\) is her reward in the case of income \(R\). So the pledgeable income is
\[(p_H + \tau)\left[1 - q_j\right]R - \frac{B}{\Delta p} \geq I(\tilde{\tau}) - A.
(\(\hat{\tau}, \tilde{\tau}\)) is a symmetric Nash equilibrium if and only if
\[(p_H + \hat{\tau})\left[1 - (p_H + \hat{\tau})\right]R - \frac{B}{\Delta p} \geq I(\tilde{\tau}) - A.
This equation yields \(\hat{A}.
• “Natural monopoly case.” Let \(\tau(A)\) be defined as in subquestion (i). Consider a candidate equilibrium in which borrower 1 selects \(\tau(A)\) and borrower 2 does not raise funds. That is,
\[A \leq \min_{\tau} \left\{I(\tau) - \left[p_H + \tau\right]\left[1 - (p_H + \tau(A))\right]R - \frac{B}{\Delta p} \right\}.
(iii) \(\{\hat{q}, \tilde{q}\}\) is a symmetric Nash equilibrium for \(A = \hat{A}.
• By choosing \(q_1 = \hat{q} + \epsilon\), borrower 1 deters entry by borrower 2.
Exercise 3.8 (equity multiplier and active monitoring). (i) See Section 3.4.
(ii) Suppose that monitoring at level \(c\) is to be induced. Two incentive compatibility constraints must be satisfied:
\[(\Delta p)R_m \geq cI \quad \text{and} \quad (\Delta p)R_0 \geq b(c)I.
Because there is no scarcity of monitoring capital, the monitor contributes \(I_m\) to the project and breaks even:
\[I_m = p_H R_m - cI = p_H \frac{c}{\Delta p} I - cI.
The equity multiplier, \(k\), is given by
\[p_H \left[R - \frac{b(c) + c}{\Delta p}\right] I = I - A - I_m\]
or
\[p_H \left[R - \frac{b(c) + c}{\Delta p}\right] = I - \frac{c}{\Delta p} I - I_m,
that is,
\[I = k(c)A,
where
\[k(c) = \frac{1}{1 + c - p_H (R - b(c)/\Delta p)} = \frac{1}{1 - \rho_0 + c + (p_H/\Delta p)[b(c) - B]}.
The project’s NPV (which includes the monitoring cost) is equal to
\[\rho_1 I - cI = (\rho_1 - 1 - c) k(c) A.
The borrower maximizes \((\rho_1 - 1 - c) k(c)\) since the other parties receive zero utility and she therefore receives the project’s NPV.
Exercise 3.9 (concave private benefit). (i) Suppose that the NPV per unit of investment is positive:
\[p_H R > 1
(otherwise there is no investment).
The entrepreneur's utility is equal to the NPV,

\[ U_{b} = (p_{H}R - 1)I, \]

and so the entrepreneur chooses the highest investment that is consistent with the investors' breakeven constraint

\[ p_{H}\left(\frac{R - B(I)}{\Delta p}\right) = I - A. \]

Because \( \lim_{\Delta p \to 0} B'(I) = B \) and \( p_{H}(R - (B/\Delta p)) < 1 \), this upper limit indeed exists.

(i) The shadow price is given by

\[ v = \frac{dU_{b}}{dA} = (p_{H}R - 1)\frac{dI}{dA}, \]

or

\[ v = \frac{1}{(p_{H}B'(I)/(p_{H}R - 1)) - 1}. \]

Hence \( v \) increases with \( A \) (since \( B'' < 0 \) and \( dI/dA > 0 \)).

**Exercise 3.10 (congruence, pledgeable income, and power of incentive scheme).** (i) Either \( R_{0} \geq B/(\Delta p) \) and the entrepreneur always behaves well. The NPV is

\[ \text{NPV} = p_{H}R - I + (1 - x)B, \]

and the financing condition

\[ p_{H}\left(\frac{R - B}{\Delta p}\right) \geq I - A. \quad (1) \]

Or \( R_{0} < B/(\Delta p) \). The NPV is then

\[ \text{NPV} = x(p_{H}R + B) + (1 - x)(p_{H}R + B) - I < \text{NPV}', \]

and the financing condition \( (R_{0} = 0 \) then maximizes the pledgeable income) is

\[ [xp_{H} + (1 - x)p_{H}]R \geq I - A. \quad (2) \]

The pledgeable income is increased only if \( x \) is sufficiently low. The high-powered incentive scheme is always preferable if (1) is satisfied; otherwise, the parties may content themselves with a low-powered scheme (provided (2) is satisfied).

(ii) Suppose that the menu offers \( (R_{b}^{k}, R_{b}^{f}) \) when interests are divergent and \( (R_{b}^{k}, R_{b}^{f}) \) when interests are aligned. The state (divergent/congruent) is not observed by the investors and so this menu must be incentive compatible (the entrepreneur must indeed prefer the incentive scheme tailored to the state of nature she faces).

The interesting case is when the incentive scheme in the divergent state is incentive compatible \( ((\Delta p) \times (R_{b}^{k} - R_{0}) \geq B; \) otherwise, setting all rewards equal to 0 is obviously optimal).

In the congruent state, the entrepreneur must not pretend interests are divergent, and so

\[ p_{H}R_{b}^{k} + (1 - p_{H})R_{b}^{f} \geq p_{H}R_{b}^{k} + (1 - p_{H})R_{b}^{f}. \]

So one might as well take \( R_{b}^{k} = R_{b}^{f} = R_{b}^{f} \). This choice yields incentive compatibility in the congruent state and maximizes the pledgeable income.

**Exercise 3.11 (retained-earnings benefit).** (i) Let us assume away any discounting for notational simplicity. The assumption on \( B^{2} \) implies that retained earnings are always needed to finance the second project, as

\[ p_{H}^{2}\left(\frac{R^{2} - B^{2}}{\Delta p^{2}}\right) \leq I^{2} \quad \text{for all } B^{2}. \]

The borrower's utility is, as a function of date-1 earnings \( R_{b}^{1} \),

\[ U_{b}(R_{b}^{1}) = \begin{cases} R_{b}^{1} & \text{if the second project is not financed}, \\ R_{b}^{1} + \text{NPV}^{1} & \text{otherwise}, \end{cases} \]

where

\[ \text{NPV}^{1} = p_{H}^{2}R^{2} - I^{2} \]

is independent of \( B^{2} \).

Let \( R_{b}^{k}(B^{2}) \) denote the required level of retained earnings when the date-2 private benefit turns out to be \( B^{2} \):

\[ p_{H}^{k}\left(\frac{R^{2} - B^{2}}{\Delta p^{2}}\right) = I^{2} - R_{b}^{k}(B^{2}). \]

This equation also defines a threshold \( B^{2}(R_{b}^{k}) \).

Thus, the expected utility is

\[ E[U_{b}(R_{b}^{1})] = R_{b}^{1} + f(B^{2}(R_{b}^{k}))\text{[NPV}^{2}\text{]}. \]

The shadow value of retained earnings is therefore

\[ \mu = \frac{dE[U_{b}(R_{b}^{1})]}{dR_{b}^{1}} = 1 + f(B^{2}(R_{b}^{k}))\left[\frac{d\hat{B}^{2}}{dR_{b}^{1}}\right][\text{NPV}^{2}]. \]

(ii) The date-1 incentive compatibility constraint is

\[ (\Delta p^{1})\left[R_{b}^{1} + f(B^{2}(R_{b}^{k}))\text{[NPV}^{2}\text{]}ight] \geq B^{1}. \]

The pledgeable income,

\[ p_{H}^{1}\left[R^{1} - \min_{R_{b}^{1}} R_{b}^{1}\right], \]
is therefore larger than in the absence of a second project. It is therefore more likely to exceed \( I^1 - A^1 \), where \( A^1 \) is the entrepreneur’s initial wealth.

**Exercise 3.12 (investor risk aversion and risk premia).** (i) This condition says that the risk-free rate is normalized at 0. In other words, investors are willing to lend 1 unit at date 0 against a safe return of 1 unit at date 1.

(ii) With a competitive capital market, the financing condition becomes

\[
p_H q_S R_1 \geq I - A.
\]

With a risk-neutral entrepreneur, the incentive compatibility constraint is unchanged:

\[
(\Delta p) R_0 \geq B.
\]

Thus, enough pledgeable income can be harnessed provided that

\[
p_H \left[ R - \frac{B}{\Delta p} \right] \geq \frac{I - A}{q_S}. \tag{1}
\]

Comparing condition (1) with condition (3.3) in Chapter 3, we conclude that obtaining financing is easier for a countercyclical firm than for a procyclical one, ceteris paribus.

(iii) The entrepreneur maximizes her utility subject to the investors’ being willing to lend

\[
\max_{\{q_S, q_F\}} \left[ p_H R_S^q + (1 - p_H) R_F^q \right] \tag{2}
\]

subject to these two constraints.

\[
q_S p_H (R - R_S^q) + q_F (1 - p_H) (-R_F^q) \geq I - A, \tag{3}
\]

\[
(\Delta p) (R_S^q - R_F^q) \geq B, \tag{4}
\]

\[
R_S^q \geq 0. \tag{5}
\]

Letting \( \mu_1 \), \( \mu_2 \), and \( \mu_3 \) denote the shadow prices of the constraints, the first-order conditions are

\[
p_H [1 - \mu_1 q_S] + \mu_2 (\Delta p) = 0 \tag{6}
\]

and

\[
(1 - p_H) [1 - \mu_1 q_F] - \mu_2 (\Delta p) + \mu_3 = 0. \tag{7}
\]

- First, note that for \( q_S \neq q_F \) at least one of constraints (4) and (5) must be binding: if \( \mu_2 = \mu_3 = 0 \), (6) and (7) cannot be simultaneously satisfied.
- Conversely, (4) and (5) cannot be simultaneously binding, except when condition (1) is satisfied with exact equality.

- Suppose that constraint (4) is not binding (\( \mu_2 = 0 \)), which, from what has gone before, implies that \( R_F^q = 0 \). Then \( \mu_1 = 1/q_S \), and (7) can be satisfied only if

\[
q_F > q_S.
\]

- In contrast, suppose that constraint (5) is not binding (\( \mu_3 = 0 \)). Constraints (6) and (7) taken together imply that

\[
q_5 > q_F.
\]

To sum up, the maximum punishment result (\( R_F^q = 0 \)) carries over to procyclical firms, because the incentive effect compounds with the “marginal rates of substitution” effect (the investors value income in the case of failure relatively more compared with the entrepreneur). But it does not in general hold for countercyclical firms. Then the investors care more about the payoff in the case of success, and the entrepreneur should keep marginal incentives equal to \( B/\Delta p \) and select \( R_F^q > 0 \) (since the firm’s income is equal to 0 in the case of failure, this requires the firm to hoard some claim at date 0 so as to be able to pay the entrepreneur even in the case of failure).

Entrepreneurial risk aversion changes the incentive constraint (4) and the objective function (2). It may be the case that \( R_F^q > 0 \) even for a procyclical firm.

**Exercise 3.13 (lender market power).** (i) If \( A \geq I \), then the “borrower” does not need the lender and just obtains the NPV (\( U_b = V \)). So let us assume that \( A < I \). The lender must respect two constraints. First, the standard incentive compatibility constraint:

\[
(\Delta p) R_0 \geq B. \tag{IC_b}
\]

Second, her net utility must be nonnegative:

\[
U_b = p_H R_b - A \geq 0. \tag{IR_b}
\]

The lender maximizes

\[
U_l = p_H [R - R_b] - (I - A)
\]

subject to these two constraints.

Let us first ignore (IC_b). The lender sets \( R_b = A/p_H \) and thus

\[
U_b = 0.
\]

The lender appropriates the entire surplus (\( U_l = V \)) as long as \( R_b = A/p_H \) satisfies the incentive
constraint, or
\[(\Delta p) \frac{A}{p_H} \geq B \iff A \geq \bar{A}.
\]
For \(A \in [\bar{A}, \bar{\bar{A}}]\), the lender cannot capture the borrower’s surplus without violating the incentive constraint; then the borrower’s net utility
\[U_b = p_H \frac{B}{\Delta p} - A\]
is decreasing in \(A\).

Lastly, the lender is willing to lend as long as
\[U_l = V - U_b \geq 0 \text{ or } A \geq \bar{A}.
\]
The borrower’s net utility is as represented in Figure 1.

The borrower is “better off” (from the relationship) if she is either very rich (she does not need the lender) or poor (she cannot be expropriated by the lender)—although, of course, not too poor!

(ii) The lender solves
\[
\begin{align*}
\max U_l &= p_H (RI - R_b) - (I - A) \\
\text{s.t.} & \\
(\Delta p)R_b & \geq BI, \quad (\text{IC}_b) \\
p_H R_b & \geq A. \quad (\text{IR}_b)
\end{align*}
\]
If \((\text{IC}_b)\) were not binding, \((\text{IR}_b)\) would have to be binding \((U_l\) is decreasing in \(R_b)\) and
\[U_l = (p_H R - 1)I\]
would yield \(I = \infty\), violating \((\text{IC}_b)\), a contradiction.

If \((\text{IR}_b)\) were not binding, \((\text{IC}_b)\) would have to be binding, and
\[U_l = \left(p_H \left(R - \frac{B}{\Delta p}\right) - 1\right)I + A,
\]
and so \(I = 0 = R_b\), contradicting \((\text{IR}_b)\).

Hence, the two constraints are binding, and so
\[I = \frac{1}{p_H B/\Delta p} A.
\]
Recall that, in the presence of a competitive market,
\[I^* = \frac{1 - p_H (R - B/\Delta p)}{1 - p_H (R - B/\Delta p)} A,
\]
and so
\[I < I^*.
\]
With variable-size investment, lender market power leads to a contraction of investment.

Exercise 3.14 (liquidation incentives). (i) Technically, the realization of \(\gamma\) is a “sufficient statistic” for inferring the effort chosen by the entrepreneur. Rewarding the entrepreneur as a function not only of \(\gamma\), but also of the realization of the final profit amounts to introducing into the incentive scheme noise over which the entrepreneur has no control. (We leave it to the reader to start with a general incentive scheme and then show that without loss of generality the reward can be made contingent on \(\gamma\) only.)

Second, it is optimal to liquidate if and only if \(\gamma = \hat{\gamma}\). Hence, one can define expected profits:
\[R^S \equiv \bar{\gamma} R \quad \text{and} \quad R^F \equiv L,
\]
where “success” (“S”) now refers to a good signal, “failure” (“F”) to a bad signal, and \(R^S\) and \(R^F\) denote the associated continuation profits.

We are now in a position to apply the analysis of Section 3.2. Let \(R^S_b\) denote the entrepreneur’s reward in the case of a good signal \((\gamma = \hat{\gamma})\) and 0 that in the case of a bad signal. Incentive compatibility requires that
\[(\Delta p)R^S_b \geq B.
\]
The NPV is
\[U_b = p_H \hat{\gamma} R + (1 - p_H)L - I,
\]
and the pledgeable income is
\[\mathcal{P} = p_H \hat{\gamma} R + (1 - p_H)L - p_H \frac{B}{\Delta p}.
\]
Financing is then feasible provided that \(A \geq \bar{A}\), where
\[p_H \left(\hat{\gamma} R - \frac{B}{\Delta p}\right) + (1 - p_H)L = I - \bar{A}.
\]
(ii) Truth telling by the entrepreneur requires that
\[ \gamma R_b > L_b > \gamma R_b. \]

The entrepreneur’s other incentive compatibility constraint (that relative to effort) is then
\[ (\Delta p)(\gamma R_b - L_b) \geq B. \]

The investors’ payoff is then
\[ p H\gamma (R - R_b) + (1 - p H)(L - L_b). \]

As expected, it is highest when \( L_b \) and \( R_b \) are as small as is consistent with the incentive constraints:
\[ L_b = \gamma R_b \quad \text{and} \quad (\Delta p)(\gamma R_b - L_b) = B. \]

And so the pledgeable income is
\[ p H\gamma (R - R_b) + (1 - p H)(L - L_b) \]
for these values of \( L_b \) and \( R_b \). Simple computations show that the financing condition amounts to
\[ p H\gamma R + (1 - p H)L - [p H\gamma + (1 - p H)\gamma] \frac{B}{(\Delta p)(\gamma y)} \geq I - A \]
or
\[ A \geq A + \gamma \frac{B}{(\Delta p)(\gamma y)}. \]

Exercise 3.15 (project riskiness and credit rationing). The managerial minimum reward (consistent with incentive compatibility) is the same for both variants:
\[ \frac{B}{\Delta p A} = \frac{B}{\Delta p B}. \]

And so the investors’ breakeven condition can be written (with obvious notation) as
\[ I - A \leq p H\left( R^A - \frac{B}{\Delta p} \right) \]
for variant A
and
\[ I - A \leq p H\left( R^B - \frac{B}{\Delta p} \right) \]
for variant B.

Because \( p H^A > p H^B \), the safer project (project A) is financed for a smaller range of cash on hand \( A \). That is, the safe project is more prone to credit rationing. Intuitively, the nonpledgeable income is higher for a safe project, since the entrepreneur has a higher chance to be successful and thus to receive the incentive payment \( B/\Delta p \).

This, however, assumes that good behavior is needed for funding either variant. Let us relax this assumption. Good behavior boosts the pledgeable income (as well as the NPV, for that matter) more when the payoff in the case of success is high, that is, for the risky project. Thus, suppose that the following conditions hold:
\[ I - A > p H^A\left( R^B - \frac{B}{\Delta p} \right), \]
\[ I - A > p H^B, \]
\[ I - A \leq p H^A R^A, \]
\[ I < p H^B R^A + B. \]

The first two inequalities state that the risky variant cannot receive financing whether good behavior or misbehavior is induced by the managerial compensation scheme (note, for example, that the second inequality is automatically satisfied if \( p H^B \) is close to its lowest feasible value \( \Delta p \)). The third states that the risky project generates enough pledgeable income when the cash-flow rights are allocated entirely to investors. Finally, the fourth inequality guarantees that the safe project’s NPV is positive.

To check that these inequalities are not inconsistent, assume, for example, that \( A = 0 \) and \( p H^A R^A = I \) (or just above); then
\[ p H^B R^B = \frac{p H^B}{p H^A} I < I. \]

Lastly, for \( B \) large enough, the first inequality is satisfied. We conclude that the risky project may be more prone to credit rationing if high-powered incentives are not necessarily called for.

Exercise 4.15 investigates a different notion of project risk, in which a safe project yields a higher liquidation value and a lower long-term payoff and is less prone to credit rationing than a risky project.

Exercise 3.16 (scale versus riskiness tradeoff). The risky project’s NPV is
\[ U^r_b = (x\rho_1 - 1)I. \]

The investors’ breakeven condition can be written as
\[ x\rho_0 I = I - A. \]

And so
\[ U^r_b = \frac{x\rho_1 - 1}{1 - x\rho_0} A = \frac{\rho_1 - 1/x}{1/x - \rho_0} A. \]

Note that this is the same formula as obtained in Section 3.4.2, except that the expected cost of bringing
1 unit of investment to completion is $1/x$ rather than 1.

Turn now to the safe project. The NPV is then

$$U^s_B = (\rho_1 - X)I,$$

and the investors’ breakeven condition is

$$\rho_0 I = XI - A.$$

Hence,

$$U^s_B = \frac{\rho_1 - X}{X - \rho_0} A.$$

The expected cost of bringing 1 unit of investment to completion is now $X$.

Thus the safe project is strictly preferred to the risky one if and only if

$$X < \frac{1}{X} \quad \text{or} \quad xX < 1.$$

**Exercise 3.17 (competitive product market interactions).** The representative firm’s investment must satisfy

$$p_{\Pi} \left[ PR - \frac{B}{\Delta p} \right] i \geq i - A, \quad (1)$$

since the manager’s reward in the case of success, $R_B$, must satisfy

$$(\Delta p) R_B \geq Bi.$$

The representative entrepreneur wants to borrow up to her borrowing capacity as long as the NPV per unit of investment is positive:

$$p_{\Pi} PR \geq 1. \quad (2)$$

In equilibrium $i = I$ and $P = P(p_{\Pi} RI)$. Let $I^*$ (the optimal level from an individual firm’s viewpoint) be given by

$$p_{\Pi} R P^* = 1 \quad \text{and} \quad P^* = P(p_{\Pi} RI^*).$$

Two cases must therefore be considered, depending on whether $\Delta$ is (a) large or (b) small:

(a) if

$$p_{\Pi} [P^* R - B/\Delta p] I^* \geq I^* - A,$$

then the borrowing constraint is not binding and $I = I^*$;

(b) if

$$p_{\Pi} [P^* R - B/\Delta p] I^* < I^* - A,$$

then (1) is binding, and so

$$I = \frac{A}{1 - p_{\Pi} [RP(p_{\Pi} RI) - B/\Delta p]}.$$

**Exercise 3.18 (maximal incentives principle in the fixed-investment model).** Recall that, because the investors break even, the entrepreneur’s expected payoff when the project is financed is nothing but the project’s NPV. The entrepreneur’s expected payoff is therefore independent of the way the investment is financed. The financing structure just serves the purpose of guaranteeing good behavior by the entrepreneur. Let $R^S_F$ and $R^F_F$ denote the (nonnegative) rewards of the borrower in the cases of success ($R^S$) and failure ($R^F$), respectively. The incentive constraint can be written as

$$(\Delta p) (R^S_F - R^F_F) \geq B. \quad (IC^B)$$

This constraint implies that setting $R^S_F$ at its minimum level (0) provides the entrepreneur with maximal incentives. So, the incentive constraint becomes

$$(\Delta p) R^S_F \geq B.$$

The pledgeable income is equal to total expected income minus the borrower’s minimum stake consistent with incentives to behave:

$$p_{\Pi} R^S + (1 - p_{\Pi}) R^F - p_{\Pi} \frac{B}{\Delta p} = p_{\Pi} \left( R - \frac{B}{\Delta p} \right) + R^F.$$

Thus the project is financed if and only if

$$p_{\Pi} \left( R - \frac{B}{\Delta p} \right) \geq I - (A + R^F). \quad (1)$$

As one would expect, the minimum income $R^F$ plays the same role as cash or collateral. It is really part of the borrower’s net worth.

The optimum contract can be implemented through a debt contract: let $D$, $R^S_F < D < R^S$, be defined by

$$p_{\Pi} D + (1 - p_{\Pi}) R^F = I - A. \quad (IR^S)$$

That is, the borrower owes $D$ to the lenders. In the case of failure ($R^F$), the borrower defaults and the lenders receive the firm’s cash, $R^S$. Equation (IR^S) then guarantees that the lenders break even.

In this fixed-investment version of the model, the debt contract is, however, in general not uniquely optimal: a small reward $R^S_F > 0$ for the borrower in the case of failure would still be consistent with (IC^B) and (IR^S) as long as condition (IR^S) is satisfied with strict inequality. By contrast, the standard debt contract is uniquely optimal in the variable-investment version of the model as it maximizes the borrower’s borrowing capacity (see Section 3.4.3).
Exercise 3.19 (balanced-budget investment subsidy and profit tax). The total investment subsidy is \( sI \) and the profit tax \( tRI \). Budget balance then requires
\[
p_{II} tRI = sI.
\]
The amount of income that is pledgeable to investors is
\[
p_{II} \left[ R - tR - \frac{B}{\Delta p} \right] I,
\]
and so the breakeven constraint is
\[
p_{II} \left[ (1 - t)R - \frac{B}{\Delta p} \right] I = (1 - s)I - A.
\]
Adding up the two equalities yields
\[
p_{II} \left[ R - \frac{B}{\Delta p} \right] I = I - A
\]
or
\[
I = \frac{A}{1 - \rho_0}.
\]
Finally, the entrepreneur receives the NPV, \((\rho_1 - 1)I\), since both the investors and the government make no surplus.

Exercise 3.20 (variable effort, the marginal value of net worth, and the pooling of equity). (i) Let \( R_b \) denote the entrepreneur’s reward in the case of success. The entrepreneur is residual claimant when she does not need to borrow:
\[
R_b = R.
\]
And so she maximizes
\[
\max_p \{ pR - \frac{1}{2} p^2 - I \}
\]
yielding
\[
p = R
\]
and
\[
U_b = \frac{1}{2} R^2 - I > 0.
\]
(ii) More generally,
\[
p = R_b.
\]
The investors’ breakeven condition is
\[
p(R - R_b) \geq I - A
\]
or
\[
R_b(R - R_b) \geq I - A.
\]
Only the region \( R_b \geq \frac{1}{2} R \) is relevant: were \( R_b \) to be smaller than \( \frac{1}{2} R \), then \( \hat{R}_b = R - R_b \) would yield the same pledgeable income, but a higher utility to the entrepreneur.

The highest pledgeable income is obtained when \( R_b = \frac{1}{2} R \). Thus a necessary condition for financing is that \( A \geq A_1 \), where
\[
\frac{1}{2} R^2 = I - A_1.
\]
It must further be the case that the project’s NPV be positive. That is, for the (maximum) value of \( R_b \) satisfying
\[
R_b(R - R_b) = I - A,
\]
then
\[
U_b = R_bR - I - \frac{1}{2} R_b^2 - A \geq 0.
\]
So, using the breakeven constraint to rewrite the NPV, let
\[
U_b = V(A) = \max_{\{R_b\}} \{ R_bR - \frac{1}{2} R_b^2 - I \}
\]
s.t.
\[
R_b(R - R_b) \geq I - A.
\]
This yields the shadow price of equity, \( V'(A) \):
\[
V'(A) = [R - R_b(A)] \left[ \frac{dR_b(A)}{dA} \right] > 0,
\]
where \( R_b(A) \) is given by the investors’ breakeven condition. For \( A > I \), we can define \( V(A) = \left( \frac{1}{2} R^2 \right) - I \). And so \( V'(A) = 0 \) (note that we discuss net utilities, so the no-agency-cost benchmark is a shadow price of cash on hand equal to 0; this benchmark is equal to 1 for gross utilities). When \( A > I \), the entrepreneur is residual claimant and exerts the socially optimal effort. For \( A < I \), \( V'(A) > 0 \), but \( V'(I) = 0 \): a local increase in the entrepreneur’s compensation just below \( R \) has only a second-order effect.

Furthermore,
\[
V''(A) < 0.
\]
Let \( A_2 < I \) satisfy
\[
V(A_2) = 0.
\]
Then
\[
\bar{A} = \max \{ A_1, A_2 \}.
\]
(iii) Let \( I \equiv I_L \). That is, we fix \( I_L \) and the corresponding \( V(\cdot) \) function. In the absence of an ex ante arrangement between the two entrepreneurs, each receives a net utility:
\[
\frac{1}{2} V(A)
\]
(the gross utility is \( \frac{1}{2} (V(A) + A) \)). For, because \( R_b R - \frac{1}{2} R_b^2 \) is concave, it is optimal for both to have
the same reward if they both invest. Thus the strategy consisting in (a) pooling cash on hand, (b) investing, and (c) setting identical reward schemes and investment, yields, for each entrepreneur,

\[ V(A - \frac{1}{2}(I_H - I_l)) \]

Alternatively, the two can pool resources but only the low-investment-cost project will be funded. The expected net utility of each is then

\[ \frac{1}{2} V(\max(2A, I_l)) \]

since, if \( 2A \geq I_l \), the low-investment-cost entrepreneur is residual claimant.

Note that

\[ \frac{1}{2} V(\max(2A, I_l)) > \frac{1}{2} V(A) \]

so pooling is always optimal.

The lucky entrepreneur cross-subsidizes the unlucky entrepreneur if and only if

\[ V(A - \frac{1}{2}(I_H - I_l)) > \frac{1}{2} V(\max(2A, I_l)) \]

The unlucky entrepreneur cross-subsidizes the lucky one if this inequality is violated. Finally, because

\[ V(A) > \frac{1}{2} V(\max(2A, I_l)) \]

the cross-subsidization is from the lucky to the unlucky for \( I_H \) below some threshold.

**Exercise 3.21 (hedging or gambling on net worth?).**

(i) Letting \( R_b \) denote the entrepreneur's stake in success (and 0 in failure), the incentive compatibility constraint is

\[ (\Delta p) R_b \geq B. \]

Financing is feasible if and only if

\[ \frac{p_H}{\Delta p} \left( R - \frac{B}{\Delta p} \right) \geq I - A. \]

The entrepreneur's date-1 gross utility is

\[ [p_H R - I] + [A - \bar{A}] \quad \text{if } A \geq \bar{A} \]

and

\[ A \quad \text{if } A < \bar{A}. \]

- If \( A_0 \geq \bar{A} \), the entrepreneur's date-0 expected gross utility is

\[ U^g_b = [p_H R - I] + A_0 \]

if she hedges.

By contrast, and letting \( F(\varepsilon) \) denote the cumulative distribution of \( \varepsilon \), her expected utility becomes

\[ U^g_b = [1 - F(\bar{A} - A_0)] [\{p_H R - I\} + m^+(\bar{A})] + F(\bar{A} - A_0) m^-(\bar{A}) < U^g_b, \]

where

\[ m^+(\bar{A}) = E[A \mid A \geq \bar{A}], \]

\[ m^-(\bar{A}) = E[A \mid A < \bar{A}], \]

\[ [1 - F(\bar{A} - A_0)] m^+(\bar{A}) + F(\bar{A} - A_0) m^-(\bar{A}) = A_0. \]

- If \( A_0 < \bar{A} \), then

\[ U^g_b = A_0 < U^g_b. \]

(ii) Ex post the entrepreneur chooses \( p \) so as to solve

\[ \max_{(p)} \left\{ p R_b - \frac{1}{2} p^2 \right\}, \]

and so

\[ p = R_b. \]

The pledgeable income is

\[ \mathcal{P} = R_b (R - R_b) \]

and the NPV, i.e., the entrepreneur's expected net utility, in the case of financing is

\[ U_b = R_b R - I. \]

Without loss of generality, assume that \( R_b \geq \frac{1}{2} R \) (if \( R_b < \frac{1}{2} R, \bar{R}_b = R - R_b \) yields the same \( \mathcal{P} \) and a higher \( U_b \)).

Assume that \( I - A_0 < \frac{1}{2} R^2 \). This condition means that the entrepreneur can receive funding if she hedges (the highest pledgeable income is reached for \( R_b = \frac{1}{2} R \)). She also receives funding even in the absence of hedging provided that the support of \( \varepsilon \) is small enough (the lower bound is smaller than \( \frac{1}{2} R^2 - (I - A_0) \) in absolute value). Let

\[ V(A) = R_b(A) R - I, \]

where \( R_b(A) \) is the largest root of

\[ R_b(R - R_b) = I - A. \]

One has

\[ \frac{dV}{dA} = R \frac{dR_b}{dA} = \frac{R}{2R_b(A) - R} > 0 \]

and

\[ \frac{d^2V}{dA^2} = - \frac{2R}{(2R_b(A) - R)^2} \frac{dR_b}{dA} < 0. \]
Hence, $V$ is concave and so

$$V(A_0) > E[V(A_0 + \epsilon)].$$

The entrepreneur is better off hedging.

(iii) The investment is given by the investors' breakeven condition:

$$p_H \left[ RI - \frac{B(I)}{\Delta p} \right] = I - A.$$

This yields investment $I(A)$, with $I' > 0$ and $I'' < 0$ if $B'' > 0$, $I'' > 0$ if $B'' < 0$. The ex ante utility is

$$U^g(b_S) = (p_H R - 1) E[I(A_0 + \epsilon)]$$

in the absence of hedging. And so $U^g(b_S) > U^g(b) = U^g(b)$ if $B'' > 0$ and $U^g(b_S) < U^g(b) = U^g(b)$ if $B'' < 0$.

(iv) When the profit is unobservable by investors, there is no pledgeable income and so

$$I = A.$$  

And so

$$U^g(b_S) = R(A_0) \quad \text{and} \quad U^g(b) = E[R(A_0 + \epsilon)] < R(A_0)$$

since $R$ is concave.

(v) Quite generally, in the absence of hedging the realization of $\epsilon$ generates a distribution $G(I)$ over investment levels $I = I(\epsilon)$ and over cash used in the project $\mathcal{A}(\epsilon) \leq A_0 + \epsilon$ such that

$$P[I(\epsilon)] \geq I(\epsilon) - \mathcal{A}(\epsilon),$$

where $P$ is the expected gross utility. And so

$$E[P[I(\epsilon)] \geq E[I] - A_0.$$  

Drawing $I$ from distribution $G(\cdot)$ regardless of the realization of $\epsilon$ and keeping $A_0 - E[\mathcal{A}(\epsilon)]$ makes the entrepreneur as well off.

In general, the entrepreneur can do strictly better by insulating her investment from the realization of $\epsilon$ (in the constant-returns-to-scale model of Section 3.4, though, she is indifferent between hedging and gambling).

Consider, for example, the case $A_0 < \bar{A}$ in subquestion (i). Then we know that gambling is optimal. The probability that the project is financed is

$$1 - F(\bar{A} - A_0) \quad \text{and} \quad [1 - F(\bar{A} - A_0)] \bar{A} < A_0.$$  

This last inequality states that there is almost surely “unused cash”: either $A_0 + \epsilon < \bar{A}$ and then there is no investment, or $A_0 + \epsilon > \bar{A}$ and then there is “excess cash” $[A_0 + \epsilon - \bar{A}]$.

Consider therefore the date-0 contract in which the date-1 income $r = A_0 + \epsilon$ is pledged to investors. The probability of funding is then $X$, which allows investors to break even:

$$A_0 = X \left[ I - p_H \left( R - \frac{B}{\Delta p} \right) \right] = X \bar{A}.$$  

Clearly,

$$X > 1 - F(\bar{A} - A_0),$$

and so the entrepreneur’s date-0 expected gross utility has increased from

$$[1 - F(\bar{A} - A_0)](p_H R - I) + A_0$$

to

$$X(p_H R - I) + A_0.$$  

Of course, this is not quite a fair comparison, since we have allowed random funding under hedging and not under gambling. But, because there is excess cash in states of nature in which $A > \bar{A}$, the same result would hold even if we allowed for random funding under gambling: when $A < \bar{A}$, the project could be funded with probability $x(A) = A/\bar{A}$. The total probability of funding under gambling would then be

$$\int_{\bar{A} - A_0}^{\bar{A}} \frac{A \, dF(A - A_0)}{\bar{A}} + \left[ 1 - F(\bar{A} - A_0) \right]$$

< $\int_{\bar{A} - A_0}^{\bar{A}} \frac{A \, dF(A - A_0)}{\bar{A}} + \int_{\bar{A} - A_0}^{\infty} \frac{A \, dF(A - A_0)}{\bar{A}} = \frac{A_0}{\bar{A}}.$$

For more on liquidity and risk management, see Chapter 5.

**Exercise 4.1 (maintenance of collateral and asset depletion just before distress).** (i) When $c = 0$ (no moral hazard on maintenance), the pledgeable income is equal to $(A + \epsilon)$

$$p_H \left( R - \frac{B}{\Delta p} \right) - c.$$  

Consider $c > 0$. First, suppose that the entrepreneur receives $R_0$ in the case of success, and $r_0$ in the case of good maintenance. That is, the two incentives are not linked together. The IC constraints are

$$(\Delta p) R_0 \geq B \quad \text{and} \quad r_0 \geq c.$$

The pledgeable income is $(A + \epsilon)$

$$p_H \left( R - \frac{B}{\Delta p} \right) - c.$$
However, and as in Diamond’s (1984) model (see Section 4.2), it is optimal to link the two incentives. Let us look for conditions that guarantee that the entrepreneur both exerts effort to raise the probability of success and maintains the collateral. We just saw that it is optimal to reward the entrepreneur only if the project is successful and the asset has been maintained. Let $R_b > 0$ denote this reward. There are three potential incentive compatibility constraints:

1. $\{\text{work, maintain}\} \succeq \{\text{shirk, maintain}\}$

   \[ p_H R_b - c \geq p_L R_b - c + B \]

or

\[ (\Delta p)R_b \geq B. \]

2. $\{\text{work, maintain}\} \succeq \{\text{shirk, do not maintain}\}$

   \[ p_H R_b - c \geq B. \]

Note that this second constraint does not bind if the first constraint is satisfied, since by assumption $p_L B / (\Delta p) \geq c$.

3. $\{\text{work, maintain}\} \succeq \{\text{work, do not maintain}\}$

   \[ p_H R_b - c \geq 0. \]

This third constraint is not binding either.

The necessary and sufficient condition for financing is

\[ p_H \left( R - \frac{B}{\Delta p} \right) \geq I - A, \]

and the NPV is

\[ U_b = [p_H R - I] + [A - c]. \]

(ii) The decision over whether to maintain the collateral now depends on the realization of the signal about the eventual outcome of the project. The entrepreneur stops maintaining the asset when learning that the project will fail. When no signal accrues, the conditional probability of success (assuming that the entrepreneur has chosen probability of success $p \in \{p_L, p_H\}$) is

\[ \frac{p}{p + (1 - p)(1 - \xi)}. \]

The borrower maintains the asset if and only if

\[ \frac{p}{p + (1 - p)(1 - \xi)} (R_b + A) \geq c. \]

The ex ante incentive compatibility condition (relative to the choice of $p$) is then (for $c$ not too large)

\[ p_H (R_b + A - c) + (1 - p_H)(1 - \xi)(-c) \geq p_L (R_b + A - c) + (1 - p_L)(1 - \xi)(-c) + B. \]

The interpretation of the term $(\Delta p)\xi c$ in the inequality in the statement of question (ii) is that if the entrepreneur works, she reduces the probability of receiving a signal that enables her to avoid maintenance benefitting the lenders.

(iii) Suppose, first, that the entrepreneur does not pledge the assets. Then the condition for financing is the familiar one (with the value of collateral, $A$, being nonpledgeable to investors):

\[ p_H \left( R - \frac{B}{\Delta p} \right) \geq I. \]

- If the entrepreneur pledges the assets in the case of failure, then the financing condition becomes

\[ p_H \left[ R - \left( \frac{B}{\Delta p} + \xi c - A \right) \right] + (1 - p_H)(1 - \xi)A \geq I. \]

Not pledging the asset in the case of failure facilitates financing if

\[ p_H \xi c > [p_H + (1 - p_H)(1 - \xi)]A, \]

which is never satisfied if $A > c$. Note that (1) the NPVs differ (the NPV is higher in the absence of pledging since the asset is then always maintained) and (2) more generally one should consider pledging only part of the asset.

Exercise 4.2 (diversification across heterogeneous activities). (i) Under specialization, the entrepreneur’s net utility is (see Section 3.4)

\[ U_0^i = \frac{\rho^1_i - 1}{1 - \rho^0_i} A \quad \text{for activity } i. \]

So, the entrepreneur prefers the low-NPV, low-agency-cost activity $\alpha$ if and only if

\[ \frac{\rho^\alpha_1 - 1}{1 - \rho^\alpha_0} > \frac{\rho^\beta_1 - 1}{1 - \rho^\beta_0}. \quad (1) \]

(ii) Let $R_2$ denote the entrepreneur’s reward if both activities succeed ($R_1 = R_0 = 0$). The entrepreneur must prefer behaving in both activities to misbehaving in both:

\[ (p_H^i - p_L^i) R_2 \geq B^\alpha I^\alpha + B^\beta I^\beta. \quad (2) \]
Now if the ratios $I^\alpha/I^\beta$ and $B^\alpha/B^\beta$ are sufficiently close to 1, a case we will focus on in the rest of the question, then the entrepreneur does not want to misbehave in a single activity either (the proof is similar to that in Section 4.2).

The entrepreneur solves

$$\max_{\{I^\alpha,I^\beta\}} \{(\rho^\alpha - 1)I^\alpha + (\rho^\beta - 1)I^\beta\}$$

subject to

$$\rho^\alpha I^\alpha + \rho^\beta I^\beta - \frac{p^\alpha}{p^\alpha - p^\beta}[B^\alpha I^\alpha + B^\beta I^\beta] \geq I^\alpha + I^\beta - A. \quad (3)$$

In contrast, the specialization solution solves the same program but with $p^\alpha/[p^\alpha - p^\beta]$ replaced by $p_H/[p_H - p_L]$, which is bigger. Let

$$\tilde{\rho}^\alpha = \frac{p_H}{p_H - p_L}B^\alpha I^\alpha - \frac{p^\alpha}{p^\alpha - p^\beta}. \quad (2')$$

Diversification reduces the agency cost. If

$$\frac{\rho^\alpha - 1}{1 - \tilde{\rho}^\alpha} < \frac{\tilde{\rho}^\alpha - 1}{1 - \tilde{\rho}^\alpha},$$

then the optimum is to have

$$I^\beta > I^\alpha.$$

But $I^\alpha = 0$ is not optimal. We need to reintroduce the incentive constraint according to which the entrepreneur does not want to shirk in activity $\beta$ only (the one that yields the highest total private benefit); condition (2) (satisfied with equality so as to maximize borrowing capacity, and now labeled $(2')$),

$$(p_H + p_L)(\Delta p)R_2 = B^\alpha I^\alpha + B^\beta I^\beta, \quad (2')$$

does not imply

$$p_H(\Delta p)R_2 \geq B^\beta I^\beta \quad (4)$$

if the ratio $I^\alpha/I^\beta$ is too small. Conditions $(2')$ and $(4)$ (satisfied with equality) together define the optimal ratio $I^\alpha/I^\beta$.

**Exercise 4.4 (“value at risk” and benefits from diversification).** Let $R_0$, $R_1$, and $R_2$ denote the entrepreneur’s reward contingent on 0, 1, and 2 successes, respectively. The NPV (given that the entrepreneur will never receive rewards strictly above $\bar{R}$, we can reason on the risk-neutral zone in $u(\cdot)$ and use the NPV) is

$$2[p_H R - I].$$

To see whether the two projects can be financed simultaneously, minimize the nonpledgeable part of this NPV,

$$\frac{1}{4}[1 + \alpha]R_2 + \frac{1}{2}[1 - \alpha]R_1 + \frac{1}{4}[1 + \alpha]R_0, \quad (1)$$

while providing incentives. To compute the entrepreneur’s expected compensation above, note that the probability of two successes is

$$\Pr(\text{project 1 succeeds | work on project 1}) \times \Pr(\text{project 2 succeeds | work on project 2 and success in project 1})$$

or $\frac{1}{4}[\frac{1}{4}(1 + \alpha)]$. And so forth.

(i) The two incentive constraints are

$$\frac{1}{4}[1 + \alpha]R_2 + \frac{1}{2}[1 - \alpha]R_1 + \frac{1}{4}[1 + \alpha]R_0 \geq 2B + R_0 \quad (2)$$

and

$$\frac{1}{4}[1 + \alpha]R_2 + \frac{1}{2}[1 - \alpha]R_1 + \frac{1}{4}[1 + \alpha]R_0 \geq B + \frac{1}{2}R_1 + \frac{1}{2}R_0. \quad (3)$$

(ii) If $\bar{R}$ is large, one can then reward the entrepreneur only in the upper tail:

$$R_2 = \frac{8B}{1 + \alpha}.$$  

This value minimizes (1) subject to (2), and also satisfies (3).

(iii) When $\bar{R} < (8B)/(1 + \alpha)$, the entrepreneur can no longer be rewarded solely in the upper tail to satisfy (2). Note that $R_0 = 0$ is optimal from (2) and (3). (2) can be satisfied by $(R_2 = \bar{R}, R_1 \leq \bar{R}, R_0 = 0)$ if and only if

$$\frac{1}{5}(3 - \alpha)\bar{R} \geq B. \quad (4)$$

The question is then whether (3) is also satisfied.

- For positive correlation ($\alpha > 0$), increasing $R_1$ makes (3) harder to satisfy. Hence, minimizing the nonpledgeable income requires choosing the lowest $R_1$ that satisfies (2). This value satisfies (3) if and only if $B \geq \frac{4}{5}R_1$, or, after substitutions,

$$B \leq \frac{1}{5}\bar{R},$$

which is more constraining than (4).

- For negative correlation ($\alpha < 0$), increasing $R_1$ makes it easier to satisfy (3). While it is still optimal to set $R_2 = \bar{R}$, the binding constraint may now be (3) (and thus the nonpledgeable income exceeds $2B = 2p_H B/\Delta p$ here). Financing may be feasible even
though it would not be so if project correlation were positive (but $\frac{1}{\lambda}(1 - \alpha)\bar{R}$ must exceed $B$).

**Exercise 4.5 (liquidity of entrepreneur’s claim).** The entrepreneur’s incentive constraint when the liquidity shock is observed by investors is

$$(1 - \lambda)(\Delta p)R_b \geq B.$$ 

The NPV is

$$U_b = \text{NPV} = \lambda(\mu - 1)r_b + p_HR - I,$$

while the breakeven constraint is

$$\lambda(\mu_0 - 1)r_b + p_HR - (1 - \lambda)p_HR_b \geq I - A.$$ 

As in the text, it is optimal to compensate the entrepreneur by providing her with liquidity (since $\mu > 1$) once $R_b$ is equal to $B/(1 - \lambda)\Delta p$. The level of liquidity, $r^*_b$, given to the entrepreneur is set by the breakeven constraint

$$\lambda(1 - \mu_0)r_b^* + [I - A] = p_H\left(R - \frac{B}{\Delta p}\right).$$

It increases when more of the proceeds of reinvestment become pledgeable.

(ii) If $\lambda$ is a choice variable, the entrepreneur faces multiple tasks. She solves

$$\max_{\lambda \in (0, \bar{\lambda}), p \in \{p_L, p_H\}} U_b(p, \lambda)$$

$$\in \{\lambda(\mu - \mu_0)r_b + (1 - \lambda)p_Rb - \lambda c + B1[p = p_L]\}.$$

The NPV is

$$U_b = \text{NPV} = \lambda(\mu - 1)r_b + p_HR - I - \lambda c.$$ 

For a given contract $(R_b, r_b)$ the entrepreneur chooses

$$\lambda = \bar{\lambda} \text{ if } (\mu - \mu_0)r_b - p_HR_b \geq c.$$ 

Note that, for $p = p_H$, the entrepreneur does not “oversearch” for new investment opportunities as long as

$$(\mu - \mu_0)r_b - p_HR_b \leq (\mu - 1)r_b \iff (1 - \mu_0)r_b \leq p_HR_b.$$ 

Suppose that one wants to implement $p = p_H$. Then

- either $\lambda = 0$, and then the outcome is the same as in the absence of a liquidity shock;
- or, more interestingly, $\lambda = \bar{\lambda}$ (which implies a fortiori that $\lambda = \bar{\lambda}$ if the entrepreneur deviates and chooses $p = p_L$):

$$U_b(p_H, \bar{\lambda}) \geq U_b(p_L, \bar{\lambda}) \iff (1 - \bar{\lambda})(\Delta p)R_b \geq B.$$ 

Furthermore,

$$U_b(p_H, \bar{\lambda}) \geq U_b(p_H, 0) \iff (\mu - \mu_0)r_b - p_HR_b \geq c.$$ 

Hence, $R_b = B/[(1 - \bar{\lambda})(\Delta p)]$, and so an added constraint with respect to subquestion (i) is

$$(\mu - \mu_0)r_b \geq c + p_H\frac{B}{(1 - \bar{\lambda})\Delta p}.$$ 

**Exercise 4.6 (project size increase at an intermediate date).** Consider first the entrepreneur’s date-1 behavior when the size has been doubled. If the entrepreneur has worked on the initial project, and using the perfect correlation between the two projects, the incentive constraint is

$$p_HR_b \geq p_LR_b + B.$$ 

If she shirked on the first project, then it is optimal to shirk again.

The date-0 incentive constraint is then

$$(1 - \lambda)p_HR_b + \lambda p_HR_b \geq B + (1 - \lambda)p_LR_b + \lambda[p_LR_b + B].$$ 

To obtain the nonpledgeable income, minimize the left-hand side of the latter inequality subject to the incentive constraints, yielding

$$R_b = \frac{B}{\Delta p} \text{ and } R_b = \frac{B}{(1 - \lambda)\Delta p}.$$ 

Thus the nonpledgeable income is

$$U_b = \lambda + \lambda p_H\frac{B}{\Delta p}.$$ 

**Exercise 4.7 (group lending and reputational capital).** (i) By assumption,

$$p_H\left(R - \frac{B}{\Delta p}\right) < p_H\left(R - \frac{B}{(1 + a)\Delta p}\right) < I - A.$$ 

Under individual borrowing, the pledgeable income is $p_H[R - (B/\Delta p)]$, and so individual borrowing is not feasible. Under group lending, let $R_b$ denote the borrower’s individual reward when both succeed. They get 0 when at least one of them fails. The idea is that a borrower is punished “twice” for her failure: she gets no reward and also suffers from the other borrower’s not receiving a reward. The incentive constraint is then

$$p_H(\Delta p)[(1 + a)R_b] \geq B,$$ 

(II)}
yielding pledgeable income per borrower
\[ P = p_H R - p_H^2 \left[ \min_{|K_p|} R_b \right] = p_H \left[ R - \frac{B}{(1 + a)\Delta p} \right]. \]
Hence, group lending is not feasible either.

(ii) If both players are altruistic with \( a = \frac{1}{2} \), they both cooperate in the unique equilibrium of the \("stage-2\" game. They have payoff \( \frac{7}{2} \), since they enjoy the monetary gain of the other agent. More precisely, the utilities in the stage-2 game are as follows:

<table>
<thead>
<tr>
<th>Agent 1</th>
<th>Agent 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>D</td>
</tr>
<tr>
<td>( \frac{7}{2} )</td>
<td>( -\frac{1}{2} )</td>
</tr>
<tr>
<td>( -\frac{1}{2} )</td>
<td>( -\frac{5}{2} )</td>
</tr>
</tbody>
</table>

Cooperating is a dominant strategy \( (\frac{7}{2} > 1 \text{ and } -1 > -\frac{3}{2}) \), and so both cooperate.

If both agents are selfish \( (a = 0) \), the payoffs given in the statement of the question are those of a standard prisoner’s dilemma and both agents defect.

(iii) The structure of payoffs is such that the altruistic agent gets nothing in the second stage if she misbehaves in the first stage. Consider the incentive constraint facing altruistic agents:
\[ p_H(\Delta p)(1 + a)R_b + \frac{7}{2} \delta \geq B \]
with \( a = \frac{1}{2} \). The pledgeable income per borrower is
\[ p_H \left[ R - \frac{2B}{3\Delta p} + \frac{\delta}{\Delta p} \right]. \]
The financing is secured if
\[ p_H \left[ R - \frac{2B}{3\Delta p} + \frac{\delta}{\Delta p} \right] \geq I - A. \]
From this, the minimum discount factor to secure financing is
\[ \delta_{\text{min}} = \frac{\Delta p}{p_H} (I - A) - (\Delta p)R + \frac{7}{2} B > 0, \]
by assumption. The intuition is that the altruistic agent behaves in order to separate herself from the selfish agent and to build a reputation for being altruistic. The term \( \delta/\Delta p \) reflects the gain from reputation and can be interpreted as the borrower’s “social collateral.”

Exercise 4.9 (borrower-friendly bankruptcy court).
(i) • Monetary returns, such as \( L \) and \( r \), that are not subject to moral hazard (or adverse selection) are optimally pledged to investors if financing is a constraint. This increases the income that is returned to investors without creating bad incentives for the entrepreneur.

• The entrepreneur’s incentive constraint is (for a given realization of \( r \))
\[ (p_H(r) - p_L(r))R_b \geq B \text{ or } (\Delta p)R_b \geq B. \]
Condition (1) in the statement of the question says that continuation always maximizes social (total) value. However, systematic continuation (continuation for all \( r \)) generates too little pledgeable income to permit financing (right-hand side of condition (2) in the statement); on the other hand, systematic liquidation would generate enough pledgeable income (left-hand side of (2)).

Financing requires liquidating inefficiently. Intuitively, there is then no point giving \( R_b(r) > B/\Delta p \) for some \( r \) in the case of continuation. The difference serves no incentive purpose and can be used to boost pledgeable income, allowing for more frequent continuation (in other words, it is more efficient to compensate the management with continuation rather than with money as long as incentives are sufficient). (Note: to prove this, generalize the optimization program in subquestion (ii) to allow for a choice of \( R_b(r) \) for \( r \geq r^* \).)

(ii) • The borrower solves
\[ \max_{|r^*|} \{ E[r] + \int_{r^*}^{r} \rho_1(r)f(r) \, dr \} \quad \text{s.t.} \quad E[r] + \int_{r^*}^{r} \rho_0(r)f(r) \, dr + \int_{0}^{r^*} Lf(r) \, dr \geq I - A. \]
Clearly, \( r^* \) is the lowest value that satisfies the breakeven constraint. Condition (2) in the statement of the question implies that \( 0 < r^* < \bar{r} \). And, of course, \( L \geq \rho_0(r^*) \).

(iii) • With a short-term debt contract, \( d = r^* \), the firm will be able to repay its debt and continue if \( r \geq r^* \). If \( r < r^* \), the lenders are entitled to use default to liquidate. The investors do not want to renegotiate since \( L \geq \rho_0(r^*) \).

• \( dr^*/dA < 0 \). A lower amount of equity calls for more pledgeable income.
Exercise 4.10 (benefits from diversification with variable-investment projects). (i) The analysis follows the lines of Section 3.4. The incentive constraint on project $i$ with size $I^i$ is

$$(\Delta p)R^i_b \geq BI^i,$$

where $R^i_b$ is the entrepreneur’s reward in the case of success in project $i$; and so the pledgeable income is $\rho_0 I^i$.

The entrepreneur allocates $A^i$ to project $i$, where

$$A^1 + A^2 = A.$$ 

Her total utility is

$$U_b = \sum_i [(\rho_1 - 1)I^i] = \sum_i (\rho_1 - 1) \left( \frac{A^i}{1 - \rho_0} \right)$$

$$= \rho_1 - \frac{1}{1 - \rho_0} A.$$

It does not really matter how the entrepreneur allocates her wealth between the two projects. In particular, there is no benefit to having a second project.

(ii) As in the case of fixed-investment projects, it is optimal to reward the entrepreneur only if the two projects succeed ($R_2 > 0, R_1 = R_0 = 0$). The two incentive constraints are

$$p^2_0 R_2 \geq p_0 p_1 R_2 + \max_{i \in \{1, 2\}} \{ BI^i \}$$

and

$$p^2_0 R_2 \geq p_0^2 R_2 + B(I^1 + I^2).$$

Let

$$I \equiv I^1 + I^2.$$ 

Then

$$U_b = \text{NPV} = \sum_I [p_0 R I^i - I^i] = (\rho_1 - 1)I$$

and the financing condition becomes

$$p_0 R I - p_0^2 R_2 \geq I - A.$$ 

Thus, everything depends only on total investment $I$, except for the first incentive constraint. For a given $I$, this constraint is relaxed by taking

$$I^1 = I^2 = \frac{1}{2} I.$$ 

The rest of the analysis proceeds as in Section 4.2. The first incentive constraint is satisfied if the second is. And so

$$U_b = \rho_1 - \frac{1}{1 - \rho_0} A.$$
Exercise 4.11 (optimal sale policy). (i) The entrepreneur maximizes NPV,
\[ \int_{s^*}^{1} (sR) f(s) \, ds + F(s^*) L, \]
subject to the investors' breakeven constraint:
\[ \int_{s^*}^{1} \left( R - \frac{B}{\Delta p} \right) f(s) \, ds + F(s^*) L \geq I - A, \quad (\mu) \]
where use is made of the fact that the proceeds from the sale should go to investors in order to maximize pledgeable income. One finds
\[ s^* \left[ R + \mu (R - B/\Delta p) \right] = L. \]
Note that \( s^* R = L \) if financing is not a constraint (\( A \) large), and
\[ s^* \left[ R - \frac{B}{\Delta p} \right] < L. \]
The optimal \( s^* \) trades off maximizing NPV (which would call for \( s^* = L/R \)) and pleasing investors (which would lead to \( s^* = L/(R - (B/\Delta p)) \)).

(Showoffs: we have assumed that it is optimal to endogenize the entrepreneur to exert effort when the firm is not liquidated. A sufficient condition for this to be the case is
\[ (s - \Delta p) R \leq \max \left\{ L, \left( s - \frac{B}{\Delta p} \right) R \right\}; \]
that is, the pledgeable income is always lowest under continuation and shirking. To see this, consider state-contingent probabilities \( x(s) \) of continuation and working, \( y(s) \) of continuation and shirking, and \( z(s) \) of liquidation.

Solve
\[ \max_{\{x(s), y(s), z(s)\}} \left\{ \int_{s}^{1} \left[ x(s) (sR) + y(s) [(s - \Delta p) R] \right. \right. \]
\[ \left. \left. + z(s) L \right] f(s) \, ds \right\} \]
s.t.
\[ \int_{s}^{1} \left[ x(s) \left( s - \frac{B}{\Delta p} \right) R \right. \]
\[ \left. + y(s) [(s - \Delta p) R] + z(s) L \right] f(s) \, ds \geq I - A \]
and \( x(s) + y(s) + z(s) = 1 \) for all \( s \).

(ii) Endogenizing \( R_b(s) \geq B/\Delta p \) for \( s \geq s^* \) (where the threshold may differ from the one obtained in (a)), the expression for the NPV is unchanged. The breakeven constraint becomes
\[ \int_{s^*}^{1} s [R - R_b(s)] f(s) \, ds + F(s^*) L \geq I - A. \]

The derivative with respect to \( R_b(s) \) is negative and so
\[ R_b(s) = B/\Delta p \quad \text{as long as } \mu > 0. \]

(iii) It is optimal to sell if \( s = s_1 \). Let \( R^*_b \) (> \( B/\Delta p \)) from the assumption made) be defined by
\[ s_2 (R - R^*_b) = I - A. \]

If
\[ B_0 < s_2 R^*_b, \]
then the "career concerns" incentives are sufficient to prevent first-stage moral hazard. The only possible issue is then renegotiation. That is, if \( s_1 (R - B/\Delta p) > L \), the two parties are tempted to renegotiate.

If in contrast \( B_0 > s_2 R^*_b \),

then even in the absence of renegotiation, there is first-stage moral hazard. Financing becomes infeasible.

Exercise 4.12 (conflict of interest and division of labor). (i) The incentive constraints are
\[ p_{H} R_b + (1 - p_{H}) \tilde{R}_b - c \]
\[ \geq p_{L} R_b + (1 - p_{L}) \tilde{R}_b - c + B \]
(no shirking on project choice)
\[ \geq p_{L} R_b \]
(no shirking on maintenance)
\[ \geq p_{L} R_b + B \]
(no shirking on either dimension).

The first two constraints can be rewritten as
\[ (\Delta p) (R_b - \tilde{R}_b) \geq B \quad \text{and} \quad \tilde{R}_b \geq \frac{c}{1 - p_{H}}. \]

The third,
\[ (\Delta p) R_b + (1 - p_{H}) \tilde{R}_b \geq B + c, \]
is guaranteed by the other two.

(ii) The nonpledgeable income is
\[ \min_{\{c\}} \{ p_{H} R_b + (1 - p_{H}) \tilde{R}_b \} = p_{H} \frac{B}{\Delta p} + \frac{c}{1 - p_{H}}. \]

The financing condition is
\[ p_{H} R + (1 - p_{H}) L - p_{H} \frac{B}{\Delta p} - \frac{c}{1 - p_{H}} \geq I - A. \]

(iii) The agent in charge of maintenance is given \( \tilde{R}_b \) conditional on failure and proper maintenance,
Answers to Selected Exercises

and 0 otherwise. Her incentive constraint is

\[(1 - p_H)\hat{R}_b \geq c.\]

So when given \(\hat{R}_b = c/(1 - p_H)\), this agent exerts care in maintaining the asset and receives no rent.

The entrepreneur's incentive constraint then becomes

\[(\Delta p)\hat{R}_b \geq B.\]

The nonpledgeable income is now

\[p_H \frac{B}{\Delta p} + (1 - p_H)\hat{R}_b = p_H \frac{B}{\Delta p} + c.\]

For more on the division of labor when multiple tasks are in conflict, see Dewatripont and Tirole (1999) as well as Review Problem 9.5

Exercise 4.14 (diversification and correlation). (i) The two incentive constraints are

\[p_H^2R_2 \geq p_H^2R_2 + 2B \quad \text{and} \quad p_H^2R_2 \geq p_Hp_LR_2 + B.\]

The first constraint can be rewritten as

\[p_H^2R_2 \geq \frac{2p_H^2B}{(p_H + p_L)\Delta p}.\]  \(\text{IC}\)

The second constraint is satisfied if the first is. The pledgeable income is

\[2p_HR - \min\{p_H^2R_2\},\]

hence the result.

(ii) The entrepreneur receives \(p_HR_2\) by behaving on both projects. When misbehaving (either on one or the two projects), the entrepreneur receives expected income \(p_LR_2\). And so she might as well misbehave in both. The incentive constraint is then

\[p_HR_2 \geq p_HR_2 + 2B.\]  \(\text{IC}\)

And so the pledgeable income is

\[2p_HR - \min\{p_H^2R_2\} = 2p_HR - 2p_H \frac{B}{\Delta p}.\]

This yields the financing condition.

(iii) The incentive constraints are

\[xp_H + (1 - x)p_L^2R_2 \geq [xp_L + (1 - x)p_L^2]R_2 + 2B\]

and

\[xp_H + (1 - x)p_H^2 \geq [xp_L + (1 - x)p_1p_H]R_2 + B.\]

The second turns out to be satisfied if the first is. The financing condition becomes

\[p_H \left[ R - \left[ \frac{1 - (1 - x)(1 - p_H)}{1 - (1 - x)(1 - p_L - p_H)} \right] \frac{B}{\Delta p} \right] \geq I - A.\]

Ex ante (before financing), \(x = 0\) facilitates financing. Ex post (after the investors have committed their funds), the entrepreneur's payoff,

\[xp_H + (1 - x)p_H^2 \geq \hat{R}_2,\]

is increasing in \(x\) and so \(x = 1\). Note that the NPV is independent of \(x\):

\[U_b = NPV = 2[p_HR - I].\]

Exercise 4.15 (credit rationing and the bias towards less risky projects). (i) Note, first, that the incentive compatibility constraint is the same regardless of the choice of project specification: letting \(R_b\) denote the entrepreneur's reward in the case of success (as usual, there is no point rewarding the entrepreneur in the case of failure), the incentive compatibility constraints are

\[(p_H^2 - p_L^2)\hat{R}_b \geq B \iff (p_H^2 - p_L^2)\hat{R}_b \geq B \iff (\Delta p)\hat{R}_b \geq B.\]

The pledgeable income is therefore

\[T^s = xp_H \left[ R - \frac{B}{\Delta p} \right] + (1 - x)L^s\]

for the safe variant, and

\[T^r = xp_H \left[ R - \frac{B}{\Delta p} \right] + (1 - x)L^r\]

for the risky one.

Because \(T^s \geq T^r\), choosing the safe variant facilitates funding. Lastly, \(\bar{A}\) is defined by

\[T^r = I - \bar{A}.\]

The NPV is otherwise the same for both variants. Hence, \(U_b\) is the same provided the project is funded.

(ii) The entrepreneur having discretion over the choice of projects adds an extra dimension of moral hazard. Providing her with “high-powered incentives” (\(R_b\) in the case of success, 0 in the case of failure) is ideal for encouraging good behavior in the case of continuation, but it also pushes the

entrepreneur to take risks, as\(^6\)

\[ xp_0 R_b > xp R_b. \]

More generally, any incentive scheme that addresses the *ex post* moral-hazard problem \((\Delta P) (R_b^0 - R_b^1) \geq B)\) encourages the choice of the risky variant unless the entrepreneur receives a reward (only) when the collateral value is high \((I^*)\). But such a reward further reduces pledgeable income and may jeopardize financing altogether when \(A < A\), but \(P^0 \geq I - A\).

**Exercise 4.16 (fire sale externalities and total surplus-enhancing cartelizations).** (i) The representative entrepreneur’s borrowing capacity \(i\) is determined by the investors’ breakeven condition:

\[ [xp_0 + (1 - x)P]i = i - A, \]

where

\[ \rho_0 \equiv p_i \left( R - \frac{B}{\Delta P} \right) \]

is the pledgeable income per unit of investment in the absence of distress.

Because it is individually optimal to resell all assets when in distress, \(J = (1 - x)I\), and so

\[ P = P((1 - x)I). \]

Furthermore, in equilibrium \(i = I\), and so

\[ I = \frac{A}{1 - [xp_0 + (1 - x)P((1 - x)I)].} \]

The representative firm’s NPV (or utility) is

\[ U_b = [xp_1 + (1 - x)P((1 - x)I) - 1]I \]

for the value of \(I\) just obtained.

(ii) In the case of cartelization, specifying that at most \(z < 1\) can be resold on the market, and so \(J = (1 - x)zI\), these expressions become

\[ I = \frac{A}{1 - [xp_0 + (1 - x)zP((1 - x)zI)]} \]

and

\[ U_b = [xp_1 + (1 - x)zP((1 - x)zI) - 1]I. \]

Let

\[ H(z, I) \equiv (1 - x)zP((1 - x)zI). \]

Then

\[ \frac{\partial H}{\partial z} = (1 - x)[P + JP']. \]

Hence, \(H\) decreases with \(z\) if and only if the elasticity of demand is greater than 1.

Let us check that an elasticity of demand greater than 1 is consistent with the stability condition (incidentally, the same reasoning applies to the more general case in which only a fraction \(z\) of the assets are put up for sale). Simple computations show that

\[ \frac{dI}{dI} = \frac{(1 - x)^2}{P + JP'}, \]

and that the conditions

\[ \frac{dI}{dI} > -1 \quad \text{and} \quad P + JP' < 0 \]

are consistent if and only if

\[ 1 > xp_0 + 2(1 - x)P. \]

This latter condition is not guaranteed by the fact that investment is finite \((1 > xp_0 + (1 - x)P)\), but is satisfied when \(x\) is large enough.

When the elasticity of demand exceeds 1,

\[ I = \frac{A}{1 - [xp_0 + H(z, I)]} \]

decreases with \(z\), and

\[ U_b = [xp_1 + H(z, I) - 1]I \]

decreases with \(z\) for two reasons: both the NPV per unit of investment and the investment decrease.

Simple computations show that

\[ (A - J^2 P') dI = (1 - x)I^2 [P + JP'] dI, \]

and so \(dI/dz < 0\).

(iii) Let \(\hat{p}_1 = xp_1 + (1 - x)zP\). The change in total surplus is given by

\[ d(U_b + S^a) = [(1 - x)(P dz + z dP)I + (\hat{p}_1 - 1) dI] \]

\[ (1 - x)I dP, \]

where the first term (in brackets) on the RHS measures the change in the entrepreneur’s utility and the second term the change in buyer surplus. And so

\[ d(U_b + S^a) = (1 - x)PI dz + (\hat{p}_1 - 1) dI. \]

6. Note that the choice of the risky project is perfectly detected in the case of liquidation, since liquidation then yields only \(I^*\) instead of the (higher) level \(I^*\). The entrepreneur is, however, protected by limited liability and therefore cannot be punished for the wrong choice of project. (For the reader interested in contract theory: if we endogenized limited liability through large risk aversion below 0, we would need to assume that the safe project yields the low liquidation value \(I^*\) at least with positive probability. Otherwise, the entrepreneur could be threatened with a negative income in the case of low liquidation value and there would be no moral hazard in the choice of project.)
The term \((1 - x)PI\) \(dz\) corresponds to a better utilization of distressed assets (which are valued \(P\) by the marginal buyer) when \(dz > 0\), while the second term (the original one from the point of view of welfare analysis) stands for the social surplus created by an increase in borrowing capacity (associated with \(dz < 0\)).

The total surplus increases when \(z\) decreases as long as

\[
\hat{\rho}_1 - 1 \geq 1 - \hat{\rho}_0 - (1 - x)^2 z^2 (A/(1 - \hat{\rho}_0)) P',
\]

where \(\hat{\rho}_0 \equiv x\rho_0 + (1 - x)zP\) and \(\eta \equiv -P'/JP\).

Note that \(\hat{\rho}_1\) can be increased without bound (by increasing \(\rho_1\) keeping \(\rho_0\) constant, i.e.,, by increasing \(B\) for a given \(\rho_0\)) without altering any other variable. So for \(\rho_1\) sufficiently large, total surplus increases.

**Exercise 4.17 (loan size and collateral requirements).** When collateral is pledged only in the case of failure, the NPV (also equal to the entrepreneur’s utility) is

\[
U_b = p_H R(I) - I - (1 - p_H)[C - \phi(C)].
\]

The entrepreneur’s incentive compatibility constraint can be written as

\[
(\Delta p)[R_b + C] \geq BI,
\]

where \(R_b\) denotes the entrepreneur’s reward in the case of success. The investors’ breakeven constraint is

\[
p_H[R(I) - R_b] + (1 - p_H)\phi(C) \geq I - A,
\]

or, if the incentive constraint is binding,

\[
p_H \left[ R(I) - \frac{BI}{\Delta p} + C \right] + (1 - p_H)\phi(C) \geq I - A.
\]

Maximizing \(U_b\) with respect to \(I\) and \(C\) subject to this latter constraint yields

\[
p_H R'(I) - 1 = \frac{\mu}{1 + \mu} \left( \frac{p_H B}{\Delta p} \right)
\]

and

\[
\phi'(C) = \frac{1}{1 + \mu} - \frac{\mu}{1 + \mu} \left( \frac{p_H}{1 - p_H} \right),
\]

where \(\mu\) is the shadow price of the investors’ breakeven constraint. As the balance sheet deteriorates, \(\mu\) increases, \(I\) decreases, and \(C\) increases. Borrowing increases if the agency cost decreases; the impact of \(A\) on net borrowing \((I - A)\) is more ambiguous.

**Exercise 5.1 (long-term contract and loan commitment).** (i) The entrepreneur wants to carry on both projects as often as possible as this maximizes NPV. The pledgeable income in a contract that pays \(R_b = B/p_H\Delta p\) in the case of two successes and continues in the case of first success is

\[
p_H(p_H R - I) + \left( p_H R - p_H \frac{B}{\Delta p} \right); \]

hence, if it is weakly larger than 0, then the investors break even and the second project is financed if the first one was successful. If it is strictly larger than 0, then with investors breaking even, the entrepreneur has some additional income; it is optimal to take it in the form of a stochastic loan commitment in period 1.

(ii) Intuitively, \(\xi\) weakly increases in \(R\), \(p_H\) and decreases in \(B\), \(I\), and \(p_L\) (as long as \(p_L\) is not too large). The optimal \(\xi\) is such that

\[
(p_H + \xi (1 - p_H)) \left( p_H R - I - p_H \frac{B}{\Delta p} \right)
+ \left( p_H R - I - \left( p_H \frac{B}{\Delta p} - (1 - \xi)(\Delta p) \frac{B}{\Delta p} \right) \right) = 0
\]

or \(\xi = 1\) if the solution to the previous equation exceeds 1.

(iii) The contract is renegotiation proof. Indeed, either \(p_H R - I - p_H B/\Delta p < 0\) and then the lenders will not invest in the second project unless obliged to, or \(\xi = 1\) and then the borrower wants to carry on the second project.

(iv) The described sequence of short-term contracts is behaviorally equivalent to the optimal long-term contract from (i).

**Exercise 5.2 (credit rationing, predation, and liquidity shocks).** (i) The incentive constraint is

\[
(\Delta p)R_b \geq B_1.
\]

Hence, expected pledgeable income is

\[
p_0^\dagger = p_H \left( R_1 - \frac{B_1}{\Delta p} \right).
\]

The entrepreneur receives funding if and only if \(p_0^\dagger \geq I_1 - A\).

(ii) • The competitor preys if the entrepreneur waits until date 1 to secure funding for the date-1 investment.
To prevent predation, the entrepreneur can (publicly) secure at date 0 a credit line equal to 
\( (I_0 - \rho_0^1 - a) \), or else obtain a guarantee that the date-1 project will be funded.

Such long-term contracts are not renegotiated because they are \textit{ex post} efficient (social surplus is maximized if the date-1 project is undertaken, as \( p_{\text{H}}R_1 > I_1 \)).

(iii) The condition implies that unconditional financing of the two projects and date-0 shirking cannot allow investors to break even.

- \( x^* \) is given by

\[
(\Delta q)(1 - x^*)(\frac{p_B B_1}{\Delta p}) \geq B_0.
\]

- Suppose that \( \rho_0^1 > I_1 \). In states of nature where the initial contract specifies that the date-1 project is not financed, investors can offer to finance the project. They and the entrepreneur then get an extra rent (for example, \( \rho_0^1 - I_1 \) and \( p_B B_1 / \Delta p \) if the investors make a take-it-or-leave-it renegotiation offer).

(iv) Termination is no longer a threat under renegotiation. The only way to induce the entrepreneur to behave at date 0 and date 1 is to give her, in the case of success at date 1, \( R_0 = B_1 / \Delta p \) if profit is equal to \( a \), and \( R_b > R_0 \) if it is equal to \( A \), such that

\[
(\Delta q)p_{\text{H}}(R_0 - R_b) \geq B_0.
\]

This reduces the date-1 pledgeable income from \( \rho_0^1 \) to

\[
\rho_0^1 - q_B p_{\text{H}}(R_b - R_0) = \rho_0^1 - q_B B_0 \frac{B_0}{\Delta q}.
\]

The condition in the statement of the exercise then implies that funding cannot be secured at date 0.

Exercise 5.3 (asset maintenance and the soft budget constraint). (i) Assume that the financiers can commit not to renegotiate the initial contract. The optimal contract for the entrepreneur maximizes the NPV,

\[
U_b = \left\{ \int_0^L \left[ F(\rho^*(L))\rho_1 - \int_0^{\rho^*(L)} \rho f(\rho) \, d\rho - 1 
+ [1 - F(\rho^*(L))] \frac{\rho B_1}{\Delta p} \right] g(L) \, dL \right\} I,
\]

subject to the financing constraint,

\[
\left\{ \int_0^L \left[ F(\rho^*(L))\rho_0 - \int_0^{\rho^*(L)} \rho f(\rho) \, d\rho 
+ [1 - F(\rho^*(L))] \frac{\rho B_1}{\Delta p} \right] g(L) \, dL \right\} I = I - A,
\]

and the incentive compatibility constraint for maintenance,

\[
\left\{ \int_0^L \left[ F(\rho^*(L)) \left( \rho_1 - \rho_0 \right) + \Delta(L) \right] g(L) \, dL \right\} I \geq B_0 I,
\]

where

\[
\ell(L) = \frac{g(L) - \tilde{g}(L)}{g(L)}
\]

is the likelihood ratio, and \( \rho_1 - \rho_0 = B / \Delta p \).

Letting \( \mu \) and \( \nu \) denote the shadow prices of these two constraints, one gets the formulæ in the statement of the question by differentiating with respect to \( \rho^*(L) \) and \( \Delta(L) \).

(ii) The function \( \rho^*(\cdot) \) obtained under commitment has slope exceeding \( -1 \) (except for very large \( L \), for which the slope is equal to \( -1 \)). This slope can be positive or negative. The soft-budget-constraint problem arises when \( \rho \) is smaller than \( \rho_0 - L \) (allowing for negative values of \( \rho \)), i.e., for \( L \) small.

Exercise 5.4 (long-term prospects and the soft budget constraint). Go through the same steps as in Exercise 5.3, replacing \( "p_1" \) by \( "p_1 + R_L" \), \( "p_0" \) by \( "p_0 + R_1" \), eliminating the liquidation values, and making the functions \( \rho^*(\cdot) \) and \( \Delta(\cdot) \) functions of \( R_L \) instead of \( L \). One finds

\[
\rho^*(R_L) = R_L + \frac{\rho_1 + \nu \rho_0}{1 + \nu} + \frac{\mu (\rho_1 - \rho_0)}{1 + \nu} \ell(R_L)
\]

and

\[
\Delta^*(R_L) = 0 \quad \text{if} \quad \nu \ell(R_L) < \nu
\]

(and if \( \Delta^*(R_L) > 0 \), then \( \rho^*(R_L) = \rho_1 + R_L \)).

Exercise 5.5 (liquidity needs and pricing of liquid assets). (i) The borrower’s utility, conditional on receiving funds, is equal to the project’s NPV. Letting \( (x_L, x_H) \in \{0, 1\}^2 \) denote the probabilities of continuation in low- and high-liquidity shock states, we have

\[
U_b = (1 - \lambda)(\rho_1 - \rho_L) x_L + \lambda(\rho_1 - \rho_H) x_H 
- (I - A) - (q - 1)(\rho_{\text{H}} - \rho_0) x_{\text{H}}.
\]
Funding is feasible if

\[(1 - \lambda)(\rho_0 - \rho_L)x_L + \lambda(\rho_0 - \rho_H)x_H \geq I - A + (q - 1)(\rho_H - \rho_0)x_H.\]

For, the borrower needs no liquidity in order to cover the low shock: because \(\rho_0 > \rho_L\), the investors are willing to let their claim be diluted in order to continue. In contrast, the borrower needs to hoard \((\rho_H - \rho_0)\) Treasury bonds if \(x_H = 1\), in order to make up the shortfall between the liquidity shock and what can be raised on the capital market by diluting existing claimholders.

Clearly, \(x_L = 1\) as this both raises the borrower’s objective function and relaxes the financing constraint. In contrast, \(x_H = 1\) raises the objective function as long as \((q - 1)(\rho_H - \rho_0) \leq \lambda(\rho_1 - \rho_H)\) but reduces the pledgeable income. If condition (2) in the statement of the exercise is satisfied, then \(x_H = 1\) is indeed optimal. Otherwise \(x_H = 0\) is optimal given the financing constraint. (Note that, were we to allow \(0 \leq x_H \leq 1\), that is, randomized liquidation, an \(x_H \in (0, 1)\) could be optimal when condition (2) is violated.)

(ii) Suppose neither (2) nor (3) is binding. Then each firm hoards \((\rho_H - \rho_0)\) Treasury bonds. But then there is excess demand for Treasury bonds as \(T < \rho_H - \rho_0\).

Next, note that, for \(\lambda\) small, condition (2) cannot bind. Hence, (3) must bind:

\[q - 1 = \lambda(\rho_1 - \rho_H) / (\rho_H - \rho_0).\]

(iii) The new asset yields no liquidity premium since it yields no income in the bad state, and so \(q' = 1 - \lambda\).

**Exercise 5.6 (continuous entrepreneurial effort; liquidity needs).** (i) The entrepreneur chooses probability of success \(p\) such that

\[
\max_p \{pR_b - \frac{1}{2}p^2\}.
\]

Hence,

\[p = R_b.
\]

The breakeven constraint is

\[p(R - R_b) = I - A \quad \text{or} \quad R_b(R - R_b) = I - A.
\]

Note that this equation is satisfied for \(R_b = \frac{1}{2}R\).

(ii) The investors’ breakeven condition is

\[I - A + \int_0^{\rho^*} \rho f(\rho) \, d\rho = F(\rho^*)R_b(R - R_b).
\]

The entrepreneur maximizes

\[F(\rho^*)R_b^2\]

subject to the breakeven condition.

**Exercise 5.7 (decreasing returns to scale).** (i) The optimal policy maximizes the entrepreneur’s expected utility, which is equal to the NPV,

\[U_b = R I + F(\rho^*)p_H R(I) - \left(\int_0^{\rho^*} p f(\rho) \, d\rho\right)I - I,
\]

subject to the investors’ breakeven constraint,

\[R I + F(\rho^*)p_H \left( R(I) - \frac{B I}{\Delta p} \right) \geq I - A + \left(\int_0^{\rho^*} \rho f(\rho) \, d\rho\right)I. \quad (IR_1)
\]

Let us assume that this constraint is binding. Taking the first-order conditions with respect to \(I\) and \(\rho^*\), we obtain, after some manipulations,

\[p_H \left[ R'(I) - \frac{R(I)}{I} \right] = \frac{1 - r - \int_0^{\rho^*} (\rho^* - \rho) f(\rho) \, d\rho}{F(\rho^*)}.
\]

(ii) The right-hand side of (1) is decreasing in the cutoff \(\rho^*\). The left-hand side of (1) is increasing in \(I\). Thus \(\rho^*\) and \(I\) comove positively. From (IR_1), when the balance sheet deteriorates (\(A\) decreases), both \(I\) and \(\rho^*\) decrease. This implies, in particular, that the firm issues more short-term debt.

**Exercise 5.8 (multistage investment with interim accrual of information about prospects).** (i) Start with variant (a) (uncertainty about \(\tau\)). The optimal contract specifies a cutoff \(\tau^*\) above which the firm should reinvest \(I_1\).

The NPV (also equal to the entrepreneur’s utility under a competitive capital market) is, for a given \(\tau^*\),

\[U_b(\tau^*) = \int_{\tau^*}^{\Delta \tau^*} [(p_H + \tau)R - I_1] f(\tau) \, d\tau - I_0.
\]

As usual, the incentive constraint (in the case of continuation) requires a minimum stake \(R_b\) in the case of success for the entrepreneur. \(R_b\) must satisfy

\[(\Delta p)R_b \geq B.
\]
So the pledgeable income
\[ P(\tau^*) = \int_{\tau^*}^{R^*} (p_{H} + \tau) \left( R - \frac{B}{\Delta p} \right) - I_1 \right] f(\tau) \, d\tau. \]
Financing requires that
\[ P(\tau^*) \geq I_0 - A. \]

U_b and \( P \) are maximized at \( \tau^*_1 \) and \( \tau^*_0 \) such that
\[ (p_{H} + \tau^*_1)R = I_1 \]
and
\[ (p_{H} + \tau^*_0) \left( R - \frac{B}{\Delta p} \right) = I_1, \]
respectively. The entrepreneur is more eager to continue than the investors.

If \( P(\tau^*_1) \geq I_0 - A \), then the firm has deep pockets and the first-best continuation threshold \( \tau^*_1 \) is consistent with financing. So \( P(\tau^*_1) = I_0 - A_0 \). Otherwise, continuation must be less frequent as \( A \) declines:
\[ P(\tau^*) = I_0 - A. \]

But at the level \( A_0 \) at which
\[ P(\tau^*_0) = I_0 - A_0, \]
there is no longer the possibility to increase pledgeable income at the expense of value. For \( A < A_0 \), financing cannot be secured.

• The analysis of variant (b) proceeds similarly, with
\[ U_b(R^*) = \int_{R^*}^{\infty} [p_{H}R - I_1] \Delta g(R) \, dR - I_0, \]
\[ P(R^*) = \int_{R^*}^{\infty} \left[ p_{H} \left( R - \frac{B}{\Delta p} \right) - I_1 \right] g(R) \, dR - I_0, \]
\[ p_{H} \left( R^*_1 + B \frac{\Delta p}{R^*} \right) = I_1, \]
\[ p_{H} \left( R^*_0 - B \frac{\Delta p}{R^*} \right) = I_1. \]

(ii) For \( A = A_0 \), the entrepreneur must give the entire pledgeable income in order to secure funding. So, she only takes
\[ R_b = B \frac{\Delta p}{R} \]
in the case of continuation, and
\[ R = \left( p_{H} + \tau \right) B \frac{\Delta p}{\Delta p} = \frac{B}{\Delta p} R' \]
where \( B/(\Delta p)R < 1 \) in variant (a), and \( R = p_{H}B/\Delta p \) in variant (b).

Exercise 5.9 (the priority game: uncoordinated lending leads to a short-term bias). (i) The first-best allocation maximizes the NPV:
\[ \max_{I_1} \{ r - I_1 + [p + \tau(I_1)]R \}, \]
yielding
\[ \tau'(I^*_1)R = 1. \]

Note that \( I^*_1 < r \) by assumption, and so an amount \( (r - I^*_1) \) can be distributed at date 1. The date-1 payouts, \( r_b \) and \( r_l \), must satisfy
\[ r_b + r_l + I^*_1 = r, \]
\[ R_b + R_l = R, \]
\[ I = r_l + [p + \tau(I^*_1)]R_l. \]

This yields one degree of freedom.

(ii) Suppose that the entrepreneur secretly proposes the following contract to a (representative) lender: the lender’s short-term claim increases by \( \delta r_l \) in exchange for the transfer of his long-term claim to the entrepreneur (by assumption, the entrepreneur is not allowed to defraud other investors of their short- or long-term claims). The lender is willing to accept this deal as long as
\[ \delta r_l \geq [p + \tau(I_1)](\delta R_l). \]

Deepening investment decreases:
\[ \delta I_1 = -\delta r_l. \]

The entrepreneur’s interim utility increases by
\[ \delta U_b = \left[ \tau'(I_1) \Delta g(R) \right] R_b + [p + \tau(I_1)] \delta R_b \]
\[ = -\tau'(I_1)R_b + 1) \delta r_l > 0 \]
when \( I_1 = I^*_1 \), since \( \tau'(I^*_1)R = 1 \) and \( R_b < R \).

Note that the incentive to sacrifice the long-term profitability by increasing short-term debt decreases as \( R_b \) increases. Thus, it is optimal for the borrower to hold the smallest possible short-term claim \( (R_b = 0) \) and the largest long-term claim consistent with the investors’ breakeven constraint and the collusion-proof constraint:
\[ I = r - I_1 + [p + \tau(I_1)](R - R_b), \]
and
\[ \tau'(I_1)R_b = 0, \]
where \( I_1 < I^*_1 \).
Exercise 5.10 (liquidity and deepening investment). (i) Let \( R_0 \) denote the entrepreneur’s reward in the case of success (she optimally receives 0 in the case of failure). The incentive constraint, as usual, is
\[
(\Delta p)R_0 \geq B.
\]
The necessary and sufficient condition for financing is that the pledgeable income exceeds the investors’ outlay:
\[
P_{\Pi}(R - \frac{B}{\Delta p}) \geq I - A.
\]
(ii) The incentive compatibility condition is not affected by a deepening investment:
\[
[(p_{\Pi} + \tau) - (p_{\Pi} + \tau)]R_0 \geq B \iff (\Delta p)R_0 \geq B.
\]
The investors’ breakeven condition is
\[
[F(\rho^*)](p_{\Pi} + \tau) + [1 - F(\rho^*)]p_{\Pi}(R - R_0) \geq I - A + \int_{\rho^*}^{\rho^*_1} \rho f(\rho) \, d\rho.
\]
(iii) The NPV (or borrower’s utility) is
\[
U_b = [F(\rho^*)](p_{\Pi} + \tau) + [1 - F(\rho^*)]p_{\Pi}R - I - \int_{\rho^*}^{\rho^*_1} \rho f(\rho) \, d\rho.
\]
This NPV is maximized at
\[
\rho^* = \tau R = \hat{\rho}_1.
\]
Because
\[
R_0 \geq \frac{B}{\Delta p},
\]
the first best is implementable only in Case 1, which follows.

Case 1:
\[
[F(\hat{\rho}_1)](p_{\Pi} + \tau) + [1 - F(\hat{\rho}_1)]p_{\Pi}(R - \frac{B}{\Delta p}) \geq I - A + \int_0^{\hat{\rho}_1} \rho f(\rho) \, d\rho
\]
\[
\iff [1 + \mu F(\hat{\rho}_1)]\rho_0 \geq I - A + \int_0^{\hat{\rho}_1} \rho f(\rho) \, d\rho.
\]
Case 2: if
\[
[1 + \mu F(\hat{\rho}_0)]\rho_0 < I - A + \int_0^{\hat{\rho}_0} \rho f(\rho) \, d\rho,
\]
financing is infeasible.

Case 3: in the intermediate case, \( \rho^* \) is given by
\[
[1 + \mu F(\rho^*)]\rho_0 = I - A + \int_0^{\rho^*} \rho f(\rho) \, d\rho.
\]
(iv) Whenever \( \rho^* > \hat{\rho}_0 \) (which is the generic case, conditional on financing), the firm must hoard liquidity in order to avoid credit rationing at the intermediate stage. The investors’ maximal return on the deepening investment, \( \mu \rho_0 \), is smaller than the total value, \( \mu \rho_1 \), of this reinvestment.

Exercise 5.11 (should debt contracts be indexed to output prices?). (i) For a given policy \( \rho^*(P) \), the NPV is
\[
U_b = \tilde{P}r + E[F(\rho^*(P))p_{\Pi}PR]
\]
\[
- I - E\left[ \int_0^{\rho^*(P)} \rho f(\rho) \, d\rho \right],
\]
where expectations are taken with respect to the random price \( P \). The investors’ breakeven constraint is
\[
\tilde{P}r + E\left[ F(\rho^*(P)) \right] p_{\Pi}\left( PR - \frac{B}{\Delta p} \right)
\]
\[
\geq I - A + E\left[ \int_0^{\rho^*(P)} \rho f(\rho) \, d\rho \right].
\]
Let \( \mu \) denote the shadow price of the budget constraint (we assume that \( \mu > 0 \)). Then, taking the derivative of the Lagrangian with respect to \( \rho^*(P) \) yields
\[
\rho^*(P) = p_{\Pi}PR - \left( \frac{\mu}{1 + \mu} \right) \frac{p_{\Pi}B}{\Delta p}
\]
(ii) To implement the optimal policy through a state-contingent debt \( d(P) \), one must have
\[
\rho^*(P) = [Pr - d(P)] + \left[ p_{\Pi}\left( PR - \frac{B}{\Delta p} \right) \right]
\]
or
\[
d(P) = Pr - \ell_0,
\]
where
\[
\ell_0 \equiv \frac{1}{1 + \mu} \left( p_{\Pi}\frac{B}{\Delta p} \right).
\]

Exercise 6.1 (privately known private benefit and market breakdown). (i) If the borrower’s private benefit \( B \) were common knowledge, then, if financed, the borrower would receive \( R_0 \) in the case of success, with
\[
R_0 \geq \frac{B}{\Delta p},
\]
so as to induce her to behave. The project would be funded if and only if the pledgeable income exceeded the investment cost:
\[
p_{\Pi}(R - \frac{B}{\Delta p}) \geq I.
Suppose that the borrower offers a contract specifying that she will receive $R_b$ in the case of success and 0 in the case of failure (offering to receive more than 0 in the case of failure would evidently raise suspicion, and can indeed be shown not to improve the borrower’s welfare). There are three possible cases:

(a) $R_b \geq B_1/\Delta p$ induces the borrower to work regardless of her type, and thus creates an information insensitive security for the lenders, who obtain

$$p_H(R - R_b) - I \leq p_H\left(\frac{R - B_1}{\Delta p}\right) - I < 0$$

using (1). So, such high rewards for the borrower cannot attract financing.

(b) $R_b < B_1/\Delta p$ induces the borrower to shirk regardless of her type. The lenders’ claim is again information insensitive, and from (2) fails to attract financing.

(c) $B_1/\Delta p \leq R_b < B_1/\Delta p$: suppose that, in equilibrium, the good borrower offers a contract with a reward in this range, and that this attracts financing. A bad borrower must then “pool” and offer the same contract; if she were to offer a different contract, her type would be revealed to the capital market and her project would not be funded. Furthermore, she receives utility from the project being funded at least equal to that of a good borrower (she receives the same payoff conditional on working and a higher payoff conditional on shirking). So, she is better off pooling with the good borrower than not being funded.

We conclude that equilibrium is necessarily a pooling equilibrium. It either involves no funding at all or funding of both types. From the study of cases (a) and (b), we also know that, in the case of funding, the good type behaves and the bad one misbehaves.

(ii) A necessary condition for funding is thus that

$$[\alpha^* p_H + (1 - \alpha^*) p_L]\left(\frac{R - B_1}{\Delta p}\right) = I.$$ 

Since $R_b \geq B_1/\Delta p$, there cannot be any lending if $\alpha < \alpha^*$, where

$$[\alpha^* p_H + (1 - \alpha^*) p_L]\left(\frac{R - B_1}{\Delta p}\right) = I.$$

Thus, if the proportion of good borrowers is smaller than $\alpha^* \in (0, 1)$, there is no lending at all. Bad borrowers drive out good ones and the loan market breaks down.

Suppose, next, that the proportion of good borrowers is high: $\alpha > \alpha^*$. The borrower may now be able to receive financing. Suppose that the borrower, regardless of her type, offers to receive $R_b^*$ in the case of success and 0 in the case of failure, where

$$\frac{\alpha^* p_H + (1 - \alpha^*) p_L}{\alpha^*} = R - I.$$

Because $\alpha > \alpha^*$, $R_b^* > B_l/\Delta p$ and so the good borrower behaves. The investors’ breakeven condition is therefore satisfied. It is an equilibrium for both types to offer contract $\{R_b^*, 0\}$ and for the capital market to fund the project.

(iii) The pooling equilibrium (which exists whenever $\alpha > \alpha^*$) exhibits no market breakdown. Indeed, there is more lending under adverse selection than under symmetric information.

- It involves an externality between the two types of borrower. The good type obtains reward

$$R_b^* = R - I/\left[\alpha^* p_H + (1 - \alpha^*) p_L\right]$$

in the case of success below that, $R - I/p_H$, that she would obtain under symmetric information. The good type thus cross-subsidizes the bad type, who would not receive any funding under symmetric information.

- The project’s NPV conditional on being funded falls from $p_H R - I$ to $\left[\alpha^* p_H + (1 - \alpha^*) p_L\right] R - I$ due to asymmetric information. The quality of lending is thus affected by adverse selection.

**Exercise 6.2 (more on pooling in credit markets).** A loan agreement specifying reward $R_b$ in the case of success, and 0 in the case of failure, induces a proportion $H(R_b/\Delta p)$ of borrowers to behave. This proportion is endogenous and increases with $R_b$. Thus

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7. The reasoning can easily be extended to allow mixed strategies by the borrower and the capital market.

8. A more formal analysis of equilibrium behavior and of the equilibrium set can be performed along the lines of Section 6.4. We prefer to stick to a rather informal presentation at this stage.

the lender’s expected profit is

\[ U_l = H(R_b \Delta p)\pi_l(R - R_b) + (1 - H(R_b \Delta p))\pi_l(R - R_b). \]

Because \( \pi_l > \pi_l \) so with only high-quality types, the level of \( R_b \) that satisfies the breakeven constraint of lenders could be larger than \( R_b \) when they face distribution \( H \) of borrowers. Thus, there is an externality among different types of borrowers.

Under a uniform distribution on \([0, B]\) and for \( \pi_l = 0 \), the level of \( R_b \) maximizing pledgeable income is given by

\[
0 = h(R_b \Delta p)\pi_l(R - R_b)\Delta p - h(R_b \Delta p)\pi_l(R - R_b)\Delta p
- H(R_b \Delta p)\pi_l(R - (1 - H(R_b \Delta p)))\pi_l
= h(R_b \Delta p)(R - (1 - H(R_b \Delta p)))\pi_l
= \frac{1}{B}(R - R_b)\pi_l^2 - \frac{R_b}{B}\pi_l^2
\]

or

\[
R_b = \frac{1}{2}R.
\]

Thus the pledgeable income is

\[
P(R_b) = \frac{1}{B}\pi_l^2 R^2,
\]

and is smaller than \( I \) for \( B \) large enough.

**Exercise 6.3 (reputational capital).** (i) In this one-period adverse-selection problem, the bad type is always more eager to go on with a project than the good type. Thus, we may only have a pooling equilibrium. The assumptions imply that if we induce the bad type to work, or if we do not induce the good type to work, then the pledgeable income will not cover investment expenses. So, the only chance to receive funding is to induce the good type to work and the bad type to shirk. Under this type of contract, the pledgeable income is

\[
[\alpha\pi_l + (1 - \alpha)\pi_l]\left( R - \frac{b}{\Delta p} \right)
= (\pi_l - (1 - \alpha)\Delta p)\left( R - \frac{b}{\Delta p} \right).
\]

(ii) First, note that the good type always works in the first period as \( b < A\Delta p_1 \).

In a pooling equilibrium, the bad type would always work. But then, the updated belief on the probability of the good type would still be \( \alpha \) in period 2, and from the first inequality of the last displayed set of inequalities, and the result in (i), the project would not be financed in period 2. But this implies that the bad type would be better off shirking in period 1. So there is no pooling equilibrium.

In a separating equilibrium, the bad type would not work in period 1. Then, after a success in period 1, the updated belief on the probability of the good type would be \( \alpha_s \), and conditional on success in period 1 the project would be financed in period 2 (by the last assumed inequality) and the payoff to the borrower in the case of success would be

\[
R - \frac{I - A}{\pi_l - (1 - \alpha_s)\Delta p}.
\]

That, however, means that the bad type strictly prefers to work in period 1. Thus, there is no separating equilibrium.

The semiseparating equilibrium requires that the bad type is indifferent between working and shirking in period 1, that is,

\[
B = (\Delta p_1)\left[ \pi_l\left( R - \frac{I - A}{\pi_l - (1 - \alpha_s)\Delta p} \right) + B \right].
\]

This determines the updated belief \( \alpha_s \) on the probability of the good type conditional on success in period 1, and thus determines the probability of the bad type working in period 1.

**Exercise 6.5 (asymmetric information about the value of assets in place and the negative stock price reaction to equity offerings with a continuum of types).** (i) The investors receive \( R_l \) in the case of success and 0 in the case of failure. The entrepreneur therefore issues equity if and only if

\[
(p + \tau)(R - R_l) \geq pR \iff \tau R \geq (p + \tau)R_l
\]

and so there indeed exists a cutoff \( p^* \in [p, \bar{p}] \) such that the entrepreneur issues equity if and only if \( p \leq p^* \).

(ii) The investors’ breakeven condition is therefore

\[
[E(p \mid p \leq p^*)]R_l = I \quad \text{or} \quad R_l = \frac{I}{m^-(p^*) + \tau}.
\]

If interior, the cutoff satisfies

\[
\tau R = (p^* + \tau)R_l \quad \text{or} \quad \frac{\tau R}{I} = \frac{p^* + \tau}{m^-(p^*) + \tau}.
\]

Note also that \( p^* > p \): if \( p^* \) were equal to \( p \), then \( m^-(p^*) = p^* \) and so types \( p \) and just above would be strictly better off issuing equity. The condition

\[
\alpha \left( \pi_i - \pi_l \right) (R - b) \geq \pi_l (R - R_l) \Delta p
\]

for \( \alpha \geq 0 \), \( \pi_i \leq \pi_l \) and \( \pi_i \) being the probability of the good type conditional on success in period 1, and \( \pi_l \) being the probability of the bad type conditional on success in period 1.
(m^-)' \leq 1$ does not suffice to guarantee uniqueness, though. Uniqueness, however, prevails if $(m^-)'$ is bounded away from 1 (for example, $(m^-)' = \frac{1}{2}$ in the case of a uniform distribution) and if $\tau R/I$ is close to 1.

For $p^* = \hat{p}$, $m^-(p^*) = E[p]$ (the prior expectation). And so the condition stated in (ii) ensures that the cutoff is interior.

Finally, if there are multiple equilibria, the one with the highest $p^*$ yields the lowest stigma for equity issues since

$$R_1 = \frac{I}{m^-(p^*) + \tau}$$

is then smallest among equilibria.

For a uniform density, the equilibrium is, as we noted, unique, and, if interior, is given by

$$\left[\frac{1}{2}(p^* + \bar{p}) + \tau\right] R = (p^* + \tau) I.$$ 

(ii) Let us now look at the stock price reaction. The market value prior to the announcement of the equity issue is equal to total value (given that in­

$$V_0 = E[p]R + F(p^*)[\tau R - I]$$

$$= [F(p^*)m^-(p^*) + [1 - F(p^*)]m^+(p^*)] R + F(p^*)[\tau R - I].$$

The ex post value of shares upon an announcement is

$$V_1 = [m^-(p^*) + \tau] R - I.$$ 

And so

$$V_0 - V_1 = [1 - F(p^*)]$$

$$\times [m^+(p^*) R - [(m^-(p^*) + \tau) R - I]].$$

In the case of an interior equilibrium,

$$V_0 - V_1 = [1 - F(p^*)] R$$

$$\times \left[ m^+(p^*) - \frac{p^*}{p^* + \tau} (m^-(p^*) + \tau) \right].$$

But

$$\frac{m^+(p^*)}{p^*} > 1 > \frac{m^-(p^*) + \tau}{p^* + \tau}.$$ 

Hence,

$$V_0 - V_1 > 0.$$ 

(iv) Let

$$H(p^*, \tau) = \frac{\tau R}{I} [m^-(p^*) + \tau] - [p^* + \tau].$$

At the Pareto-dominant, interior equilibrium,

$$H_{p^*} < 0$$

(where the subscript denotes a partial derivative). Furthermore, and using the fact that $H = 0$ at an equilibrium,

$$H_{\tau} = [m^-(p^*) + \tau] R \frac{p^* - m^-(p^*)}{m^-(p^*) + \tau} > 0.$$ 

Hence, $p^*$ increase with $\tau$. So does the volume $[1 - F(p^*)] I.$

Exercise 6.6 (adverse selection and rating). (i) • Condition (1) means that the pledgeable income of a good (bad) borrower exceeds (is lower than) the investors’ investment $I - A$. The pledgeable income is equal to the expected income, $p_t R$, minus the entrepreneur’s incompressible share, $p_t b/\Delta p$ (or $p_t B/\Delta p$).

- To see that no lending occurs in equilibrium, note that the bad type (type B) always derives a (weakly) higher surplus from being financed than a good type (type b). Hence, contracts that provide financing to a good type will also provide financing to a bad one (pooling behavior).

Condition (1) implies that one cannot offer a breakeven contract that induces the bad type to work. So any breakeven contract must induce misbe­

havior by the bad type. But condition (2) in turn implies that pooling contracts with stakes for the bor­

rower in the interval $[b/\Delta p, B/\Delta p]$ generate a loss for the investors.

(ii) • In a separating equilibrium the good type chooses $x$ and then offers $R_b$, and the bad type, which is recognized, chooses $x = 0$ and, from condi­tion (1), receives no funding. Were the bad type to mimic the good type, she would get funding with probability $1 - x$; for, either the signal reveals the type and then she gets no funding, or the signal reveals nothing and the investors still believe they face a good type (we here use the fact that the equilibrium is separating).

Letting $R_{G_b}$ denote the good type’s “full information” (with net capital $A - rx$) contract (given by $p_t (R - R_{G_b}) = I - A + rx$), it must be the case that the bad type does not want to mimic the good type and prefers to keep her capital $A$ instead. That is,

$$A \geq (1 - x) [p_t R_{G_b} + B] + x(A - rx)$$
or
\[ A \geq x(A - rx) + (1 - x) \left[ \frac{\pi_l}{p_H} \left( R - I - A + rx \right) + B \right], \]
which yields the condition in the question. This condition is satisfied with equality at the separating equilibrium (see the chapter).

**Exercise 6.7 (endogenous communication among lenders).** (i) First, consider date 1. The assumption \([\alpha p + (1 - \alpha)q]R - I + \delta[\alpha p + (1 - \alpha)q](R - I) < 0\) implies that a foreign bank would not lend at date 1 even if it faced no competition at date 1 and it remained a monopoly at date 2 and hence could offer \(R_b = 0\) in either period (with probability \(\alpha p + (1 - \alpha)q\), the borrower would be known to be successful at date 2).

Thus, only the local bank will lend at date 1. Furthermore, the condition
\[ qR - I + \delta q(R - I) < 0 \]
implies that it would not lend to a bad type even if it faced no competition in either period. Hence, the local bank lends only to the good type. It offers \(R_b = 0\).

In the absence of information sharing, foreign banks do not know whether the borrower succeeded at date 1, and therefore at date 2 (they put probability \(p\) on the borrower’s being successful at date 2).

Note that the foreign banks do not want to make offers to the local borrower at date 2: suppose that they offer \(R_b < R\). Either the borrower will succeed and then the local, incumbent bank will offer a bit more \((R_b + \varepsilon)\), or it will fail and then the incumbent will not bid. Hence, a foreign bank can win the contest for the local firm only if the latter will fail. Hence, they do not bid, and the incumbent bank bids \(R_b = 0\) if the borrower is successful (and does not finance otherwise). The local bank’s profit (and thus each bank’s profit since banks do not make profits in foreign markets) is
\[ \pi_{ns} = \alpha[pR - I + \delta p(R - I)], \]
where “ns” means “no sharing.”

The borrower’s *ex ante* utility is
\[ U_{b}^{\text{ns}} = 0. \]
Suppose now that banks share their information. They are then Bertrand competitors at date 2 and make no profit at that date. But the local bank still lends at date 1 if the borrower’s type is \(p\); the profits and utilities are
\[ \pi^s = \alpha[pR - I] \quad \text{and} \quad U_{b}^s = \delta \alpha p(R - I). \]

Hence, banks do not want to share their information.

(ii) Suppose now that \(\alpha\) is endogenous. Then \(C(\alpha)\) needs to be subtracted from the borrower’s previous utility (which is now a gross utility) in order to obtain the net utility.

In the absence of information sharing, the borrower is held up by the local bank, and so
\[ \alpha_{ns} = \pi_{ns} = U_{b}^{\text{ns}} = 0. \]

Under information sharing, the borrower’s investment is given by
\[ \max_{\alpha} \{\delta \alpha p(R - I) - C(\alpha)\}, \]
and so, for an interior solution,
\[ C'(\alpha^*) = \delta p(R - I). \]

Then
\[ \pi^s = \alpha^* [pR - I] > \pi_{ns} \]
and
\[ U_{b}^s = \delta \alpha^* p(R - I) - C(\alpha^*). \]

**Exercise 6.8 (pecking order with variable investment).** (i) The separating program is
\[ \max_{\{p_H R_b^g + (1 - p_H) R_b^f\}} \]
\[ \{p_H (R^g I - R_b^g) + (1 - p_H) (R^f I - R_b^f)) \geq I - A, \quad (I_RI) \]
\[ q_H R_b^g + (1 - q_H) R_b^f \leq U_{b}^{\text{SI}}, \quad (M) \]
\[ (\Delta p) (R_b^g - R_b^f) \geq BI. \quad (I_Cb) \]

Note that \((IC_b)\) implies that the bad borrower works if she mimics the good one.

(ii) The key observation is that the solution to the separating program satisfies
\[ R_b^f = 0. \]

That is, the good borrower receives nothing in the case of failure. In particular, if \(R^f I\) stands for the salvage value of the leftover assets, this salvage value is entirely transferred to the investors in the case of failure.
The proof of this observation is instructive. Suppose that $R^b_b > 0$. Consider a small increase $\delta R^b_S > 0$ in the borrower’s reward in the case of success and a small decrease $\delta R^b_F < 0$ in her reward in the case of failure such that

$$ p_H(\delta R^b_S) + (1 - p_H)(\delta R^b_F) = 0. $$

This change alters neither the objective function nor the investors’ profit from the good borrower (see (IR_1)), but it relaxes the moral-hazard constraint (IC_b), and interestingly the mimicking constraint as well since $q_H < p_H$. In words, a good borrower, who has a higher probability of success, cares relatively more about her income in the case of success and relatively less about her income in the case of failure than a bad borrower.

(iii) Because the weak monotonic-profit assumption is satisfied, Proposition 6.2 in the supplementary section implies that the separating allocation is the unique perfect Bayesian equilibrium allocation if and only if prior beliefs lie below some threshold $\alpha^*$. 

Exercise 6.9 (herd behavior). Entrepreneur 1, who moves first, chooses his best project, regardless of the state of nature. The investors then attach probability of success

$$ m = \alpha p + (1 - \alpha)q $$

to the project. They are willing to go along with compensation $R^1_b$ such that

$$ m(R - R^1_b) = I. $$

Now consider entrepreneur 2. In the unfavorable environment, she has no choice but choosing the strategy that gives a probability of success. Suppose now that she herds with entrepreneur 1 in the favorable environment. Her overall probability of success when she selects the same strategy as entrepreneur 1 is

$$ \partial p + (1 - \partial) r. $$

So let $R^u_2$ and $R^f_b$ denote the second entrepreneur’s compensation in the case of success depending on whether the environment is unfavorable or favorable, respectively:

$$ q(R - R^u_2) = 1 \quad \text{and} \quad (\partial p + (1 - \partial) r)(R - R^f_b) = 1. $$

Herd behavior requires that

$$ R^u_2 > p R^f_b $$
or

$$ R - \frac{I}{\partial p + (1 - \partial) r} \geq p \left[ R - \frac{I}{q} \right]. $$

This condition requires in particular that, despite herding, the choice of the same strategy by both entrepreneurs is sufficiently good news about the environment $(\partial p + (1 - \partial) r > q)$ and therefore brings about much better financing terms for entrepreneur 2. It is satisfied, for example, if the project is hardly creditworthy in the unfavorable environment $(qR = I)$ and $r$ is not too small.

Exercise 6.10 (maturity structure). In this simple example the good borrower can costlessly separate from the bad one by not hoarding any liquidity (i.e., setting short-term debt $d = r$). Because $\rho^G > \rho$, the good borrower knows that she will be able to find sufficient funds by going to the capital market at date 1 and diluting existing external claims. By contrast, the project will be stopped at date 1 for the bad borrower in the absence of liquidity hoarding, which would not be the case if the borrower resorted to hoarded liquidity rather than to the capital market to meet the liquidity shock.

This example is very special but it conveys the basic intuition: going back to the capital market is less costly for a good borrower than for a bad one if information about the firm’s quality accrues in between. What is special about the example is that signaling by not hoarding liquidity is costless to the good borrower. Suppose that the liquidity shock is random and may exceed $\rho^G$. Then we know from Chapter 5 that it is optimal for the good borrower to hoard liquidity under symmetric information. So, signaling may involve insufficient continuation in general.
Exercise 7.1 (competition and vertical integration). (i) The project can be financed because there is enough pledgeable income from condition (1).
   - Feasible contracts:
     \[ R^F + \theta_1 M \geq I \quad \text{and} \quad (\Delta p)(1 - \theta_1)M \geq B. \]
   For example, the debt contract,
   \[ R^F = R^E \quad \text{and} \quad \theta_1 = (I - R^E)/M \]
   (which amounts to a debt \( D = I \), is an optimal contract. To obtain it as the unique optimal contract, one could, for example, add variable investment.
   (ii) The entrepreneur obtains
   \[ U_b = R^E + M - I \]
   under an exclusive contract with the supplier. By contrast, the industry profit when the rival obtains the enabling technology is
   \[ 2(R^F + D - I) + K < R^E + M - I \]
   from condition (2) and the profit-destruction effect. Because neither the supplier's nor the rival's rent (which is 0 under exclusivity) can decrease, the entrepreneur cannot gain from nonexclusivity.
   - The supplier will not find it profitable to supply the enabling technology to the rival if and only if
     \[ R^F + \theta_1 M \geq R^E + \theta_1 D + \left[ R^E + \left( D - \frac{B}{\Delta p} \right) (I - K) \right] \]  \( (3) \)
   or
   \[ \theta_1 (M - D) \geq R^E + \left( D - \frac{B}{\Delta p} \right) (I - K). \]
   The term in square brackets in (3) is the difference between the rival's pledgeable income and the extra investment cost \( I - K \). The solution is thus to offer enough equity to the supplier. Note that the borrower can always achieve this while maintaining borrower incentives: \((\Delta p)(1 - \theta_1)D \geq B\). (If the borrower chose effort after observing the supplier's action, the incentive constraint would become \((\Delta p)(1 - \theta_1)M \geq B\).)

Remark. For some parameter values an optimal debt/equity mix might involve a larger expected payment for the supplier than the investment \( I \), but that is not a problem as the entrepreneur may demand a lump-sum payment equal to the difference up front, thus leaving the supplier with no rent.

Exercise 7.2 (benefits from financial muscle in a competitive environment). (i) If \( \rho > \rho_0(R) \), then the entrepreneur will not be able to withstand the liquidity shock if it occurs. Hence, it needs a liquidity cushion, perhaps in the form of a credit line.
   - The NPV is
     \[ (1 - \lambda)[\rho_1(R)] + \lambda[\rho_1(R) - \rho]z - I, \]
     where \( z = 1 \) if the firm withstands the liquidity shock, and \( z = 0 \) otherwise. Hence,
     (a) \( z = 0 \) if \( \rho \geq \rho_1(R) \);
     (b) \( z = 1 \) if \( \rho < \rho_1(R) \) and there is enough pledgeable income to “secure a credit line,”
     \[ \rho_0(R) \geq I - A + \lambda \rho \]
     or
     \[ (1 - \lambda)\rho_0(R) - (I - A) \geq \lambda[\rho_1(R) - \rho]; \]  \( (5) \)
     (c) \( z = 0 \) if (5) is not satisfied and
     \[ (1 - \lambda)\rho_0(R) \geq I - A; \]
     (d) no investment takes place if
     \[ (1 - \lambda)\rho_0(R) < I - A. \]
   (ii) Simultaneous choices: under simultaneous choices, there is no commitment effect. Condition (1) and question (i) imply that the incumbent does not want to withstand her liquidity shock regardless of the existence of the entrant. The left inequality in (2) then implies that the entrant has enough pledgeable income to obtain financing if the incumbent does not build financial muscle (and wants to be financed from (3)); while the right inequality prevents the incumbent from investing \((I - A) > \rho_0(C) > (1 - \lambda)\rho_0(C)\).
   - Sequential choices: suppose now that the incumbent chooses her financial structure first. The analysis of the simultaneous choice case shows that the incumbent cannot obtain financing without financial muscle. By contrast, condition (2) shows that the incumbent deters entry if she commits to withstand her liquidity shock. Condition (4) then implies that the incumbent has enough pledgeable income in a monopoly situation even if she withstands the costly liquidity shock.
Exercise 7.3 (dealing with asset substitution). (i) • The liquidation value \( L_0 \) is fully pledgeable. By contrast, only \( R - R_b \) is pledged in the case of success, where
\[
p_{H}R_b \geq p_{H}R_b + B.
\]
Hence, the left-hand side of (1) is the pledgeable income.

• With a competitive capital market, the entrepreneur’s utility is the NPV:
\[
U^* = (1 - x)L_0 + x(p_{H}R - I).
\]

• Optimal contracts must satisfy
\[
(1 - x)(L_0 - r_b) + p_{H}(R - R_b) = I - A,
\]
with
\[
R_b \geq B/\Delta p.
\]
For \( A = \overline{A} \) the optimal contract is necessarily a debt contract (\( r_b = 0 \)).

(ii) • Interpretation of equation (2). The NPV is
\[
(1 - x)L + x[p_{H} + \tau(L)]R - I.
\]
Hence, \( L = L_0 \) maximizes the NPV, which is then equal to \( U^*_b \).

• Consider a “step-function” contract: in the case of liquidation, the entrepreneur receives
\[
0 \quad \text{if} \ L < L_0,
\]
\[
r_b \quad \text{if} \ L \geq L_0.
\]
Furthermore, the entrepreneur receives \( R_b = B/\Delta p \) in the case of continuation and success (this value minimizes both the nonpledgeable income and the incentive to cut down on maintenance to raise future profit). With this incentive scheme, the entrepreneur’s utility
\[
(1 - x)r_b(L) + x[p_{H} + \tau(L)]R_b
\]
is maximized either at \( L = L_0 \) or at \( L = 0 \). One therefore needs
\[
(1 - x)r_b + x(p_{H} + \tau(0))B/\Delta p \geq x[p_{H} + \tau(0)]B/\Delta p.
\]
The threshold for financing that does not encourage asset substitution is given by
\[
I - A^* = (1 - x)(L_0 - r_b) + x(p_{H}R - B/\Delta p),
\]
where \( r_b \) is given by the first inequality satisfied with equality.

Exercise 7.4 (competition and preemption). Let us first compute the first date \( t_1 < t_0 \) at which lenders are willing to finance an entrepreneur who will later on be a monopolist:
\[
I - e^{-r(t_0-t_1)}A = e^{-r(t_0-t_1)}p_{H}\left(M - \frac{B}{\Delta p}\right).
\]
Thus no financing is feasible before date \( t_1 \).

Next, compute the earliest date \( t_b < t_0 \) at which the entrepreneur prefers to invest (as a monopolist) rather than just consuming her endowment:
\[
NPV = e^{-r(t_0-t_b)}p_{H}M - I = 0,
\]
where the NPV is computed from date \( t_b \) on.

The condition in the statement of the question,
\[
p_{H}M \geq p_{H}\left(M - \frac{B}{\Delta p}\right) + A,
\]
is equivalent to
\[
t_b \geq t_1.
\]
Note that \( t_b < t_1 \) if \( A = 0 \).

(a) If \( t_b \geq t_1 \), then the equilibrium involves rent equalization, as in Fudenberg and Tirole (1985). Only one entrepreneur invests, and this at date \( t_b \). (See Fudenberg and Tirole (1985) for a more rigorous description of the strategies.) This entrepreneur does not enjoy any rent relative to the entrepreneur who does not invest.

(b) If \( t_b < t_1 \), then we are back to a situation similar to the static game. Entrepreneurs are unable to invest before \( t_1 \), even though, starting from \( t_b \), they would like to preempt their rival. (Again, we refer to Fudenberg and Tirole (1985) for more details about this type of situation.)

Exercise 7.5 (benchmarking). (i) Let us write the NPV, the breakeven constraint, and the incentive constraint. First, the NPV accounts for deadweight losses due to negative incomes:
\[
U_b = \text{NPV} = \rho(p_{H}D - (1 - p_{H})\theta b_2)
\]
\[
+ (1 - \rho)(p_{H}^2D + p_{H}(1 - p_{H})(M - \theta b_1)
\]
\[
- (1 - p_{H})^2\theta b_2)] - I.
\]

The breakeven constraint is
\[
\rho[p_1(D - a_2) + (1 - p_1)b_2] \\
+ (1 - \rho)[p_2(D - a_2) + p_1(M - a_1 + b_1)] \\
+ (1 - p_1)\Delta b_2 \geq I - A.
\]

Lastly, the incentive constraint is
\[
\rho[a_2 + (1 + \theta)b_1] \\
+ (1 - \rho)[p_1[a_2 + (1 + \theta)b_1] \\
+ (1 - p_1)[a_1 + (1 + \theta)b_1]] \geq \frac{B}{\Delta p}.
\]

To show that one can set \( a_2 = b_2 = 0 \) without loss of generality, write the Lagrangian and the first-order condition. Equivalently, if \( a_2 > 0 \), we can decrease \( a_2 \) and increase \( a_1 \) so as to keep both \( (IR_1) \) and \( (IC_b) \) unchanged, and note that these two variables do not enter into the expression of the NPV; while, if \( b_2 > 0 \), we can decrease it and increase \( b_1 \) so that \( (IR_2) \) and the NPV are kept intact, but \( (IC_b) \) is then not binding.

The diagrammatic representation of the problem in the \((a_1, b_1)\)-space is as in Figure 4.

(ii) • When \( \rho \) tends to 1: \( b_1 \) going to infinity has almost no cost in terms of NPV. Thus \((IC_b)\) becomes costless to satisfy, as in Section 7.1.1 in the case of perfect correlation.

• When \( \theta \) goes to 0, then punishments are almost costless, and so again \((IC_b)\) can be satisfied without jeopardizing \((IR_1)\). Again there is basically no agency cost (as in the case in which firms have a large amount of collateral that the lenders value almost as much as the borrower).

**Exercise 7.7 (optimal contracts in the Bolton–Scharfstein model).** Consider a more general long-term contract in which the entrepreneur’s reward contingent on different events is \( r_b^S \) if date-0 profit is \( D \) but there is no refinancing at date 1 (with probability \( z^S \)); and, if refinanced, \( R_b^{FS} (R_p^{FS}) \) when the entrepreneur succeeds in both periods (when she fails at date 0, but succeeds at date 1, respectively). When reinvesting at date 1, to “commit to” high effort, the entrepreneur should keep a high enough stake, i.e., \( R_b^{SS} \) and \( R_p^{FS} \geq B/\Delta p \).

Fixing the continuation policy \( z^S \) and \( z^F \), as long as the high effort is guaranteed the predation deterrence constraint is not affected by this enrichment of the contract space:

\[
D \geq (z^S - z^F)(M - D).
\]

The date-0 incentive compatibility constraint and investor’s breakeven constraint, however, need to be modified:

\[
\begin{align*}
&z^S r_b^{SS} + (1 - z^S)r_b^S \\
\geq & B_0 + p_1[z^S R_b^{SS} + (1 - z^S)r_b^S] + (1 - p_1)z^F R_b^{FS} \\
\iff & (\Delta p)[z^S R_b^{SS} - z^F R_b^{FS} + (1 - z^F)r_b^S] \geq B_0
\end{align*}
\]

and

\[
I - A \leq z^S(D + D - R_b^{SS} - I) + (1 - z^S)(D - r_b^S) \\
\iff I - A \leq D + z^S(D - I - R_b^{SS}) - (1 - z^S)r_b^S.
\]

The entrepreneur’s expected utility is

\[
U_b = z^S R_b^{SS} + (1 - z^S)r_b^S - A = \text{NPV} = D - I + z^S(D - I),
\]

as usual, when \((IR')\) is binding.

As in Section 7.1.2, suppose \((PD)\) is binding. \((IC')\) is binding; for, if it were not, \( z^S \) could be increased to relax \((PD) \) without violating \((IC') \).

Then one can show that

• \( R_b^{SS} \geq R_p^{FS} \geq B/\Delta p \); if \( R_b^{SS} < R_p^{FS} \), then \( R_p^{FS} \) could be reduced so as to relax \((IC') \), which would
contradict the fact that (IC') is binding. And so \( R_{b}^{FS} = B/\Delta p \).

* \( r_{b}^{5} = 0 \): suppose \( z^{5} \in (0,1) \) and \( r_{b}^{5} > 0 \) (if \( z^{5} = 1 \), we could simply set \( r_{b}^{5} = 0 \)). From (PD) being binding, the incentive constraint can be written as

\[
z^{5}(R_{b}^{SS} - R_{b}^{FS}) + (1 - z^{5})r_{b}^{5} + \frac{D}{M - D}R_{b}^{FS} = B_{b}/\Delta p.
\]

Keeping \( z^{5} \) unchanged, we can decrease \( r_{b}^{5} \) and increase \( R_{b}^{SS} \) so that \( z^{5}R_{b}^{SS} + (1 - z^{5})r_{b}^{5} \) remains the same, i.e., in the case of date-0 success, one rewards the entrepreneur only in the case of continuation. There is no loss of generality in doing so since no constraint is affected, nor is the entrepreneur’s objective function.

Exercise 7.8 (playing the soft-budget-constraint game vis-à-vis a customer). (i) At date 2, given success and in the absence of a date-1 contract, the customer would offer a purchasing price equal to 0 (or any arbitrarily small but positive amount) and the entrepreneur would accept. In this event, the entrepreneur and the investors get zero profit. Therefore, by playing wait-and-see, the customer would enjoy expected payoff \( p_{L}v \), since the entrepreneur would shirk under this strategy. The same outcome prevails if the customer offers \( R = 0 \) at date 1.

Given that the entrepreneur has obtained funding at date 0, to induce a high probability of success at date 1 the customer needs to offer a price \( R = R_{l} + B/\Delta p \). This is more profitable for the customer than offering a contract that is not incentive compatible:

\[
p_{L}(v - R_{l} - B/\Delta p) > p_{L}v.
\]

When this inequality holds, the NPV is

\[
p_{H}(R_{l} + B/\Delta p) - I,
\]

which is smaller than \((\Delta p)v - I\). On the other hand, if the condition above is violated, it is optimal for the customer to offer \( R = 0 \). But in this case the entrepreneur shirks and the project is not financed at date 0.

(ii) Suppose now that the entrepreneur issues short-term debt \( r_{1} \) at date 0. At time 1 the customer has to cover \( r_{1} \) in order for the firm to continue. It is as if date 1 were an initial financing stage at which the customer finances an investment with size \( r_{1} \). The short-term debt can be chosen such that the customer finances the project only if the entrepreneur works, i.e.,

\[
p_{L}v < r_{1}.
\]

Then, to induce the high effort, the customer offers a transfer price \( R = B/\Delta p \), on top of \( r_{1} \). The customer gets

\[
p_{H}(v - B/\Delta p) - r_{1}.
\]

By assumption \( p_{H}(v - B/\Delta p) > p_{L}v \). It is possible to extract the full surplus from the customer by setting

\[
\gamma = p_{H}(v - B/\Delta p).
\]

This amount is greater than \( I - A \) by assumption and so investors are willing to finance the project at date 0. The entrepreneur then gets

\[
p_{H}B/\Delta p - A + [\gamma - (I - A)],
\]

which is equal to the NPV, \( p_{H}v - I \). This is intuitive since both the initial investors and the customer get zero profit.

Exercise 7.9 (optimality of golden parachutes). Consider the following class of contract: when the entrepreneur reports a signal \( s \in \{r,q\} \), the probability of continuation is \( z^{s} \). She is paid \( R_{b}^{s} \) in the case of continuation and success, and \( T^{s} \) in the case of termination. In the latter event, the investors get \( L_{1}^{s} = L - T^{s} \leq L \).

In the case of continuation, in order to overcome the moral-hazard problem, both \( r_{b}^{5} \) and \( R_{b}^{5} \) must exceed \( B/\Delta p \). For the \( q^{s} \)-type entrepreneur, the (NM) constraint is now

\[
z^{q^{s}}(R_{b}^{SS} - R_{b}^{FS}) + (1 - z^{q^{s}})R_{b}^{FS} + (1 - z^{q^{s}})T^{q^{s}} - I_{q^{s}}.
\]

The investors’ breakeven condition is

\[
I - A \leq \alpha[z^{q^{s}}R_{b}^{SS} + (1 - z^{q^{s}})(L - T^{q^{s}})] + (1 - \alpha)[z^{q^{s}}q_{H}(R_{b}^{SS} + (1 - z^{q^{s}})T^{q^{s}}) - A] - I,
\]

and the entrepreneur gets expected payoff

\[
U_{b} = \alpha[z^{q^{s}}R_{b}^{SS} + (1 - z^{q^{s}})T^{q^{s}}] + (1 - \alpha)[z^{q^{s}}q_{H}R_{b}^{SS} + (1 - z^{q^{s}})T^{q^{s}}] - A
\]

\[
= \text{NPV}
\]

\[
= \alpha[z^{q^{s}}R_{b}^{SS} + (1 - z^{q^{s}})L] + (1 - \alpha)[z^{q^{s}}q_{H}R + (1 - z^{q^{s}})L] - I,
\]

under the investors’ breakeven condition.
We claim that the following properties hold:

- (NM) is binding. Otherwise, we could decrease either \( R_0^q \) or \( T_d \) and increase the pledgeable income unless \( R_0^q = B/\Delta p \) and \( T_d = 0 \). But, in the latter case, from (NM)’ being slack, we must have \( z^d > 0 \), then from \( L > q_h(R - B/\Delta p) \) the pledgeable income can be increased by reducing \( z^d \).
- \( R_0^q = B/\Delta p \): if \( R_0^q > B/\Delta p \), decreasing it boosts pledgeable income and relaxes (NM).
- \( T_d = 0 \): suppose \( T_d > 0 \) and \( z^d < 1 \) (when \( z^d = 1 \), we can simply set \( T_d = 0 \)). Following the logic of Section 7.2.1, a simultaneous change of \( T_d \) and \( z^d \) that keeps the pledgeable income constant must satisfy

\[
\left[ r_h \left( R - \frac{B}{\Delta p} \right) - L + T_d \right] d z^d = (1 - z^d) \ d T_d.
\]

By doing so, the LHS of (NM)’ changes by an amount equal to

\[
\left[ r_h \left( R - \frac{B}{\Delta p} \right) - L + (q_h - \tau) \frac{B}{\Delta p} \right] dz^d;
\]

(NM)’ is relaxed by a simultaneous decrease of \( T_d \) and \( z^d \). (If \( z^d = 0 \), we could instead decrease \( T_d \) to relax (NM)’ and increase the pledgeable income.)

Incorporating these findings, the program becomes

\[
\begin{align*}
\max \{ \text{NPV} = & \alpha[L + z^d (r_h R - L)] \\
& + (1 - \alpha) \{ [L + z^d (q_h R - L)] - I \} \}
\end{align*}
\]

s.t. \( z^d (q_h - \tau) \frac{B}{\Delta p} = z^d q_h R_0^q + (1 - z^d) T_d \), (NM)’

\[
I - A = \tau = \alpha \{ L + z^d [r_h (R - B/\Delta p) - L]] \\
& + (1 - \alpha) \{ [L + z^d (q_h R - R_0^q)] - L \} \\
& - (1 - z^d) T_d \}
\]

(\( \text{IR}' \))

- When \( q_h R > L \), it is optimal not to adopt the golden parachute policy, \( T_d = 0 \): suppose \( T_d > 0 \). First, note that to satisfy (NM)’ as an equality, \( T_d < q_h B/\Delta p \leq q_h R_0^q \) as long as \( \tau > 0 \). Therefore, an increase in \( z^d \) relaxes (NM)’ and increases the NPV. Consider a simultaneous change in \( z^d \) and \( T_d \) that leaves (NM)’ unchanged:

\[
(q_h R_0^q - T_d) \ dz^d = -(1 - z^d) \ d T_d.
\]

Since \( T_d < q_h R_0^q \), a decrease in \( T_d \) comes with an increase in \( z^d \), which increases the NPV. This change is feasible since the pledgeable income is increased:

\[
\begin{align*}
d T &= [q_h (R - R_0^q) - L + T_d] \ d z^d -(1 - z^d) \ d T_d \\
& = (q_h R - L) \ d z^d > 0.
\end{align*}
\]

- When \( q_h R < L \), a golden parachute is optimal, \( T_d > 0 \) and \( z^d = 0 \). From \( T_d < q_h R_0^q \), the relevant part in the pledgeable income can be written as

\[
L + z^d (q_h R - L - (q_h R_0^q - T_d)) - T_d,
\]

therefore decreasing \( z^d \) raises both the pledgeable income and the NPV. At the optimum \( z^d = 0 \), and the optimal \( T_d \) is determined by (NM):

\[
T_d = \tau (q_h - \tau) \frac{B}{\Delta p}.
\]

It is also easy to check for both cases that the (NM) constraint of the \( r \)-type entrepreneur is not binding.

**Exercise 7.10 (delaying income recognition).** We look for a “pooling equilibrium” in which the entrepreneur keeps a low profile (\( y_1 = 0 \)) when successful (\( y_1 = R_1 \)). To this end, let us compute the posterior probability \( \alpha_{\text{LB}} \) (where “LB” stands for “late bloomer”) that the entrepreneur has high ability at date 2 \((H_2)\) following (reported) profit 1 at date 1 and (actual and reported) profit 2 at date 2:

\[
\alpha_{\text{LB}} = \text{Pr}(H_2 | (0, R_2)) = \frac{A + B}{C + D}
\]

where \( A = \alpha \rho (r + r \tau), \)

\( B = (1 - \alpha) \rho (r + q \tau), \)

\( C = \alpha \rho r + (1 - \alpha) q + r \tau, \)

\( D = (1 - \alpha) (1 - \alpha) q + q \tau. \)

The numerator represents the probability that the entrepreneur has ability \( H_2 \) and succeeds at date 2: with probability \( \alpha \rho \), she had high ability at date 1 and still has high ability and so has average probability of success \( r + r \tau \) (due to the date-1 hidden savings made when she is successful at date 1, which has probability \( r \)); with probability \( (1 - \alpha) (1 - \rho) \) she had low ability at date 1 (and therefore had hidden savings with probability \( q \) and became expert in the task (and so has probability of success \( r + q \tau \)). The denominator represents the total probability of date-2 success in this pooling equilibrium, and is computed in a similar way.

By contrast, the probability that the entrepreneur has type \( H_2 \) when she fails at date 2 is

\[
\alpha_0 = \frac{E + F}{G + H} < \alpha_{\text{LB}},
\]

where \( E = \alpha \rho r, \)

\( F = (1 - \alpha) \rho (r + q \tau), \)

\( G = \alpha \rho r + (1 - \alpha) q + r \tau, \)

\( H = (1 - \alpha) (1 - \alpha) q + q \tau. \)
where \( E = \alpha \rho [1 - (r + r \tau)] \), \( F = (1 - \alpha)(1 - \rho)[1 - r - r q \tau] \), \( G = \alpha[1 - (\rho r + (1 - \rho)q + r \tau)] \), and \( H = + (1 - \alpha)(1 - [(1 - \rho)r + \rho q + q r \tau]) \).

Suppose now that the entrepreneur reports \( \hat{y}_1 = R_1 \). Let

\[
\alpha_{EB} \equiv \Pr(H_2 \mid (R_1, R_2)) = \frac{I}{J + K}
\]

(where \( I = [\alpha \rho r + (1 - \alpha)(1 - \rho)q] r \), \( J = [\alpha \rho r + (1 - \alpha)(1 - \rho)q] r \), and \( K = [\alpha(1 - \rho)r + (1 - \alpha)\rho q] q \))

and

\[
\beta_{EB} \equiv \Pr(H_2 \mid (R_1, 0)) = \frac{M}{N + O}
\]

(where \( M = [\alpha \rho r + (1 - \alpha)(1 - \rho)q](1 - r) \), \( N = \{\alpha \rho r + (1 - \alpha)(1 - \rho)q\}(1 - r) \), and \( O = [\alpha(1 - \rho)r + (1 - \alpha)\rho q]q \}) denote the posterior beliefs when such an “early bloomer” (EB) succeeds and fails at date 2, respectively. It can be checked that a good report at date 1 improves one’s reputation for an arbitrary date-2 performance,

\[
\alpha_{EB} > \alpha_{LB} \quad \text{and} \quad \beta_{EB} > \alpha_{F},
\]

and that

\[
\alpha_{LB} > \beta_{EB}.
\]

Intuitively, a late success is more telling than an early one if either the type has a reasonable probability to evolve or if an early success confirms what one already knows, namely, that the entrepreneur has high ability.

Now assume that

\[
\alpha_{LB} > \alpha_{LB} > \alpha_{F} > \beta_{EB} > \alpha_{F}.
\]

Then, the entrepreneur keeps her job at date 3 if and only if she succeeds at date 2. Keeping a low profile at date 1 when \( y_1 = R_1 \) is then the optimal strategy because it increases the probability of date-2 success by \( \tau \).

**Exercise 8.1 (early performance measurement boosts borrowing capacity in the variable-investment model).** In the variable-investment model, the private benefit of shirking is \( BI \), and the income in the case of success \( RI \). Using the notation of Section 8.2.2, the incentive compatibility constraint is

\[
(\sigma_{IIH} - \sigma_{II}) R_0 \geq BI,
\]

where \( R_0 \) is the entrepreneur’s reward in the case of success. The borrowing capacity is then given by the investors’ breakeven constraint:

\[
p_H R I - \sigma_{IIH} \frac{BI}{\sigma_{IIH} - \sigma_{II}} = I - A.
\]

And so

\[
U_b = \sigma_{IIH} R_0 - A = (p_H R - 1)I
\]

\[
= 1 - \frac{p_H - 1}{\rho_1} \frac{\rho_1 - 1}{1 - (\rho_1 - \sigma_{IIH} B / (\sigma_{IIH} - \sigma_{II})).}
\]

In the absence of an intermediate signal, the expression is the same except that \( \sigma_{IIH} / (\sigma_{IIH} - \sigma_{II}) \) is replaced by \( p_H / [p_H - p_L] \).

**Exercise 8.2 (collusion between the designated monitor and the entrepreneur).** When the signal is high, there is no collusion. In the absence of collusion, the entrepreneur obtains \( R_b \) since it is in the interest of the monitor to exercise his options. Furthermore, the entrepreneur cannot receive more than \( R_b \) from the assumption that the entrepreneur cannot receive income without being detected.

Suppose therefore that the signal is low. In the absence of collusion, the entrepreneur and the monitor both receive 0. Suppose that the entrepreneur instead offers to tunnel resources to the monitor. For a given choice of \( \tau \), the monitor agrees to collude if and only if his loss from exercising the options is compensated by the diverted resources:

\[
s[p_H - (\nu_L - \tau)] R < T(\tau).
\]

There is no collusion provided that

\[
H(s) \equiv \max_{\tau} \{T(\tau) - s[p_H - (\nu_L - \tau)]\} \leq 0.
\]

Because \( \partial H / \partial s < 0 \), there is no collusion provided that \( s \) exceeds some threshold.

**Exercise 9.1 (low-quality public debt versus bank debt).** Consider the three possible financing options.

**High-quality public debt.** Such debt has probability \( p_H \) of being reimbursed. As usual, the incentive constraint is

\[
(\Delta p) R_0 \geq B, \quad p_H \left(R - \frac{B}{\Delta p}\right) \geq I - A,
\]

and so such financing is doable only if

\[
\Rightarrow A_3 = I - p_H \left(R - \frac{B}{\Delta p}\right).
\]
The entrepreneur’s utility is then the NPV:

\[ U_3 = p_H R - I > 0. \]

**Low-quality public debt.** Such debt corresponds to the case in which the entrepreneur has too low a stake to behave; and this debt is repaid with probability \( p_L \):

\[ (\Delta p)_b < B \quad \text{and} \quad p_L (R - R_b) = I - A. \]

Hence,

\[ A_1 = I - p_L R. \]

The entrepreneur’s utility is then

\[ U_1 = p_L R + B - I > 0. \]

**Monitoring.** Follow the treatment in Chapter 9. To secure such financing with stake \( R_m \) for the monitor:

\[ (\Delta p) R_m \geq c \quad \text{and} \quad p_H R_m - c = I_m. \]

And so a necessary and sufficient condition is

\[ p_H \left( R - \frac{b}{\Delta p} \right) - c \geq I - A, \]

yielding threshold

\[ A_2 = I + c - p_H \left( R - \frac{b}{\Delta p} \right), \]

and NPV

\[ U_2^c = p_H R - I - c. \]

Summing up, under the assumptions made in the statement of the exercise:

\[ U_3 > U_2^c > U_1 > 0 \quad \text{and} \quad A_3 > A_2 > A_1. \]

So, financing is arranged as described in the statement of the question.

(A similar framework is used by Morrison\(^{12}\), except that the monitor is risk averse (which makes it more costly to hire). Morrison allows the monitor to contract with a “protection seller” in the credit derivative market in order to pass the default risk on to this third party and to thereby obtain insurance. This reduces the monitor’s incentive to monitor.)

**Exercise 9.2 (start-up and venture capitalist exit strategy).** (i) When the date-2 payoff can be verified at date 1, and there is no active monitor, the entrepreneur’s reward, \( R_b \), in the case of success must ensure incentive compatibility and allow investors to recoup their date-0 outlay:

\[ (\Delta p)_b \geq B \quad \text{and} \quad p_L (R - R_b) \geq I - A. \]

Because

\[ I - p_H \left( R - \frac{B}{\Delta p} \right) > A, \]

these two conditions are mutually inconsistent.

Suppose, in contrast, that an active monitor receives \( R_A \) in the case of success. We now have two incentive compatibility conditions and one breakeven condition:

\[ (\Delta p)_b \geq b, \]

\[ (\Delta p)_A \geq c_A, \]

and

\[ p_H (R - R_b - R_A) \geq I - A. \]

Because

\[ A > I - p_H \left( R - \frac{b + c_A}{\Delta p} \right), \]

these inequalities are consistent. The second and the third inequalities then bind, and so the NPV for the entrepreneur (which is equal to the total value created by the project minus the rent received by the monitor) is

\[ p_H R_b - A = p_H \left[ R - \frac{c_A}{\Delta p} \right] - I. \]

(ii) The conditions are

\[ p_H s[R - P] \geq c_p \]

(the speculator makes money when he acquires information and exercises his call option in the case of good news),

\[ (\Delta p)_s P \geq c_A \]

(this is the previous IC constraint with \( R_A = sP \)), and

\[ s \geq p_H R \]

(the speculator cannot make money by refusing to monitor and purchasing the shares at price \( P \)).

Ignoring the last constraint yields the condition in the statement of the exercise. The third constraint requires that

\[ \frac{c_A}{c_p} \geq \frac{1 - p_H}{p_H (\Delta p)}. \]

If this condition is not satisfied, the speculator does not have enough incentives to acquire the information when only the shares of the active monitor are

brought to the market at date 1. This means that the active monitor should be granted the right to “drag along” the shares (or some of the shares) of the limited partners in order to ensure the stock receives enough attention.

Exercise 9.3 (diversification of intermediaries).
(i) Straightforward. Follows the lines of Chapters 3 and 4.
(ii) Similar to Chapter 4’s treatment of diversification.

The venture capitalist obtains \( R_m \) if both projects succeed. The incentive constraints are
\[
p_{II}^2 R_m \geq p_{II} p_L R_m + c
\]
(no shirking on monitoring one firm)
\[
\geq p_{II}^2 R_m + 2c
\]
(no shirking on monitoring both firms).

As usual, it can be checked that only the latter constraint is binding. So
\[
R_m \geq \frac{2c}{(\Delta p)(p_{II} + p_L)}.
\]
The nonpledgeable income (aggregated over the two firms) is
\[
2 \left[ p_{II} \frac{b}{\Delta p} + p_L \left( \frac{p_{II}}{p_{II} + p_L} \frac{c}{\Delta p} \right) \right].
\]

Exercise 9.4 (the advising monitor model with capital scarcity). The entrepreneur’s utility when enlisting a monitor is now equal to the NPV minus the rent derived by the monitor:
\[
U_{b}^m = (p_{II} + q_{II}) \left( R - \frac{c}{\Delta q} \right) - I.
\]
Note that \( U_{b}^m \) may no longer exceed
\[
U_{b}^{\text{nom}} = p_{II} R - I,
\]
even when \( (\Delta q) R > c \).

Funding with a monitor on board is feasible if and only if
\[
(p_{II} + q_{II}) \left( R - \frac{B}{\Delta p} - \frac{c}{\Delta q} \right) \geq I - A.
\]
The presence of a monitor facilitates funding if and only if
\[
(p_{II} + q_{II}) \left( R - \frac{B}{\Delta p} - \frac{c}{\Delta q} \right) > p_{II} \left( R - \frac{B}{\Delta p} \right)
\]
or
\[
q_{II} R > c + p_{II} \frac{c}{\Delta q} + q_{II} \frac{B}{\Delta p}.
\]

The left-hand side is the increase in expected revenue; the right-hand side is the sum of the monitoring cost and the extra rents for the two agents.

Exercise 9.5 (random inspections). (i) Suppose first that the entrepreneur behaves with probability 1; then there is no gain from monitoring and so \( y = 1 \). But, in the absence of monitoring, the entrepreneur prefers to misbehave:
\[
(\Delta p) R_b < B,
\]
a contradiction. Conversely, suppose that the entrepreneur misbehaves with probability 1; because
\[
\nu R_m > c,
\]
the monitor monitors for certain \( (y = 0) \). But then the entrepreneur prefers to behave as
\[
p_{II} R_b > 0.
\]
Hence, the entrepreneur must randomize. For her to be indifferent between behaving and misbehaving, it must be the case that
\[
p_{II} R_b = y(p_{II} R_b + B) + (1 - y) \cdot 0
\]
or
\[
y = \frac{p_{II} R_b}{p_{II} R_b + B}.
\]
Similarly, the monitor must randomize. Indifference between monitoring and not monitoring implies that
\[
(1 - x) p_{II} R_m + x(p_L + \nu) R_m - c
\]
\[
= (1 - x) p_{II} R_m + x p_L R_m
\]
or
\[
x \nu R_m = c \iff x = \frac{c}{\nu R_m}.
\]
(ii) Assume that \( p_{II}(R - B/\Delta p) < I - A \), so that financing is not feasible in the absence of a monitor. As usual, one should be careful here: because the monitor has no cash and thus cannot be asked to contribute to the investment and gets a rent, the borrower’s utility differs from the NPV,
\[
U_b = (1 - x) p_{II} R_b + x y(B + p_L R_b) - A
\]
\[
= p_{II} R_b - A,
\]
using the indifference condition for the entrepreneur. The uninformed investors’ breakeven condi-
tion is
\[ P = (1 - x)p_H(R - R_b - R_m) \\
+ \gamma y p_L(R - R_b - R_m) \\
+ (1 - y)(p_L + \nu)(R - R_m) \]
\[ \geq I - A. \]

Note that \( y = 0 \) maximizes \( P \). First, if \( x > 0 \), a smaller \( y \) increases the amount of money returned to uninformed investors when correcting misbehavior. Second, it raises managerial discipline (reduces the level of \( R_b \) necessary to obtain incentive compatibility); indeed \( R_b \) can be taken equal to 0! (Note this would no longer hold if the entrepreneur could capture private benefit \( b \in (0, B] \) before being fired.) The pledgeable income is then
\[ P = [(1 - x)p_H + x(p_L + \nu)] \left[ R - \frac{c}{x\nu} \right]. \]

Noting that \( \partial P/\partial x > 0 \) at \( x = 0 \) and \( \partial P/\partial x < 0 \) at \( x = 1 \), the pledgeable income is maximized for \( x \) between 0 and 1. (The optimum does not, of course, involve \( R_b = 0 \). We are just computing what it takes to obtain financing.)

(iii) We know from Chapter 8 that the entrepreneur is best rewarded on the basis of a sufficient statistic for her performance. Here, the monitor’s information is not garbled by exogenous noise, unlike the final outcome. Hence, it would in principle be better to reward the management on the basis of information disclosed (in an incentive-compatible way) by the monitor. We leave it to the reader to derive the optimal contract when one allows the monitor to report on his observation of the entrepreneur’s choice of effort.

**Exercise 9.6 (monitor’s junior claim).** Let \( R_b^S \) and \( R_b^F \) denote the entrepreneur’s rewards in the cases of success and failure. We are interested in situations in which the entrepreneur would choose the bad project if left unmonitored:

\[ (\Delta p)(R_b^S - R_b^F) < B. \]

Under monitoring, incentive compatibility requires that

\[ (\Delta p)(R_b^S - R_b^F) \geq b, \]

where \( \Delta p = p_H - p_L \).

Similarly, the monitor’s compensation scheme must satisfy

\[ (\Delta p)(R_m^S - R_m^F) \geq c. \]

The uninformed investors are willing to lend if and only if

\[ p_H(R_b^S - R_b^F - R_m^F) + (1 - p_H)(R_F - R_b^F - R_m^F) \geq I - A. \]

Finally, the borrower’s utility is

\[ p_H R_b^F + (1 - p_H) R_m^F. \]

It is therefore in the borrower’s interest to minimize the monitor’s rent,

\[ p_H R_b^F + (1 - p_H) R_m^F - c, \]

subject to his incentive constraint,

\[ (\Delta p)(R_m^S - R_m^F) \geq c. \]

This yields

\[ R_m^F = 0 \quad \text{and} \quad R_m^S = \frac{c}{\Delta p}. \]

A necessary and sufficient condition for the borrower to have access to financing is

\[ p_H \left( R_b^S - \frac{b + c}{\Delta p} \right) + (1 - p_H) R_F \geq I - A. \]

**Exercise 9.7 (intertemporal recoupment).** (i) **Long-term contracts.** The potential NPV is

\[ V = 2p_H R - (I_1 + I_2) - 2c. \]

Under competition among monitors, the borrower can obtain \( V \), for example, by proposing a contract specifying that the selected monitor at date \( t \), \( t = 1, 2 \), contributes \( I_m^t \) and receives \( R_m^t \) in the case of success (and 0 in the case of failure) such that

\[ p_H(I_m^1 + R_m^1) = I_m^1 + I_m^2 + 2c, \]

\[ (\Delta p)R_m^1 = c. \]

(The reader familiar with Sections 4.2 and 4.7 will notice that considering two incentive constraints, one per period, is in general not optimal. More on this later. However, we here show that the upper bound on the borrower’s utility can be reached, and so we do not need to enter the finer analysis of “cross-pledging.”)

Similarly, giving a stake \( R_b^t \) in the case of success (and 0 in the case of failure) such that

\[ (\Delta p)R_b^t \geq b \]
suffices (but is not necessary) to ensure borrower incentive compatibility.

Uninformed investors are then willing to finance the rest of the investments provided that

$$
\sum_{t=1}^{2} \left( p_H[R - R_b^t - R_m^t] \right) \geq \sum_{t=1}^{2} [I_t - I_m^t]
$$
or

$$
p_H[2R - R_b^1 - R_b^2] \geq I_1 + I_2 + 2c.
$$

The second condition in the statement of the exercise ensures that this condition can be met while satisfying the entrepreneur's incentive compatibility.

Under monopoly in monitoring, the same reasoning applies, with a few twists. First, the entrepreneur is rewarded only in the case of two successes. From Chapter 4, we know that she then gets $R_b$ such that

$$
[(p_H)^2 - (p_L)^2]R_b \geq 2b.
$$

(Two remarks. First, we do not allow renegotiation-proof anyway. Second, one can check that the monitor's incentive scheme can be designed so as to induce monitoring in both periods.) Second, the monitor then receives the NPV minus the entrepreneur's rent, i.e.,

$$
V - \frac{(p_H)^2}{(p_H)^2 - (p_L)^2} 2b = V - 2 \left( \frac{p_H}{p_H + p_L} \right) \left( \frac{p_Hb}{\Delta p} \right).
$$

(ii) **Short-term contracts.** Under competition, each monitor obtains no profit at date 2. The condition

$$
I_1 + c > p_H \left( R - \frac{b}{\Delta p} \right)
$$

implies that no lending is feasible at date 1.

Under monopoly, the monitor will secure

$$
p_H \left( R - \frac{b}{\Delta p} \right) - I_2 - c > 0
$$
at date 2, if he helps the firm obtain funding at date 1. His intertemporal profit is then

$$
2p_H \left( R - \frac{b}{\Delta p} \right) - (I_1 + I_2) - 2c > 0
$$

(which is smaller than that under commitment because of the absence of cross-pledging across periods).

**Exercise 10.1 (security design as a disciplining device).** (i) $R_b^*$ is the maximal entrepreneurial stake in the firm's payoff in the case of continuation that is consistent with the investors' breaking even. The entire short-term income ($r$ in the case of success and $L$ in the case of failure) is pledged to investors, and the project continues only in the case of date-1 success. The three conditions say that if the entrepreneur is rewarded $R_b^*$ in the case of date-2 success, then

- $R_b^* \geq B/\Delta p$: her date-2 incentive compatibility constraint is satisfied;
- $p_H(R - R_b^*) > L$: interference reduces the investors' income; and
- $(p_H^r - p_H^d)(p_HR_b^*) \geq B_0$: the entrepreneur's date-1 incentive compatibility constraint is also satisfied.

(ii) From the definition of $R_b^*$, the project is financed, and from the three conditions, high efforts in both periods are guaranteed. Although there is an efficiency loss in terminating the project in the case of date-1 failure, this relaxes the date-1 incentive constraint and is optimal if $p_H^r$ is large enough, that is, if the probability of interference is low enough.

The incentive scheme offered to the entrepreneur is that she is rewarded $R_b^*$ if and only if she is successful in both periods; and the project is terminated if the date-1 income is equal to 0.

To implement this incentive scheme, the entrepreneur can issue two kinds of securities with different cash flow and control rights:

- short-term debt $d \in (0, \min\{L/p_H, r\})$: debt-holders receive control if $d$ is not repaid at date 1; and
- long-term equities associated with control at time 1 if $d$ is paid, and the following cash-flow rights: at date 1 equityholders receive the residual revenue $(r - d$ in the case of a date-1 success, and $\max\{0, L - d\}$ in the case of a date-1 failure); at date 2 they receive $R - R_b^*$ in the case of success.

Debt-holders interfere and terminate the project if there is no date-1 income, since

$$
p_Hd < \min\{L, d\}.
$$

Equityholders, when in control, do not interfere and so the project continues.

(iii) Suppose $R_b^* = B/\Delta p$, and all three conditions still hold. Now if the entrepreneur is also paid
Answers to Selected Exercises

Let \( r_b \in (0, r] \) in the case of date-1 success, the date-1 incentive constraint is relaxed:

\[
(p^1_b - p^1_0)[r_b + p^0_1R^*_b] \geq B_0.
\]

But given that it is satisfied for \( r_b = 0 \), there is no benefit to boosting incentives in this way. Indeed, a positive \( r_b \) reduces the pledgeable income. The breakeven constraint of investors becomes more stringent:

\[
I - A \leq p^1_0(r - r_b) + (1 - p^1_0)L + p^1_0[p^0_1(R - R^*_b)].
\]

A positive \( r_b \) is not optimal as it makes the financing more difficult to arrange but has no incentive effect.

In general, a short-term bonus reduces the pledgeable income, while incentives are best provided by vesting the manager's compensation.

Exercise 10.2 (allocation of control and liquidation policy). (i) As usual, if financing is a binding constraint it is optimal to give 0 to the entrepreneur in the case of failure and to allocate the entire liquidation value \( L \) to investors in the case of liquidation. This increases the pledgeable income without perverse incentive effects or destruction of value. The entrepreneur maximizes her expected utility,

\[
U_b = E_\omega[x(L, U^0_b)p^1_1R_b + [1 - x(L, U^0_b)]U^0_b],
\]

subject to the incentive constraint,

\[
(\Delta p)R_b \geq B,
\]

and the investors' breakeven constraint,

\[
E_\omega[x(L, U^0_b)p^1_1(R - R_b) + [1 - x(L, U^0_b)]L] \geq I - A.
\]

The interesting case is when both the incentive and the participation constraints are binding. Let us rewrite the program as

\[
\max E_\omega[x(L, U^0_b)(\rho_1 - \rho_0) + [1 - x(L, U^0_b)]U^0_b]
\]

s.t.

\[
E_\omega[x(L, U^0_b)\rho_0 + [1 - x(L, U^0_b)]L] = I - A.
\]

Let \( \mu \geq 1 \) denote the multiplier of the participation constraint. We obtain

\[
x^{SB}(\omega) = 1 \quad \text{if and only if} \quad \rho_1 - U^0_b \geq -(\mu - 1)\rho_0 + \mu L,
\]

where "SB" stands for "second best."

As one would expect, continuation is less desirable when the liquidation value and the entrepreneur's alternative employment become more attractive (and, because of the difficulty of attracting financing, the liquidation value receives a higher weight than the entrepreneur's fallback option).

(ii) The first-best continuation rule is given by

\[
x^{FB}(\omega) = 1 \quad \text{if and only if} \quad \rho_1 - U^0_b \geq L
\]

(that is, \( \mu = 1 \)). \( \Omega^{SB} \) is included in \( \Omega^{FB} \), as described in Figure 5. More generally, \( \Omega^{SB} \) shrinks as \( A \) decreases (\( \mu \) increases).

To show this, note that for \( L < \rho_0 \), everyone prefers to continue. So the interesting region is \( L > \rho_0 \).

(iii) When the entrepreneur has control, the entrepreneur can guarantee himself \( \rho_1 - \rho_0 \) by choosing to continue. Second, renegotiation always leads to the first-best efficient outcome:

(a) Continuation is first-best efficient. If the initial contract makes the entrepreneur want to continue in the absence of renegotiation, there is nothing to renegotiate about (a necessary condition for renegotiation is the existence of gains from trade). If the entrepreneur prefers to liquidate (because of the existence of a golden parachute), the investors will want to compensate the entrepreneur to induce him to continue (the split of the gains from renegotiation depend on the relative bargaining powers).

(b) Liquidation is first-best efficient. Again, if the entrepreneur prefers to liquidate in the absence of renegotiation there is nothing to renegotiate about. Otherwise, the investors will "bribe" the entrepreneur to liquidate.

So

\[
\Omega^{EN} = \Omega^{FB}.
\]

Compare the investors' return with the pledgeable income derived in question (i). In \( \Omega^{SB} \) and outside
The decision rule is unchanged, and the investors cannot get more than $\rho_0$ and $L$, respectively. In $\Omega^{FB} - \Omega^{SB}$, the investors get at most $\rho_0$, while they were getting $L > \rho_0$. Thus, the project cannot be financed.

(iv) Under investor control, and in the absence of a golden parachute,

$$x^\IN(\omega) = 1 \quad \text{if and only if } \rho_0 \geq L.$$  

If $\rho_0 < L$, then investors cannot get more than under liquidation (there is no way the entrepreneur can compensate them). If $\rho_0 > L$, but $p_\Omega R_1 < L$, then the entrepreneur can offer a reduction of her stake in the case of success (while keeping $R_b \geq B/\Delta p$).

The project is financed since the investors get the same amount as in (i), except when $L > \rho_0$ and $\omega \in \Omega^{SB}$ for which they get more ($L$ instead of $\rho_0$).

(v) In the absence of renegotiation, the investors liquidate if and only if

$$L - r_b \geq \rho_0.$$  

The policy is renegotiated (toward liquidation) if

$$(\rho_1 - \rho_0) - U_b^0 \leq L - \rho_0 < r_b.$$  

In contrast, if

$$(\rho_1 - \rho_0) - U_b^0 > L - \rho_0 > r_b,$$

then there is no renegotiation and there is (inefficient relative to the first best) liquidation.

A small golden parachute increases the NPV while continuing to satisfy the financing constraint (an alternative would be to ask the investors to finance more than $I - A$ and let the entrepreneur save so as to be able to “bribe” the investors to induce continuation).

**Exercise 10.3 (large minority blockholding).** If $\xi > (\tau + \mu) s_2 R$, then the large shareholder and the uninformed (majority) investors have aligned interests. The majority shareholders therefore always follow the large shareholder’s recommendation.

Let us therefore assume that $\xi > (\tau + \mu) s_2 R$. Let us look for an equilibrium in which the entrepreneur makes her suggestion “truthfully” (just announces her preferred modification). In state 2, the large shareholder seconds the entrepreneur’s proposal. He makes a counterproposal in states 1 and 3.

The majority shareholders then go along with the joint proposal (in state 2). In the case of disagreement, the majority shareholders select the entrepreneur’s proposal, that of the large shareholder, or the status quo so as to solve

$$\max\{-\beta \mu + \tau (1 - \kappa), \beta \tau - \mu (1 - \beta)(1 - \kappa), 0\}.$$  

Note that in the equilibrium under consideration both the entrepreneur and the minority blockholder have incentives to report their preferences truthfully (and that there are other equilibria where this is not the case).

**Exercise 10.4 (monitoring by a large investor).** Let $U_b(x) = p_{\Omega l} R + [\xi + (1 - \xi) x] [\tau R - y] - c_m(x) - I$ denote the NPV (the NPV is equal to the borrower’s utility because there is no scarcity of monitoring capital, and therefore no rent to be left to the monitor). Let

$$P(x) = [p_{\Omega l} + [\xi + (1 - \xi) x] \tau] \left( R - \frac{B}{\Delta p} \right) - c_m(x)$$

denote the income that can be pledged to investors given that (a) the entrepreneur’s stake must exceed $B/\Delta p$ in order to elicit good behavior, and (b) the monitor’s expected income must compensate him for his monitoring cost. Concerning the last point, the monitor’s reward $R_m$ in the case of success and investment contribution $I_m$ must satisfy the following breakeven and incentive conditions:

$$p_{\Omega l} R_m = I_m + c_m(x) \quad \text{and} \quad (1 - \xi) \tau R_m = c_m'(x).$$

Note that

$$U_b(x) - P(x) = [\xi + (1 - \xi) x] \left( \tau \frac{B}{\Delta p} - y \right) + \text{constant},$$

and so is decreasing in $x$.

If there is a shortage of pledgeable income, the optimal monitoring level given by (10.11) and maximizing the NPV,

$$c_m'(x^\star) = (1 - \xi)(\tau R - y),$$

is no longer adequate. Indeed

$$U_b'(x^\star) = 0 \quad \Rightarrow \quad P'(x^\star) > 0.$$  

Thus, the monitoring intensity must increase beyond $x^\star$:

$$c_m'(x) > (1 - \xi)(\tau R - y).$$
If funding is feasible, then \( x \) is given by (the smallest value satisfying)
\[
P(x) = I - A.
\]
Let \( \hat{x} (> x^*) \) be defined by
\[
c_m(\hat{x}) \equiv (1 - \xi)\tau \left( R - \frac{B}{\Delta p} \right).
\]
Because the pledgeable income no longer increases above \( \hat{x} \), funding is feasible only if
\[
P(\hat{x}) \geq I - A.
\]

**Exercise 10.5 (when investor control makes financing more difficult to secure).** (i) The incentive constraint is as usual
\[
p_H R_b \geq p_L R_b + B, \tag{1}
\]
yielding pledgeable income
\[
P_1 \equiv p_H \left( R - \frac{B}{\Delta p} \right).
\]
The entrepreneur can receive funding if and only if
\[
P_1 \geq I - A.
\]
(ii) Assume entrepreneur control. Either
\[
\nu R_b \leq \gamma,
\]
and then the entrepreneur does not engage in damage control when shirking. The relevant incentive constraint remains (1), or
\[
\nu R_b > \gamma,
\]
and the incentive constraint becomes
\[
p_H R_b \geq (p_L + \nu) R_b + B - \gamma. \tag{2}
\]
If
\[
\nu \left( \frac{B}{\Delta p} \right) \leq \gamma,
\]
then the incentive constraint is unchanged when \( R_b = B/\Delta p \), and so the pledgeable income (the maximal income that can be pledged to investors while preserving incentive compatibility) is still \( P_1 \).

(iii) Under investor control, the damage-control action is selected, and so the incentive constraint becomes
\[
p_H R_b - \gamma \geq (p_L + \nu) R_b + B - \gamma \tag{3}
\]
or
\[
(\Delta p - \nu) R_b \geq B.
\]
The new pledgeable income is
\[
P_2 = p_H \left( R - \frac{B}{\Delta p} - \nu \right),
\]
and is smaller than under entrepreneur control.

**Exercise 10.6 (complementarity or substitutability between control and incentives).** (i) As usual, this condition is
\[
p_H \left( R - \frac{B}{\Delta p} \right) \geq I - A.
\]
(ii) Under entrepreneur control, the profit-enhancing action is not chosen in combination with the high effort since
\[
(p_H + \tau_H) R_b - \gamma < p_H R_b \tag{3}
\]
(since \( \tau_H R_b < \tau_H R < \gamma \)).

Thus, to induce the high effort, \( R_b \) must satisfy \((\Delta p) R_b \geq B\).

But then it is also optimal for the entrepreneur not to misbehave and choose the profit-enhancing action simultaneously:
\[
(p_L + \tau_L) R_b + B - \gamma \leq p_H R_b + \tau_L R_b - \gamma
\]
\[
< p_H R_b,
\]
since \( R_b < R \). The analysis is therefore the same as in (i).

Under investor control, it is a dominant strategy for the investors to select the profit-enhancing action. Hence, the manager’s incentive constraint becomes
\[
(p_H + \tau_H) R_b \geq (p_L + \tau_L) R_b + B
\]
or
\[
(\Delta p + \Delta \tau) R_b \geq B.
\]
The pledgeable income increases with investor control if and only if
\[
(p_H + \tau_H) \left( R - \frac{B}{\Delta p + \Delta \tau} \right) > p_H \left( R - \frac{B}{\Delta p} \right).
\]
This condition is necessarily satisfied if \( \Delta \tau \geq 0 \) (complementarity or separability). But it may fail if \( \Delta \tau \) is sufficiently negative.

**Exercise 10.7 (extent of control).** The NPV is larger under limited investor control:
\[
(p_H + \tau_L) R - \gamma_L > (p_H + \tau_H) R - \gamma_H.
\]
We will assume that these NPVs are positive.
So the entrepreneur will grant limited control as long as this suffices to raise funds, i.e.,
\[(p_H + \tau_\lambda)\left(R - \frac{B}{\Delta p}\right) \geq I - A.\]

If this condition is not satisfied, the entrepreneur must grant extended control in order to obtain financing. Financing is then feasible provided that
\[(p_H + \tau_B)\left(R - \frac{B}{\Delta p}\right) \geq I - A.\]

Lastly, note that
\[\tau_\lambda R - y_\lambda \geq 0\]
is a sufficient condition for ruling out entrepreneurial control (but entrepreneurial control may be suboptimal even if this condition is not satisfied; for, it may conflict with the investors’ breakeven condition).

**Exercise 10.8 (uncertain managerial horizon and control rights).** (i) The assumption
\[(p_H + \tau)\left(\frac{B}{\Delta p}\right) \geq y\]
means that the new manager is willing to take on the job even if control is allocated to investors. Because his reward \(R_b\) must satisfy
\[(\Delta p)R_b \geq B,\]
regardless of who has control, the new manager receives rent
\[(p_H + \tau y)\left(\frac{B}{\Delta p}\right) - y y\]
(smaller than the rent, \(p_H B/\Delta p\), that he would receive if he were given control rights).

The entrepreneur’s utility is (if the project is undertaken)
\[U_b = (1 - \lambda)[(p_H + \tau x)R - y x] + \lambda(p_H + \tau y)\left(R - \frac{B}{\Delta p}\right) - I.\]

The financing condition is
\[(1 - \lambda)(p_H + \tau x)\left(R - \frac{B}{\Delta p}\right) + \lambda(p_H + \tau y)\left(R - \frac{B}{\Delta p}\right) \geq I - A.\]

(ii) Clearly, \(y = 1\) both maximizes \(U_b\) and facilitates financing.

Also, a necessary condition for \(U_b\) to be positive is that \(\lambda\) not be too big.

Letting \(p_0 \equiv p_H[R - B/\Delta p]\), if financing is feasible for \(x = 0\): \((1 - \lambda)p_0 + \lambda \rho^*_b \geq I - A\), then \(x = 0\) is optimal. The entrepreneur invests if and only if \(U_b \geq 0\), or
\[(1 - \lambda)p_0 + \lambda \rho^*_b \geq I.\]

If \((1 - \lambda)p_0 + \lambda \rho^*_b < I - A\), then, in order to obtain financing, the entrepreneur must set \(x\) in the following way:
\[(1 - \lambda)p_0 + \lambda \rho^*_b + \tau x\left(R - \frac{B}{\Delta p}\right) = I - A.\]

Financing then occurs if and only if \(U_b \geq 0\) for this value of \(x\).

**Exercise 10.9 (continuum of control rights).** (i) Let \(R_b\) denote the entrepreneur’s reward in the case of success. The entrepreneur maximizes her utility, which is equal to the NPV,
\[\max_{[x(\cdot), \cdot]} \{[p_H + E_f[tx(t,g)]][R - I] - E_f[gx(t,g)]\},\]
subject to the constraint that investors break even,
\[[p_H + E_f[tx(t,g)]][R - R_b] \geq I - A,\]
and to the incentive compatibility constraint,
\[(\Delta p)R_b \geq B.\]

Clearly, \(R_b = B/\Delta p\) if the investors’ breakeven constraint is binding. Let \(\mu\) denote the shadow price of this constraint. Writing the Lagrangian and taking the derivative with respect to \(x(t,g)\) for all \(t\) and \(g\) yields
\[x(t,g) = 1 \iff tR - g + \mu \left[t\left(R - \frac{B}{\Delta p}\right)\right] \geq 0.\]

This defines a straight line through the origin in the \((t,g)\)-space under which \(x = 1\) and over which \(x = 0\).

(ii) When \(A\) decreases, more pledgeable income must be harnessed. So the straight line must rotate counterclockwise (add \(t > 0\) realizations and subtract \(t < 0\) ones). In the process, both \(\tau\) and \(y\) increase.

(iii) If \(x(t,g) = 1\) and \(t > 0\), the control right can be given to investors. If \(x(t,g) = 1\) and \(t < 0\) (which implies \(g < 0\)): the decision yields a private benefit to
the entrepreneur), then the control can be allocated to the entrepreneur. Because
\[ |g| > |t|R > |t|R_0, \]
the entrepreneur chooses \( x(t, g) = 1 \). Furthermore, \( x(t, g) = 1 \) is not renegotiated since it is first-best efficient.

One proceeds similarly for \( x(t, g) = 0 \).

(iv) Assume that \( g \) is the same for all rights and is positive. The optimal rule becomes
\[ t \geq t^* = \frac{g}{R + \mu(R - B/\Delta p)}. \]

Let \( H(t) \) denote the cumulative distribution function over \( t \):
\[ y = g[1 - H(t^*)], \]
\[ \tau = \int_{t^*}^{\infty} t \, dH(t). \]

Hence,
\[ \frac{dy}{d\tau} = \frac{g}{t^*} \quad \text{and} \quad \frac{d^2y}{d\tau^2} > 0. \]

One can, as earlier, envision that \( \tau \) increases as \( A \) decreases, for example.

**Exercise 12.1 (Diamond–Dybvig model in continuous time).** To provide consumption \( c(t) \) to consumers whose liquidity need arises between \( t \) and \( t + dt \) (in number \( f(t) \) dt), one must cut \( x(t) \) dt, where
\[ x(t)R(t) \, dt = c(t)f(t) \, dt. \]

Together with the fact that the total number of trees per representative depositor is 1, this implies that the first-best contract solves
\[
\max \left\{ \int_0^1 u(c(t))f(t) \, dt \right\}
\]
subject to
\[
\int_0^1 \frac{c(t)}{R(t)}f(t) \, dt \leq 1.
\]

The first-order condition is then, for each \( t \),
\[
\left[ u'(c(t)) - \frac{\mu}{R(t)} \right] f(t) = 0,
\]
where \( \mu \) is the shadow price of the constraint.

(ii) Take the (log-) derivative of the first-order condition:
\[ u'(c(t))R(t) = \mu \Rightarrow c \frac{u''}{u'} \frac{\dot{c}}{c} + \frac{\dot{R}}{R} = 0. \]

Because the coefficient of relative risk aversion exceeds 1,
\[ \frac{\dot{c}}{c} < \frac{\dot{R}}{R}. \]

Note that, from the constraint, the average \( c/R \) is equal to 1. The existence of \( t^* \) follows (drawing a diagram may help build intuition).

(iii) Suppose that a depositor who has not yet suffered a liquidity shock withdraws at date \( \tau \). Reinvesting in the technology, she will obtain \( c(\tau)R(t - \tau) \) if the actual date of the liquidity shock is \( t > \tau \). Withdrawing is a “dominant strategy” (that is, yields more regardless of the future events) if
\[ c(\tau)R(t - \tau) > c(t) \quad \text{for all} \quad t > \tau. \]

The log-derivative of \( (c(\tau)R(t - \tau)/c(t)) \) with respect to \( t \) is, for \( \tau \) close to 0,
\[ \frac{\dot{R}(t - \tau)}{R(t - \tau)} - \frac{\dot{c}(t)}{c(t)} > 0. \]

We thus conclude that the first-best outcome is not incentive compatible.

**Exercise 12.2 (Allen and Gale (1998) on fundamentals-based panics).** (i) Let \( i_1 \) and \( i_2 \) denote the investments in the short- and long-term technologies. The social optimum solves
\[
\max \left\{ \lambda u(c_1(R)) + (1 - \lambda) u(c_2(R)) \right\}
\]
s.t.
\[
\begin{align*}
\lambda c_1(R) &\leq i_1, \\
(1 - \lambda)c_2(R) &\leq i_1 - \lambda c_1(R) + R i_2, \\
i_1 + i_2 &= 1.
\end{align*}
\]

This yields
(a) \( c_1(R) = c_2(R) = i_1 + R i_2 \)

for \( R \leq \frac{(1 - \lambda)i_1}{\lambda i_2} = R^* \),

(b) \( c_1(R) = c_1(R^*), \)

and
\[ c_2(R) = \frac{R i_2}{1 - \lambda} \geq c_1(R) \quad \text{for} \quad R \geq R^*. \]

For low long-term payoffs, \( \lambda c_1(R) < i_1 \) and the impatient types share risk with the patient types, as their short-term investment can be rolled over to

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Exercise 12.4 (random withdrawal rate). (i) This follows along standard lines. Asset maturities should match those of consumptions, \( \lambda c_1 = i_1 \) and \((1 - \lambda)c_2 = i_2 R:\)

\[
\max_{c_1} \left\{ \lambda u(c_1) + (1 - \lambda) u \left( \frac{1 - \lambda c_1}{1 - \lambda} R \right) \right\}
\]

implies

\[
u'(c_1) = Ru'(c_2).
\]

For CRRA utility, \( c_1/c_2 = R^{-1/\gamma} \). So \( i_1 \) grows and \( i_2 \) decreases as risk aversion \( (\gamma) \) increases.

(ii) The optimal program solves

\[
\max_{(i_1, i_2, y, z_1)} \left\{ \beta \left[ \lambda_1 u \left( \frac{i_1 y_1 + i_2 z_1}{\lambda_1} \right) \right] + (1 - \lambda_1) u \left( \frac{i_1 (1 - y_1) + i_2 R (1 - z_1)}{1 - \lambda_1} \right) \right. \\
+ (1 - \beta) \left[ \lambda_2 y_2 + i_2 z_1 \right] \left( \frac{i_2 (1 - y_2) + i_2 R (1 - z_2)}{1 - \lambda_2} \right) \}
\]

Clearly, \( z_{\omega} = 0 \Rightarrow y_{\omega} = 1 \) and \( y_{\omega} < 1 \Rightarrow z_{\omega} = 0 \).

Also, \( y_L = 1 \) implies \( y_H = 1 \), and \( z_H = 0 \) implies \( z_L = 0 \).

For \( \ell = 0 \), the optimum has \( z_\omega = 0 \). It may be optimal to roll over some of \( i_1 \) in state L. For \( \ell \) close to 1, \( i_2 \) serves to finance date-1 consumption in state H.

Exercise 13.1 (improved governance). (i) The pledgeable income is \( p_H (R - B/\Delta p) \). The financing constraint is

\[
(1 + r) (I - A) \leq p_H \left( R - \frac{B}{\Delta p} \right).
\]

(ii) The cutoff \( A^* \) is given by

\[
(1 + r) (I - A^*) = p_H \left( R - \frac{B}{\Delta p} \right).
\]

Market equilibrium:

\[
S(r) + \int_0^{A^*} A g(A) \, dA = \int_{A^*(r)}^{I} (I - A) g(A) \, dA
\]

or, equivalently,

\[
S(r) + \int_0^{A^*} A g(A) \, dA = [1 - G(A^*(r))]I.
\]

(Note that entrepreneurs with weak balance sheets, \( A < A^* \), would demand a zero rate of interest from their preferences. However, they receive the equilibrium market rate.)

Because \( A^* \) increases with the interest rate and with the quality of investor protection (here, \(-B\)), an increase in investor protection raises the equilibrium interest rate.
Exercise 13.2 (dynamics of income inequality).
(i) See Section 13.3:

\[ U_t(y_t) = y_t. \]

(ii) The incentive constraint is

\[ (\Delta p) R^I_t \geq BI_t, \]

and so the pledgeable income is

\[ p^H_t \left( R - \frac{B}{\Delta p} \right) I_t = \rho_0 I_t, \]

yielding an investment level given by

\[ \rho_0 I_t = (1 + r)(I_t - A_t) \quad \text{or} \quad I_t = \frac{A_t}{1 - \rho_0/(1 + r)}. \]

A project’s NPV is

\[ [p^H_t R - (1 + r)] I_t = [\rho_1 - (1 + r)] I_t. \]

By assumption, \( \rho_1 \geq 1 + r \), and so entrepreneurs prefer to invest in a project rather than lending their assets. Income is

\[ y_t = [\rho_1 - \rho_0] I_t, \]

and so

\[ A_{t+1} = a \frac{\rho_1 - \rho_0}{1 - \rho_0/(1 + r)} A_t + \hat{A}, \]

which converges to \( A_\infty \) as \( t \) tends to \( \infty \).

(iii) The threshold is given by

\[ \frac{A_0^*}{1 - \rho_0/(1 + r)} = I. \]

The limit wealth of poor dynasties is the limit point of the following first-order difference equation:

\[ A_{t+1} = a (1 + r) A_t + \hat{A} \]

or

\[ A_0^L = \frac{\hat{A}}{1 - a (1 + r)}. \]

(iv) If \( \rho_1 = 1 + r \), individuals are indifferent between being investors and becoming entrepreneurs. Note that wealths are equalized at

\[ A_\infty = \frac{\hat{A}}{1 - a \rho_1}, \]

corresponding to investment

\[ I_\infty = \frac{A_\infty}{1 - \rho_0/(1 + r)} = \frac{\rho_1 \hat{A}}{(1 - a \rho_1) (\rho_1 - \rho_0)}. \]

Equilibrium in the loan market requires that

\[ \kappa A_\infty = (1 - \kappa) (I_\infty - A_\infty) \]

or

\[ \kappa (\rho_1 - \rho_0) = (1 - \kappa) \rho_0. \]

- If \( \rho_1 > (1 + r) \), then lenders must be unable to become entrepreneurs and so have wealth \( A_\infty^L \). Thus

\[ \kappa A_\infty^L = (1 - \kappa) (I_\infty - A_\infty), \]

where \( I_\infty \) was derived in question (ii).

Exercise 13.3 (impact of market conditions with and without credit rationing).
(i) The representative entrepreneur’s project has NPV (equal to the entrepreneur’s utility)

\[ U_b = p^H_t PR(I) - I - K, \]

and the scale of investment \( I \) can be financed as long as the pledgeable income exceeds the investors’ initial outlay:

\[ P(I) \equiv p^H_t \left[ PR(I) - \frac{BI}{\Delta p} \right] \geq I + K - A \]

(this is the financing condition).

In the absence of any financing constraint (i.e., when \( B = 0 \)), the representative entrepreneur would choose a first-best (FB) policy:

\[ p^H_t PR'(I^{FB}) = 1 \quad \text{or} \quad p^H_t P\alpha(I^{FB})^{\alpha-1} = 1, \]

provided that the fixed cost \( K \) is not too large, i.e., \( K < p^H_t PR(I^{FB}) - I^{FB} \). (Otherwise, the optimal investment is equal to 0.)

- When does the financing constraint bind?

Simple computations show that

\[ P(I^{FB}) - I^{FB} = (1 - \alpha) \left[ \frac{1}{\alpha} - \frac{p^H_t B / \Delta p}{1 - \alpha} \right] I^{FB}. \]

Let us assume that the agency cost is not too large:

\[ \frac{p^H_t B}{\Delta p} < \frac{1 - \alpha}{\alpha} \]

(otherwise the financing constraint is necessarily binding).

Because \( I^{FB} \) is increasing in the product price \( P \), the financing constraint is binding for low prices, as illustrated in Figure 7, where \( I^{SB} \) denotes the solution to the financing condition (taken with equality).

(ii) Thus, there is at least some region (to the left of \( P_0 \) in the figure) in which the expansionary impact of the product price (the contractionary impact of past investment) is stronger in the presence of credit rationing, i.e., when the presence of \( B \) makes the financing condition binding.
(iii) To conclude this brief analysis, we can now endogenize the product price by assuming the existence of a prior investment $I_0$ by, say, a mass 1 of the previous generation of entrepreneurs. Then, $P$ is a decreasing function of total effective investment, i.e., total output:

$$P = P(p_0[I(R(I) + R(I_0))]), \quad \text{with } P' < 0.$$ 

When $I_0$ increases, $I$ must decrease (if $I$ increases, then $P$ decreases, and so $I$ decreases after all): this is the crowding-out effect; furthermore, total output must increase (if it decreased, then $P$ would increase and so would $I$; and thus $p_0[I(R(I) + R(I_0))$ would increase after all).

**Exercise 14.2 (alternative distributions of bargaining power in the Shleifer–Vishny model).** Entrepreneur $i$’s utility (or, equivalently, firm $i$’s NPV) is

$$U_{bi} = [xP_1 + (1-x)(1-\nu)P - 1]I_i + x(1 - \alpha)[(\rho_1 - \rho_0) + (\rho_0 - P)]I_j = \hat{\alpha}I_i + \hat{\kappa}I_j,$$

where

$$\hat{\alpha} = \alpha - (1-x)(1-\nu)(\rho_0 - P)$$

and

$$\hat{\kappa} = \kappa + x(1-\mu)(\rho_0 - P).$$

Recalling that $(1-x)(1-\nu) = x(1-\mu)$, note that $\hat{\alpha} + \hat{\kappa} = \alpha + \kappa$, as it should be from the fact that a change in bargaining power induces a mere redistribution of wealth for given investments.

Firm $i$’s borrowing capacity is now given by

$$[x(\rho_0 + (1-x)(1-\nu)P]I_i + x(1 - \mu)(\rho_0 - P)]I_j = I_i - A_i$$

or

$$I_i = \frac{A_i + x(1 - \mu)(\rho_0 - P)I_j}{1 + (1-x)(1-\nu)(\rho_0 - P) - \rho_0[x + (1-x)(1-\nu)]}.$$

In symmetric equilibrium $(A_1 = A_2 = A; I_1 = I_2)$$

$$I = \frac{A}{1 - \rho_0[x + (1-x)(1-\nu)]}$$

is independent of $P$.

**Exercise 14.3 (liquidity management and acquisitions).** (i) Suppose that the acquirer expects price demand $P$ for the assets when the risky firm is in distress (which has probability $1-\nu$). The NPV for a given cutoff $\rho^*$ is given by

$$U^s_b = (\rho_1 - 1)I + (1-\nu)J\int_0^{\rho^*} [\rho_1 - (P + \rho)]dF(\rho).$$

The borrowing capacity in turn is given by

$$\rho_0I + (1-\nu)J\int_0^{\rho^*} [\rho_0 - (P + \rho)]dF(\rho) = I - A.$$ 

And so

$$U^s_b = (\rho_1 - 1)\frac{A - (1-x)J\int_0^{\rho^*} [(P + \rho) - \rho_0]dF(\rho)}{1 - \rho_0} + (1-\nu)J\int_0^{\rho^*} [\rho_1 - (P + \rho)]dF(\rho).$$

Maximizing with respect to $\rho^*$ and simplifying yields

$$\rho^* = 1 - P.$$ 

And so

$$\rho_0 + L^* = P + \rho^* = 1.$$ 

(ii) Anticipating that the safe firm has extra liquidity $L^*$, the seller chooses price $P$ so as to solve

$$\max_P \{F(\rho_0 + L^* - P)\},$$

since the acquirer can raise funds only when $P + \rho \leq \rho_0 + L^*$.

The derivative of this objective function is

$$-f(\rho^*)P + F(\rho^*) = -f(1 - P)P + F(1 - P).$$

Note that this derivative is positive at $P = 0$ and negative at $P = 1$. Furthermore, $-P + F(1 - P)/f(1 - P)$ is a decreasing function of $P$ from the monotone hazard rate condition and so the equilibrium price is unique and belongs to $(0, 1)$. 

![Figure 7](image-url)
Suppose next that \( L \) increases for some reason (and that this is observed by the seller). The first-order condition then becomes
\[
-P + \frac{F(p_0 + L - P)}{f(p_0 + L - P)} = 0
\]
and so
\[
-\left[ 1 + \left( \frac{F}{f} \right)' \right] \frac{dP}{dL} + \left( \frac{F}{f} \right)' = 0.
\]
Because \((F/f)' > 0,\)
\[
0 < \frac{dP}{dL} < 1.
\]
This implies that the cutoff, and thus the probability of a sale, increases despite the price adjustment.

(iii) Suppose that the distribution \( F \) converges to a spike at \( \hat{\rho} \). Consider thus a sequence \( F_n(\rho) \) with
\[
\lim_{n \to \infty} F_n(\rho) = 0 \quad \text{for} \quad \rho < \hat{\rho}
\]
and
\[
\lim_{n \to \infty} F_n(\rho) = 1 \quad \text{for} \quad \rho > \hat{\rho}.
\]
Let us give an informal proof of the result stated in (iii) of the question. Choosing a price \( P \) that triggers a cutoff that is smaller than \( \hat{\rho} \) and does not converge with \( n \) to \( \hat{\rho} \) would yield (almost) zero profit, and so choosing an alternative price that leads to a cutoff a bit above \( \hat{\rho} \) would yield a higher profit. Conversely, if the cutoff is above \( \hat{\rho} \) and does not converge to \( \hat{\rho} \), then \( P F_n = 0 \) and \( F_n = 1 \), and so the first-order condition is not satisfied. (This proof is loose. A proper proof must consider a subsequence having the former or latter property.)

Exercise 14.4 (inefficiently low volume of asset reallocations). At the optimum, firm 1's assets are resold in the secondary market if and only if
\[
\rho_0 < \rho_0^*.
\]
Furthermore, it is optimal for the contract to specify that the proceeds from the sale to firm 2 go to the investors in firm 1 (so as to maximize the pledgeable income). And so the investment \( I \) is given by the investors' breakeven constraint:
\[
\left[ F(\rho_0^*) \hat{\rho}_0 + \int_{\rho_0}^{\rho_0^*} \rho_0 \ dF(\rho_0) \right] I = I - A,
\]
which yields
\[
I = I(\rho_0^*).
\]
The entrepreneur's utility is
\[
U_b = \text{NPV} = \left[ F(\rho_0^*) \hat{\rho}_0 + \int_{\rho_0}^{\rho_0^*} (\rho_0 + \Delta \rho) \ dF(\rho_0) \right] I(\rho_0^*).
\]
The optimal cutoff maximizes \( U_b \) and satisfies
\[
\hat{\rho}_0 - \Delta \rho < \rho_0^* < \hat{\rho}_0.
\]

Exercise 15.1 (downsizing and aggregate liquidity).
(i) The incentive constraint is
\[
(\Delta p) R_0^I \geq BI
\]
in the case of no shock, and
\[
(\Delta p) R_0^I \geq BJ
\]
in the presence of a liquidity shock. So the pledgeable incomes are \( p_{1I}(R(I) - BI/\Delta p) \) and \( p_{1I}(R(J) - BJ/\Delta p) \), respectively.

The investors’ breakeven constraint is
\[
(1 - \lambda) p_{1I} \left[ R(I) - \frac{BI}{\Delta p} \right] + \lambda \left[ p_{1I} \left( R(J) - \frac{BJ}{\Delta p} \right) - \rho J \right] \geq I - A. \quad (1)
\]
The entrepreneur's utility is equal to the NPV:
\[
U_b = (1 - \lambda) p_{1I} R(I) + \lambda [ p_{1I} R(J) - \rho J ] - I. \quad (2)
\]
Let \( \mu \) denote the shadow price of constraint (1). Maximizing \( U_b \) subject to (1) (and ignoring the constraint \( J \leq I \)) yields first-order conditions with respect to \( I \) and \( J \):
\[
[(1 - \lambda) p_{1I} R'(I) - 1] [1 + \mu] - \mu (1 - \lambda) p_{1I} \frac{B}{\Delta p} = 0
\]
or
\[
p_{1I} R'(I) = \frac{1}{1 - \lambda} + \frac{\mu}{1 + \mu} p_{1I} \frac{B}{\Delta p}, \quad (3)
\]
and
\[
\lambda [ p_{1I} R'(J) - \rho ] [1 + \mu] - \lambda \mu p_{1I} \frac{B}{\Delta p} = 0
\]
or
\[
p_{1I} R'(J) = \rho + \frac{\mu}{1 + \mu} p_{1I} \frac{B}{\Delta p}. \quad (4)
\]
Comparing (3) and (4), one observes that ignoring the constraint \( J \leq I \) is justified if and only if
\[
\rho > \frac{1}{1 - \lambda},
\]
that is, when the cost of continuation in the state of nature with a liquidity shock exceeds the cost of
one more unit of investment in the state without. This simple comparison comes from the fact that the per-unit agency cost is the same in both states of nature. Let \((I^*, J^*)\) denote the solution (obtained from (1), (3), and (4)).

(ii) • Under perfect correlation, no inside liquidity is available. So, in order to continue in the case of a liquidity shock, each firm requires

\[
L = \rho J^*.
\]

Hence, \(L^* = \rho J^*\).

• If \(L < L^*\), then

\[
J = \frac{L}{\rho} < J^*.
\]

\(J^*\) is available. So, in order to continue in the case of a liquidity shock, each firm requires

\[
L = \rho J^*.
\]

The liquidity premium is obtained by solving the modified program in which the extra cost associated with the liquidity premium, \((q - 1)\rho J\), is subtracted in \(U_b\) in (2), and added to the right-hand side of (1), yielding a modified investor breakeven constraint—let us call it (1'). Equation (3) is unchanged, while (4) becomes

\[
 p_H (J) = \rho \left( 1 + \frac{q - 1}{\lambda} \right) + \frac{\mu}{1 + \mu} p_H J \rho \frac{B}{\Delta p}.
\]

So \(J < I\) a fortiori.

The liquidity premium is obtained by solving (1'), (3), (4'), and (5).

(iii) • Under independent shocks, exactly a fraction \(\lambda\) of firms incur no shock. Assuming \(q = 1\) for the moment, (1) yields (provided \(I > A\))

\[
V = (1 - \lambda) p_H \left[ R(I) - \frac{BI}{\Delta p} \right] + \lambda p_H \left[ R(J) - \frac{BJ}{\Delta p} \right]
\]

\[
> \lambda \rho J.
\]

\(V\) is the value of the stock index after the shocks have been met. And so the corporate sector, as a whole, can by issuing new claims raise enough cash to meet average shock \(\lambda \rho J\). So there is, in principle, no need for outside liquidity.

• This, however, assumes that liquidity is not wasted. If each entrepreneur holds the stock index, then, when facing a liquidity shock, the entrepreneur can raise \(p_H [R(J) - BJ/\Delta p]\) by issuing new claims on the firm.

Meeting the liquidity shock then requires that

\[
p_H \left[ R(J) - \frac{BJ}{\Delta p} \right] + [V - \lambda \rho J] \geq \rho J
\]

or

\[
(1 - \lambda) p_H \left[ R(I) - \frac{BI}{\Delta p} \right] \geq (1 + \lambda) \left[ \rho J + p_H \left[ R(J) - \frac{BJ}{\Delta p} \right] \right],
\]

which is not guaranteed.

It is then optimal to pool the liquidity, for example, through a credit line mechanism.

**Exercise 15.2 (news about prospects and aggregate liquidity).**

(i) \[
\text{NPV} = \int_{y^*}^1 y \, dG(y) - \left[ 1 - G(y^*) \right] J - I.
\]

Investors' net income

\[
= \int_{y^*}^1 y \, dG(y) - \left[ 1 - G(y^*) \right] [J + R] - [I - A].
\]

(ii) • The NPV is maximized for \(y^* = y^*_0 = J\). So, if

\[
\int_{y^*}^1 y \, dG(y) - \left[ 1 - G(y^*) \right] [J + R] \geq I - A \iff A \geq A^*_0,
\]

then \(y^* = J\).

Otherwise, by concavity of the NPV, the contract raises \(y^*\) so as to attract investment:

\[
\int_{y^*}^1 y \, dG(y) - \left[ 1 - G(y^*) \right] [J + R] = I - A.
\]

The pledgeable income can no longer be increased when \(y^* = y^*_1 = J + R\).

So, for \(A < A^*_0\), no financing is feasible.

• If \(A > A^*_0\), then \(y^* < J + R\). Hence, for \(y^* \leq y < J + R\), investors have negative profit from continuation, and the firm cannot obtain financing just by going back to the capital market.

(iii) If productivities are drawn independently, the financing constraint,

\[
\int_{y^*}^1 y \, dG(y) - \left[ 1 - G(y^*) \right] [J + R] = I - A,
\]

implies

\[
\int_{y^*}^1 y \, dG(y) - \left[ 1 - G(y^*) \right] [J + R] > 0,
\]

and so, collectively, firms have enough income to pledge when going back to the capital market.

(iv) • Suppose, in a first step, that there exists a large enough quantity of stores of value, and so
Answers to Selected Exercises

$q = 1$ (there is no liquidity premium). Then the breakeven condition can be written as

$$E[\int_{y^*}^{1} (y - J - R) dG(y | \theta)] \geq I - A.$$

- *Maximize*

$$E[\int_{y^*}^{1} (y - J) dG(y | \theta)] - I$$

subject to the financing constraint (let $\mu$ denote the multiplier of the latter). Then

$$y^*(\theta) - J + \mu[y^*(\theta) - J - R] = 0 \Rightarrow y^*(\theta) = J + \frac{\mu}{1 + \mu} R.$$

- *The lowest amount of pledgeable income,*

$$\min_{\theta} \int_{y^*}^{1} (y - J - R) dG(y | \theta),$$

may be negative. It must then be complemented by an equal number of stores of value delivering one for certain, say.

- *If there are not enough stores of value, then they trade at a premium ($q > 1$).*

**Exercise 15.3 (imperfectly correlated shocks).** A shortage of liquidity may occur only if the fraction $\theta$ of correlated firms faces the high shock (the reader can follow the steps of Section 15.2.1 to show that in the other aggregate state there is no liquidity shortage).

The liquidity need is then, in aggregate,

$$[\theta + (1 - \theta) \lambda](\rho_H - \rho_0)I.$$

The net value of shares in the healthy firms is

$$(1 - \theta)(1 - \lambda)(\rho_0 - \rho_H)I.$$

Using the investors’ breakeven condition and the assumption that liquidity bears no premium:

$$[1 - \lambda](\rho_0 - \rho_L) - \lambda(\rho_H - \rho_0)I = I - A.$$

And so the corporate sector is self-sufficient if

$$(1 - \theta)(1 - \lambda)(\rho_0 - \rho_H)I \geq [\theta + (1 - \theta) \lambda](\rho_H - \rho_0)I$$

or

$$(1 - \theta)(I - A) \geq \theta(\rho_H - \rho_0)I.$$

**Exercise 15.4 (complementarity between liquid and illiquid assets).** The NPV per unit of investment is equal to

$$(1 - \lambda + \lambda x)\rho_1 - [1 + (1 - \lambda)\rho_L + [\lambda\rho_H + (q - 1)(\rho_H - \rho_0)]x].$$

We know that this NPV is negative for $x = 0$. Thus, either its derivative with respect to $x$ is nonpositive,

$$\lambda \rho_1 \leq \lambda \rho_H + (q - 1)(\rho_H - \rho_0),$$

and then there is no investment ($I = 0$). The absence of corporate investment implies that there is no corporate demand for liquidity, and so $q = 1$, which contradicts the fact that $\rho_1 > \rho_H$. Hence, the derivative with respect to $x$ must be strictly positive:

$$\lambda \rho_1 > \lambda \rho_H + (q - 1)(\rho_H - \rho_0),$$

implying that $x = 1$.

For a low supply of liquid assets, this in turn implies that

(a) investment is limited by the amount of liquid assets,

$$L^S = (\rho_H - \rho_0)I;$$

(b) the entrepreneurs compete away the benefits associated with owning liquid assets, and so they are indifferent between investing in illiquid and liquid assets and not investing at all,

$$\rho_1 = 1 + \hat{\rho} + (\hat{q} - 1)(\rho_H - \rho_0).$$

Furthermore, for a low supply of liquid assets, entrepreneurs do not borrow as much as their borrowing capacity would allow them to. This borrowing capacity, denoted $\bar{I}$, is given by

$$\rho_0 \bar{I} = [1 + \hat{\rho} + (\hat{q} - 1)(\rho_H - \rho_0)]\bar{I} - A,$$

$$= \rho_1 \bar{I} - A.$$

When $L^S$ reaches $\bar{L}^S$, given by

$$\bar{L}^S = \frac{\rho_H - \rho_0}{\rho_1 - \rho_0} A,$$

then $I = \bar{I}$. For $L^S > \bar{L}^S$, $q$ decreases with $L^S$ and investment,

$$I = \frac{A}{1 + \hat{\rho} + (\hat{q} - 1)(\rho_H - \rho_0) - \rho_0} = \frac{L^S}{\rho_H - \rho_0},$$

increases until $L^S = \bar{L}^S$ (i.e., $q = 1$), after which it is no longer affected by the supply of liquid assets.
Exercise 16.1 (borrowing abroad). (i) Investing abroad is inefficient since \( \mu < 1 \). So it is optimal to prevent investment abroad. Letting \( R_1 \) denote the return to investors in the case of success, the incentive compatibility constraint is

\[
p(RI - R_1) \geq \mu I.
\]

The breakeven constraint is

\[
pR_1 = I - A.
\]

The NPV,

\[
U_b = (pR - 1)I,
\]

is maximized when \( I \) is maximized subject to the incentive compatibility and breakeven constraints, and so

\[
I = \frac{A}{1 - (pR - \mu)},
\]

and so \( U_b = \frac{pR - 1}{1 - (pR - \mu)} A \).

This is a reinterpretation of the basic model with

\[
p_H = p, \quad p_L = 0, \quad B = \mu.
\]

Investing abroad brings the probability of success of the domestic investment down to 0. And because investors are unable to grab any of the diverted funds, their proceeds are but a private benefit for the entrepreneur.

(ii) One has

\[
p[(1 - \tau)RI - R_1] \geq \mu I
\]

and

\[
pR_1 + (1 - p)\sigma R_1 = I - A.
\]

The government’s breakeven constraint is

\[
p\tau RI = (1 - p)\sigma R_1.
\]

The borrowing capacity is unchanged, because the pledgeable income is unaffected.

In contrast, when public debt \( D \) (per entrepreneur) is financed through corporate taxes,

\[
p\tau RI = D,
\]

then

\[
I = \frac{A - D}{1 - (pR - \mu)}
\]

and

\[
U_b = \frac{pR - 1}{1 - (pR - \mu)} (A - D).
\]

(iii) In the case of government commitment, \( \mu = \mu_L \) maximizes \( U_b \). In the absence of commitment, suppose that investors expect \( \mu = \mu_H \). Then the entrepreneurs receive

\[
p(RI - R_1) = \mu_L I \quad \text{if} \quad \mu = \mu_L
\]

and

\[
\max(p(RI - R_1), \mu_H I) = \mu_H I \quad \text{if} \quad \mu = \mu_H.
\]

Hence, \( \mu = \mu_H \). And \( U_b \) is decreased.

(iv) The exchange rate is given at date 2 by

\[
eR = pR_1;
\]

(Assuming that there is no excess supply of tradables \( R \); otherwise \( e \equiv 1 \).) One has

\[
p(RI - R_1) = \mu_I
\]

and

\[
\frac{pR_1}{e} = I - A.
\]

Then

\[
I = R + A = \frac{A}{1 - (pR - \mu)/e}.
\]

\( e \geq 1 \) is equivalent to \( (1 + A/R)(pR - \mu) \geq 1 \).

Exercise 16.2 (time-consistent government policy).

(i) The incentive constraint is

\[
[(p_H + \tau)R - (p_L + \tau)]R_b \geq BI.
\]

And so the investors’ breakeven condition is

\[
(p_H + \tau)\left(R - \frac{B}{\Delta \rho}\right)I = I - A.
\]

This yields \( I(\tau) \).

The government maximizes

\[
[(p_H + \tau)R - y(\tau)]I.
\]

Hence,

\[
y'(\tau^*) = R.
\]

(ii) \( \max\{[(p_H + \tau)R - 1 - y(\tau)]I\} \)

\[
\Rightarrow [y'(\tau^*) - R]I = [(p_H + \tau)R - 1 - y(\tau^*)] \frac{dI}{d\tau}.
\]

(iii) \( \tau < \tau^* \) then.
Exercise 16.3 (political economy of exchange rate policies). (i) \( d^* = p_R R^* \) and \( d = p_R R^S + (1 - p_R) R^F \).

(ii) The entrepreneur’s incentive constraint (expressed in tradables) is
\[
(\Delta p) \left[ R^S_b + \frac{R^S_b - R^F_b}{e} \right] \geq BI.
\]

The foreign investors’ breakeven constraint can be written as
\[
d^* + \frac{d}{e} = p_R R^S + \frac{p_R R^S + (1 - p_R) R^F}{e} \geq I - A.
\]
And so, adding up these two inequalities,
\[
p_R \left( R - \frac{B}{\Delta p} \right) I + \frac{p_R SI + (1 - p_R) R^F - p_R R^F_b}{e} \geq I - A.
\]
Thus, if the NPV per unit of investment is positive (which we will assume), it is optimal to set
\[
R^F_b = 0 \quad \text{and} \quad R^F_l = SI.
\]
The investment is therefore
\[
I(e) = \frac{A}{1 - [(S/e) + p_R]}.
\]
(1)

It decreases as the exchange rate depreciates because part of the firm’s production is in nontradables.

(iii) **Commitment.** Suppose, first, that the government chooses \( g^* \) before entrepreneurs borrow abroad.

The representative entrepreneur has expected utility
\[
[S^I - d] + p_R R^S + \max_{c^*_l} [u(c^*_l) - ec^*_l] + v(g^*)
\]
In the end, the entrepreneur’s average consumption of nontradables is
\[
SI
\]
and the (average and individual) consumption of tradables is
\[
R^* - g^* + [p_R R - 1]I + A
\]
since the NPV, \( (p_R R - 1)I + SI \), must accrue to them from the investors’ breakeven condition.

Hence, the government chooses \( g^* \) so as to solve
\[
\max_{g^*} \{ SI + u(R^* - g^* + [p_R R - 1]I + A) + v(g^*) \}
\]
subject to (1) and the market-clearing equation,
\[
p_R RI(e) + R^* - g^* = c^*_l(e) + [I(e) - A].
\]
(2)
The first-order condition is (using \( u' = e \))
\[
v'(g^*) = e \left[ 1 - \left( \frac{S}{e} + (p_R R - 1) \right) \frac{dI}{de} \frac{de}{dg^*} \right] > e.
\]

**Noncommitment.** Under noncommitment, investment is fixed at some level \( \tilde{I} \) at the date at which \( g^* \) is chosen. So the government solves
\[
\max_{g^*} \left\{ \tilde{S} + u \left( R^* - g^* + p_R \tilde{I} - d^* - \frac{d}{e} \right) + v(g^*) \right\}
\]
and so
\[
v'(g^*) = e \left[ 1 - \frac{d}{e^2} \frac{de}{dg^*} \right] < e.
\]

(iv) Note that under noncommitment \( g^* \) increases as the debt expressed in nontradables, \( d \), increases. Overspending imposes a negative externality on foreigners when their claims are in nontradables and therefore can be depreciated.

Each borrower would be better off if the other borrowers issued fewer claims in nontradables. But each borrower also has an individual incentive to use nontradables as collateral so as to maximize borrowing capacity.
Review Problems

Review Problem 1 (knowledge questions). Answer the following subquestions:

(a) How does theory account for the sensitivity of investment to cash flow? What does it predict concerning the impact of balance-sheet strength on this sensitivity?

(b) What are the costs and benefits of issuing senior debt?

(c) Describe the main ingredients and conclusions of a model of signaling and term structure of debt.

(d) Explain the control approach to the diversity of securities.

(e) Why does the initial owner issue several securities (rather than just 100% equity) in the Gorton–Pennachi paper?

(f) Discuss the costs and benefits of a liquid market for stocks. According to your discussion, are subsidiaries more or less likely to be publicly traded?

(g) When does diversification boost borrowing capacity? Why?

(h) What determines the allocation of formal control rights between an entrepreneur and investors?

(i) What is a credit crunch? Who suffers most from a credit crunch?

(j) Explain a firm’s demand for liquidity.

(k) How does corporate liquidity demand affect the pricing of assets in general equilibrium?


(m) True or false?

- Speculators acquire too little information.
- Firms with the strongest balance sheets suffer more from a credit crunch.
- The “cross-pledging”/diversification benefit is highest when the borrower can secretly choose the extent of the correlation between her different activities.
- Speculative monitoring boosts pledgeable income by improving performance measurement.

(n) What is market timing? What is the theoretical take on this notion?

(o) Discuss briefly the implications of the entrepreneur’s having private information when issuing claims (type of securities issued, etc.).

(p) Explain the theory of free cash flow.

(q) Is there a liquidity–accountability tradeoff?

(r) Borrowers often sacrifice value (in the sense of NPV) so as to increase the income that can be pledged to the investors and to thereby obtain financing. Give four illustrations of this general phenomenon.

(s) Give the intuition for the existence of a corporate demand for liquidity, and why it is optimal to hoard some liquidity but not enough to allow for all reinvestments smaller than the continuation NPV.

(t) Consider an adverse-selection context in which the borrower has two possible types. What is the low-information-intensity optimum? When is the equilibrium unique?

(u) “In the pure theory of takeovers, the latter are more likely when the incumbent is credit constrained at the initial stage”: true or false? Why?

(v) Are borrowers with weak or strong balance sheets the stronger supporters of strong contracting institutions? Do their preferences in the matter
change over the firm’s life cycle? Give some examples.

(w) Discuss property rights institutions. Are there externalities in the allocation of investors among existing securities?

(x) Give a couple of reasons why a monitor may overmonitor.

(y) How can an entrant in a market reduce the probability of predation by an incumbent?

(z) Define the notion of financial muscle. When do firms accumulate too much or too little financial muscle in the context of mergers and acquisitions?

Review Problem 2 (medley). An entrepreneur, who has no cash and no assets, wants to finance a project which costs \( I > 0 \). The project yields \( R \) with probability \( p \) and 0 with probability \( 1 - p \). A loan contract specifies a reward \( R_b \) for the entrepreneur if the income is \( R \) and 0 if the income is 0. If financed, the probability of success (that is, income \( R \)) depends on the (noncontractible) effort \( e \in \{ \bar{e}, \bar{e}_c \} \) chosen by the entrepreneur: it is equal to \( p_1 \) if \( e = \bar{e} \) and \( p_L \) if \( e = \bar{e}_c \), where

\[
1 > p_H > p_L = 0.
\]

The entrepreneur enjoys private benefit \( B > 0 \) if \( e = \bar{e}_c \) and 0 if \( e = \bar{e} \). There is a competitive loan market and the economy’s rate of interest is equal to 0.

(i) Show that the project is financed if and only if

\[
p_H R \geq B + I.
\]  

Interpret condition (1).

Subquestions (ii)–(iv) modify subquestion (i) in a single direction

(ii) (Debt overhang.) Suppose that before this project comes up the entrepreneur owes debt \( D > 0 \) to some initial creditors. This debt is senior and cannot be diluted. Furthermore, the initial debtholders cannot be reached before the investment is financed. Show that (1) must be replaced by

\[
p_H R \geq B + I + p_H D.
\]

If (2) is not satisfied, what should be done to prevent this debt overhang problem?

(iii) (Inalienability of human capital.) Suppose (à la Hart and Moore 1994\(^{14}\); see also Section 4.5) that,

just before income \( R \) is realized (which can happen only if \( e = \bar{e}_c \) and there is a “good state of nature”), both parties learn that the project is about to be successful (provided that the entrepreneur completes it, which she can do at no additional cost). The entrepreneur can then force her lenders to renegotiate “à la Nash,” that is, to split the pie, because she is indispensable for the completion of the project. How is the analysis in question (i) modified?

(iv) (Intermediation.) Suppose that (1) is not satisfied but \( p_H R > I \). Introduce a monitoring technology: the entrepreneur can go to a bank. By spending \( c > 0 \), the bank can catch the entrepreneur if \( e = \bar{e}_c \) and reverse the decision to \( e = \bar{e} \); in this case the entrepreneur is punished: she receives no income and does not enjoy her private benefit. There is no scarcity of monitoring capital (and therefore no rent for the monitor in equilibrium). All borrowing is from the monitor; that is, there are no uninformed investors (unlike in Chapter 9). So \( I_m = I \) and \( R_b + R_m = R \), where \( I_m \) and \( R_m \) denote the monitor’s investment contribution and stake in success (if the entrepreneur does not misbehave, otherwise the monitor appropriates the entire return).

The bank and the entrepreneur choose simultaneously whether to monitor (for the bank), and whether to select \( \bar{e}_c \) (for the entrepreneur). The expected-payoff matrix for this game is thus

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<td>( M )</td>
<td>( (p_H R_m - c, p_H (R - R_m)) )</td>
</tr>
<tr>
<td>( DNM )</td>
<td>( (p_H R_m, p_H (R - R_m)) )</td>
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where \( P \) is the payment to the bank, “M” is “Monitor,” “DNM” is “Do not monitor,” and where the first payoff is that of the bank.

- Show that the equilibrium is in mixed strategies: the entrepreneur chooses \( e \) with probability \( z = c / (p_H R) \). The bank does not monitor with probability \( y = p_H (R - R_m) / B \).
- Argue that the project is financed if and only if

\[
p_H R \geq c + I.
\]  

(iii) (Debt overhang.) Suppose that before this project comes up the entrepreneur owes debt \( D > 0 \) to some initial creditors. This debt is senior and cannot be diluted. Furthermore, the initial debtholders cannot be reached before the investment is financed. Show that (1) must be replaced by

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\[\text{just before income } R \text{ is realized (which can happen only if } e = \bar{e}_c \text{ and there is a “good state of nature”), both parties learn that the project is about to be successful (provided that the entrepreneur completes it, which she can do at no additional cost). The entrepreneur can then force her lenders to renegotiate “à la Nash,” that is, to split the pie, because she is indispensable for the completion of the project. How is the analysis in question (i) modified?}

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The bank and the entrepreneur choose simultaneously whether to monitor (for the bank), and whether to select } \bar{e}_c \text{ (for the entrepreneur). The expected-payoff matrix for this game is thus}

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where } P \text{ is the payment to the bank, } “M” \text{ is “Monitor,” } “DNM” \text{ is “Do not monitor,” and where the first payoff is that of the bank.}

- Show that the equilibrium is in mixed strategies: the entrepreneur chooses } e \text{ with probability } z = c / (p_H R) \text{. The bank does not monitor with probability } y = p_H (R - R_m) / B \text{.}
- Argue that the project is financed if and only if

\[p_H R \geq c + I.\]
Review Problem 3 (project choice and monitoring). Consider the fixed-investment model with two alternative projects: the two projects have the same investment cost \( I \) and the same payoffs, \( R \) in the case of success and 0 in the case of failure. The entrepreneur has initial wealth \( A \) and must collect \( I - A \) from risk-neutral investors who demand a rate of return equal to 0. Project 1 has probability of success \( p_1 \) if the entrepreneur works and \( p_L = p_1 - \Delta p \) if she shirks. Similarly, the probabilities of success are \( q_1 \) and \( q_L = q_1 - \Delta q \) for project 2, where

\[
\Delta q = \Delta p.
\]

Project 1 (respectively, 2) delivers private benefit \( B \) (respectively, \( b \)) when the entrepreneur shirks; no private benefit accrues in either project if the entrepreneur works. We assume that project 1 has a higher probability of success

\[
p_1 > q_1,
\]

and that

\[
p_1 \left( R - \frac{B}{\Delta p} \right) < q_1 \left( R - \frac{b}{\Delta p} \right) < I.
\]

Assume that at most one project can be implemented (because, say, the entrepreneur has limited attention), and (except in question (iii)) that the investors can verify which project, if any, is implemented.

(i) Divide the set of possible net worths \( A \), \([0, \infty)\), into three regions \([0, \bar{A}_p]\), \([\bar{A}_q, \bar{A}_p]\), and \([\bar{A}_p, \infty)\) and show that the equilibrium investment policies in these regions are “not invest,” “invest in project 2,” and “invest in project 1.” Verify that

\[
\bar{A}_p - \bar{A}_q = (p_1 - q_1)R - \frac{[p_1B - q_1b]}{\Delta p}.
\]

(ii) In this question only, suppose that the investors cannot verify which project the entrepreneur is choosing (they only observe success/failure). Argue that nothing is altered if \( A \geq \bar{A}_p \). Show that if \( A \in [\bar{A}_q, \bar{A}_p) \), then financing may be jeopardized unless the entrepreneur must incur private cost \( \psi \) in order to substitute project 1 for project 2, where

\[
\psi > B - (q_1 - p_1)b/\Delta p.
\]

(iii) Suppose now that the private benefit of shirking on project 1 can be reduced from \( B \) to \( b \) by using an active monitor. This active monitor has private cost \( c \) of monitoring and demands monetary rate of return \( \chi \) (where \( \chi \geq p_1/p_1 \)): \( p_1R_m = \chi M \).

What is the cost \( M \) of hiring the active monitor? Assuming \( (B - b)/(\Delta p) > M \), solve for the equilibrium policies as in question (i), assuming that

\[
q_1R < p_1R - M < q_1R + \frac{b}{\Delta p}.
\]

(iv) Ignoring active monitoring, suppose now that both projects can be implemented simultaneously (at cost \( 2I \)) and that they are statistically independent. Assume that \( q_L = 0 \), that only total profit is observed, that the entrepreneur is rewarded only in the case of overall success \( (R_2 > 0, R_1 = R_0 = 0) \), and that “work” must be induced on both projects. Describe the three incentive compatibility constraints and argue that one of them is irrelevant. Distinguish two cases depending on

\[
\frac{B}{B + b} \geq q_1.
\]

Determine the threshold \( \bar{X}_{pq} \) over which the entrepreneur can thus diversify.

Review Problem 4 (exit strategies). An entrepreneur has cash \( A \) and wants to finance a project involving investment cost \( I > A \). The project yields \( R \) with probability \( p \) and 0 with probability \( 1 - p \). The entrepreneur may either behave and enjoy no private benefit, in which case the probability of success is \( p_H \), or misbehave and enjoy private benefit \( B \), in which case the project fails for certain (\( p_L = 0 \)).

Assume that \( p_H R > I \) and \( B < I \) (the NPV is positive if and only if the entrepreneur behaves).

(i) Define the notion of pledgeable income. Show that the entrepreneur can obtain financing if and only if

\[
p_1R - I \geq B - A.
\]

Show that the entrepreneur’s utility is

\[
U_b = p_1R - I.
\]

(ii) Suppose now that the entrepreneur, with probability \( \lambda \) (\( 0 < \lambda < 1 \)), has an interesting outside investment opportunity. To profit from this opportunity, the entrepreneur must receive cash (exactly) equal to \( r > 0 \) before the final outcome on the initial project is realized. With probability \( 1 - \lambda \), no such opportunity arises. Whether the opportunity arises is not observable by the investors (so the entrepreneur can “fake” a liquidity need and strategically exit). If
Review Problems

Consider a country with a continuum of identical firms (of mass 1). The representative firm is described as in Section 3.4. That is, it has initial wealth $A$ and has a variable-investment project. As usual, let
\[ \rho_0 \equiv \frac{p_H(R - \frac{B}{\Delta p})}{\Delta p} < 1 < \rho_1 \equiv \frac{p_H R}{\Delta p}. \]

To finance their investment, the domestic firms must borrow from domestic residents and from foreign investors. Domestic residents have limited savings $S_0 < \rho_0 A/(1 - \rho_0)$. Foreign investors have unlimited amounts of money to lend at the market rate of return. The rate of return demanded by foreigners and domestic residents is 0.

After the financing has been secured, the country’s government chooses a tax rate $t \geq 0$ on income received by investors. This tax rate does not apply to the entrepreneurs and does not discriminate between domestic and foreign investors.

To close the model, assume that the government transforms tax proceeds $t S_2$ (for an amount of investor income $S_2$) into $B_0(t) I$, where
\[ B_0(t) > 0, \quad B'_0(t) < 0, \quad \text{and} \quad B_0(0) = 1. \]

(That is, tax collection is wasteful here.) Assume that these benefits $B_0(t) I$ are returned to entrepreneurs

the opportunity arises and the entrepreneur is able to invest $r$ in it, the entrepreneur receives $\mu r$, where $\mu > 1$. This payoff is also unobservable by investors. The timing is as follows:

Stage 0. The investors bring $I - A$ (provided they are willing to finance the project), and investment occurs.

Stage 1. The entrepreneur chooses between $p_H$ and $p_L$.

Stage 2. The entrepreneur privately learns whether she faces an investment opportunity (and therefore needs cash $r$ in order not to forgo the opportunity).

Stage 3. The project’s outcome ($R$ or 0) is publicly observed. The entrepreneur receives $\mu r$ if she faced an investment opportunity (and only if she faked an investment opportunity) at stage 2 and invested $r$.

Consider a contract in which the entrepreneur is offered a choice for stage 2 between

(a) receiving $r$ at stage 2 and nothing at stage 3, and
(b) receiving nothing at stage 2 and $R_b$ in the case of success (and 0 in the case of failure) at stage 3.

(This class of contracts is actually optimal.) The menu is designed so that she chooses option (a) at stage 2 if and only if she has an investment opportunity.

- Show that the incentive constraint at stage 1 is
\[ (1 - \lambda)(p_H R_b - r) \geq B. \]

To prove this, argue that, were the entrepreneur to misbehave, she would always select option (a), while, if she behaves, then necessarily $p_H R_b > r$.

(iii) Keeping within the framework of question (ii) and assuming that
\[ \mu r \geq r + \frac{B}{1 - \lambda}, \]  \hspace{1cm} (3)

show that the project is financed if and only if
\[ p_H R - I \geq B - A + r \]  \hspace{1cm} (4)

and the entrepreneur’s utility is then
\[ U^*_H = p_H R - I + \lambda (\mu - 1) r \]  \hspace{1cm} (5)

(the superscript “L” stands for the fact that the entrepreneur has a liquid claim).

Compare (4) and (5) with (1) and (2), and conclude on the desirability and feasibility of liquid compensation contracts. What is the interpretation of (3)?

(iv) Suppose now that, at some cost $c$, a signal can be obtained at stage 2. So, if the entrepreneur claims she needs cash $r$ at stage 2 (which has probability $\lambda$), a signal is obtained, which takes one of two values: good or bad. The probability of a good signal is $q_H$ if the entrepreneur has behaved and $q_L < q_H$ if she misbehaved. The entrepreneur receives $r$ at stage 2 only if the good signal accrues. Option (b) is unchanged. Show that the project is financed if and only if
\[ p_H R + \lambda q_H (\mu - 1) r - B - q_H (\lambda \mu + 1 - \lambda) r \geq I - A + c. \]

(Show that the incentive constraint is $\lambda q_H \mu r + (1 - \lambda) p_H R_b \geq B + q_H (\lambda \mu + 1 - \lambda) r$.) What is $U^*_H$? What do you infer about the desirability of acquisition of this signal?

Review Problem 5 (property rights institutions and international finance). Consider a country with a...
in proportion to the tax income collected. Thus the representative entrepreneur’s equilibrium NPV is

$$(\rho_1 - 1)I - t^\ast \rho_0 I + B_0(t^\ast)\rho_0 I.$$  

The government maximizes the sum of the welfares of the entrepreneurs and of the domestic investors, and puts no weight on foreign investors (see Figure 1).

(i) Solve for a rational expectation equilibrium $$(I^\ast, t^\ast, \theta^\ast)$$, where $$I^\ast$$ is the representative entrepreneur’s investment, $$t^\ast$$ is the equilibrium tax rate, and $$\theta^\ast$$, equal to $$S_D/(I^\ast - A)$$, is the fraction of external financing brought by domestic residents.

(ii) How does the entrepreneur’s welfare change with domestic savings $$S_D$$?

(iii) What tax rate would prevail if the government were able to commit on the tax rate before the financing stage?

(iv) How would your answer to question (i) change if the government were still unable to commit to a tax rate and furthermore could discriminate between domestic and foreign investors?

**Review Problem 6 (inside liquidity).** Consider the variable-investment model with two possible values of liquidity shocks ($$0$$ and $$\rho$$ per unit of investment). The timing is described in Figure 2.

Investors and entrepreneurs are risk neutral, the entrepreneur is protected by limited liability and the rate of interest in the economy is 0. If the firm is in distress (suffers a liquidity shock), a reinvestment $$\rho x I$$ allows it to salvage a fraction $$x \in [0, 1]$$ of the investment (so there is no constraint to salvage all or nothing, even though, as we will see, the solution will be a “corner solution”).

Continuation is subject to moral hazard. The probability of success is $$p_H$$ if the entrepreneur behaves and $$p_L$$ if she misbehaves. The private benefit of misbehaving is $$B x I$$. The project yields $$R x I$$ in the case of success and 0 in the case of failure. Let

$$\rho_0 \equiv p_H \left( R - \frac{B}{\Delta p} \right) < c  
\equiv \min \left( 1 + \lambda \rho_1, \frac{1}{1 - \lambda} \right) < \rho_1 \equiv p_H R.$$

In the first two questions, one will assume that there is a costless outside store of value (there are assets that at per-unit cost $$q = 1$$ at date 0 yield a return equal to 1 at date 1).

(i) Show that, when choosing $$x$$, the entrepreneur can borrow up to

$$I = \frac{A}{(1 + \lambda \rho x) - (1 - \lambda + \lambda x)\rho_0}.$$  

(ii) Compute the borrower’s utility and show that

$$x = 1 \quad \text{if and only if} \quad (1 - \lambda) \rho \leq 1$$

(and $$x = 0$$ otherwise).

(Hint: write the borrower’s utility as a function of the “average unit cost of preserved investment.”)

(iii) Suppose now that there is no outside store of value. There is mass 1 of (ex ante identical) entrepreneurs. The only liquidity in the economy is the inside liquidity created by the securities issued by the firms. One will assume that $$\rho_0 < \rho < 1/(1 - \lambda)$$. Is there enough liquidity if the firms’ liquidity shocks
are perfectly correlated? If not, what is the level of the liquidity shortage?

(iii) Consider question (ii) except that the liquidity shocks faced by the entrepreneurs are independently distributed. Show that the firms’ holding the stock index may not be optimal. What should be done?

Review Problem 7 (monitoring). (i) A borrower has assets $A$ and must find funds $I - A$. The project yields $R$ or 0 and the borrower is protected by limited liability. Shirking yields probability of success $p_L$ and private benefit $B$, while working yields probability of success $p_H = p_L + \Delta p > p_L$ and private benefit 0. Assume that

$$I - A > p_H (R - \frac{B}{\Delta p}).$$

There is one potential monitor, who, at private cost $c(x)$ ($c' > 0$, $c'' > 0$, $c'(0) = 0$), can with probability $x$ reduce the private benefit of shirking from $B$ to $b < B$. The borrower learns what her private benefit is (that is, whether monitoring was successful) before choosing his effort.

Compute the optimal fraction $\alpha$ of the final return $R$ in the case of success that should be held by the large monitor. Show that if the large monitor holds all outside shares (i.e., all shares not belonging to the borrower), there is overmonitoring. Explain.

(ii) Suppose that initially the large monitor is not around, but it is known that one will appear (before the borrower selects her effort decision). So outside shares are initially held by small, uninformed shareholders. (The timing is: at date 0, the borrower issues the securities to the small, uninformed investors. Between dates 0 and 1, the potential large monitor appears and may try to purchase shares from the initial investors; at date 1, the monitoring and effort decisions are selected. The return, if any, accrues at date 2.)

Suppose that the large shareholder makes a tender offer (bid $P$) for a fraction or all the investors’ shares (the tender offer is unrestricted and unconditional: the large shareholder purchases all shares that are tendered to him at the price offer $P$).

- One usually believes that the supply function in competitive financial markets is perfectly elastic. Show that the "supply function" $\alpha(P)$ (the number of shares tendered) is here upward sloping.

Give the intuition for this result.

- Compute the large shareholder’s ex ante payoff for arbitrary bids $P$.

- Conclude. Is the borrower able to raise funds at date 0?

(iii) Discuss informally the implications of question (ii). How would private benefits of control of large shareholders affect the analysis?

Review Problem 8 (biotechnology research agreements). Lerner and Malmendier study biotechnology research collaborations. Almost all such contracts in their sample specify termination rights. These may be conditional on specific events (50% of the contracts in their sample of 584 biotechnology research agreements) or at the complete discretion of the financier (39%). The financing firm may in the case of termination acquire broader licensing rights than it would have in the case of continuation. These broad licensing rights can be viewed as costly collateral pledging that both increase the income of the financier and boost the R&D firm’s incentive to achieve a good performance on the project.

Lerner and Malmendier’s empirical finding is that such an assignment of termination and broad licensing rights is more likely when it is hard to specify a lead product candidate in the contract (and so entrepreneurial moral hazard is particularly important) and when the R&D firm is highly constrained financially. This review problem builds on their analysis.

There are three dates, $t = 0, 1, 2$, and two players, a biotechnology entrepreneur or borrower and a financier (pharmaceutical company).

At date 0, the risk-neutral biotechnology entrepreneur has a project involving initial investment cost $I$. The entrepreneur has initial wealth $A$, and so the (risk-neutral) financier must contribute $I - A$. The market rate of interest in the economy is 0 and the capital market is competitive. If the research activity is noncontractible, the entrepreneur exerts unobservable date-0 effort $e = 0$ or 1. (When it is contractible, then necessarily $e = 1$.) A high effort is to be interpreted as focusing on the project while a


16. See Section 4.3.4 for the theoretical foundations of this assertion.
Review Problems

Date 0

Financing contract between entrepreneur and financier.

\[ e \in \{0, 1\} \ (\text{if research activity noncontractible}) \]

### Schedule

- **Date 0**: Financing contract between entrepreneur and financier.
- **Date 1**: Signal \( \tau \) accrues. Allocation of rights.
- **Date 2**: Project succeeds \((R)\) with probability \( p + \tau \) or fails \((0)\).

### Figure 3

- **Date 0**: Financing contract between entrepreneur and financier.
- **Date 1**: Signal \( \tau \) accrues. Allocation of rights.
- **Date 2**: Project succeeds \((R)\) with probability \( p + \tau \) or fails \((0)\).

### Review Problems

low effort corresponds to paying more attention to alternative or adjacent activities, whose value is \( C_e \) if they are later pursued by the entrepreneur and only \( \beta C_e \) if they are seized and pursued by the financier. These payoffs are noncontractible and will accrue to the owner (entrepreneur or financier) of the corresponding rights. Furthermore,

\[ C_0 > C_1. \]

At date 1, a publicly observed signal \( \tau \in [\tau, \bar{\tau}] \) accrues. The cumulative distribution is \( F(\tau) \) if \( e = 1 \) and \( G(\tau) \) if \( e = 0 \), with densities \( f(\tau) \) and \( g(\tau) \) satisfying the monotone likelihood ratio property:

\[ \frac{f(\tau)}{g(\tau)} \text{ is increasing in } \tau. \]

Contingent on the realization of the signal, the project can be terminated or continued. Termination yields 0 on this specific project (while the value of the alternative activities, \( C_e \) or \( \beta C_e \) depending on the owner, are independent of the signal). Continuation requires the financier to reinvest \( J \) into the project. Success brings a verifiable profit \( R \), failure yields no profit. The probability of success at date 2 is then \( p + \tau \). Regardless of the signal \( \tau \), \( p \) is determined by entrepreneurial moral hazard at date 1: if the entrepreneur behaves, she receives no private benefit and \( p = p_H \); if she misbehaves, she receives private benefit \( B > 0 \) and \( p = p_L = p_H - \Delta p \), where \( \Delta p > 0 \).

The timing is summarized in Figure 3.

We assume that at date 0 the entrepreneur offers a contract to the financier (nothing hinges on this assumption about relative bargaining power). A financing contract specifies 17

- \( e = 1 \) if the research activity is contractible and contingent on the realization of the signal \( \tau \);
- a probability \( x(\tau) \) of continuation;
- a probability \( y(\tau) \) that the entrepreneur keeps the rights on the adjacent activities;
- a reward \( R_b(\tau) \) for the entrepreneur in the cases of continuation and success.

We assume that the entrepreneur is protected by limited liability and so the latter reward must be non-negative. Because the entrepreneur is risk neutral, there is no loss of generality in assuming that the entrepreneur receives no reward if either the project is interrupted at date 1 or if it fails at date 2.

**Assumption 1.** The project has positive maximum NPV relative to that, \( C_0 \), obtained in the absence of financing if and only if \( e = 1 \). Let \( T_{FB} \) be defined by

\[ [p_H + \tau_{FB}]R = J, \]

then

\[ \int_{\tau_{FB}}^{\tau} [(p_H + \tau - J) dF(\tau) + C_1 - I > C_0], \]

\[ \int_{\tau_{FB}}^{\tau} [(p_H + \tau - J) dG(\tau) + C_0 - I < C_0]. \]

(i) Suppose, first, that the research activity is contractible: the contract can specify \( e = 1 \) and so there is no moral hazard at date 0. Show that the optimal contract falls into one of the four following regions, as \( A \) decreases: (1) high payment, no reversion, first-best termination; (2) termination rights for the financier; (3) termination and reversion rights for the financier; and (4) no funding.

(ii) Solve for the optimal contract when the effort is noncontractible.

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17. At the optimum contract, \( x \) and \( y \) will take values 0 or 1 only.
Review Problem 9 (conflict of interest in multitasking). An R&D entrepreneur has an idea for a new product. To market this product, the entrepreneur must first develop a technology. The technology, if developed, then allows the product to be marketed, yielding profit $R$. No profit is made if no technology is developed.

There are two possible and independent research strategies. Each is described as follows. The probability that the entrepreneur succeeds in developing the technology is $p_H$ if she behaves (and then receives no private benefit) and $p_L = p_H - \Delta p$ if she misbehaves (and then receives private benefit $B$). Assume all along that the incentive contract must induce good behavior. Each research strategy involves investment cost $I$ if developed. The technologies are substitutes (the profit is $R$ whether one or two technologies have been developed). They are independent in that the success or failure of one technology conveys no information about the likelihood of success of the other.

The entrepreneur has cash on hand $A$ and is risk neutral and protected by limited liability. The investors are risk neutral and the market rate of interest is equal to 0.

(i) Suppose that the entrepreneur and the investors decide that the entrepreneur will pursue a single research strategy. Show that the project can be funded if and only if

$$p_H \left( R - \frac{B}{\Delta p} \right) \geq I - A.$$  

(ii) Suppose that

$$p_H (1 - p_H) R > I.$$  

Interpret this inequality.

Consider funding the two research strategies. The investment cost is then equal to $2I$. Assume that the managerial reward $R_b$ can be contingent only on the firm’s profit (which is equal to $R$ whether one or two technologies have been developed). Thus, $R_b$ cannot be made contingent on the market of successfully developed substitute technologies.

Show that the nonpledgeable income is equal to

$$[1 - (1 - p_H)^2] \frac{B}{(1 - p_H) \Delta p}.$$  

What is the necessary and sufficient condition for investors to be willing to finance the two research strategies?

(iii) Show that the entrepreneur (who owns the research strategies) may want to hire a second and identical entrepreneur to perform the second research strategy, even if it means leaving an agency rent to the new entrepreneur (this will be shown to occur whenever $A < p_H B / \Delta p$). One will assume that the entrepreneurs are rewarded on the basis of their own profit and that if both technologies succeed, each “division” receives $R$ with probability $\frac{1}{2}$. 
Answers to Selected Review Problems

Review Problem 2 (medley). (i) The incentive constraint is
\[(p_H - p_L)R_b \geq B\]
or, since \(p_L = 0\),
\[p_H R_b \geq B.\]
Hence the nonpledgeable income is equal to \(B\) and the pledgeable income is
\[p_H R - B.\]

Because the entrepreneur has no cash \((A = 0)\), this pledgeable income must exceed the investment cost \(I\).

(ii) (Debt overhang.) Assume that \(D < R\) (otherwise, new investors will never receive any income). The income that can be pledged to new investors is
\[p_H (R - D) - B.\]
Hence condition (2) must hold in the absence of renegotiation with initial lenders. The only way to raise funds if (2) is violated is to renegotiate the initial lender’s debt to a level \(R_1 \leq d\), where \(d \in (0, D)\) satisfies
\[p_H (R - d) = B + I.\]

(iii) (Inalienability of human capital.) The threat of renegotiation implies that the borrower can demand \(\frac{1}{2}R\) whenever \(R\) is about to accrue. And so a new constraint must be added to the funding program:
\[R_0 \geq \frac{1}{2}R.\]
Let \(R_0^*\) denote the borrower’s stake in the absence of negotiation (i.e., in question (i)):
\[p_H (R - R_0^*) = I\]
with
\[p_H R_0^* \geq B\]
for incentive compatibility.

Either \(R_0^* \geq \frac{1}{2}R\) and there is no renegotiation, and the outcome is as in question (i), or \(R_0^* < \frac{1}{2}R\), and then rewards that allow investors to recoup their initial outlay (i.e., \(R_0 \leq R_0^*\) are renegotiated up to \(\frac{1}{2}R\) just before success. Anticipating this, investors do not want to lend:
\[p_H (R - \frac{1}{2}R) - I < p_H (R - R_0^*) - I = 0.\]

(iv) (Intermediation.) The entrepreneur works if she is monitored with probability 1; but then the monitor does not want to monitor. Conversely, if the entrepreneur works with probability 1, the monitor does not monitor and the entrepreneur shirks if \(p_H R_0 < B\), which we will assume (we will assume that (1) is not satisfied). Hence the equilibrium must be in mixed strategies. Let us first write the monitor’s indifference equation:
\[(1 - z)(p_H R_m) + z(p_H R - c) = (1 - z)p_H R_m\]
\[= I_m = I.\]
Hence
\[zp_H R = c.\]
Similarly, \(y\) is given by the entrepreneur’s indifference equation:
\[p_H (R - R_m) = yB.\]
The monitor is willing to finance \(I\) if and only if
\[(1 - z)p_H R_m = \left(1 - \frac{c}{p_H R}\right)p_H R_m \geq I\]
or, because \(R_m \leq R\),
\[p_H R \geq c + I.\]

Finally, consider the entrepreneur’s utility. There are two ways of writing it. First,
\[U_b = p_H (R - R_m) = p_H R - \frac{I}{1 - z}\]
\[= p_H R - \frac{I}{1 - c/p_H R}.\]
Alternatively, $U_b$ is equal to the NPV:

$$U_b = \text{NPV} = p_R - yz(R - B) - (1 - y)c - I.$$  
Replacing $z$ and $yB$ by the values found above, we have

$$U_b = p_R - c - I + \frac{c}{p_R} (R - R_m)$$

or

$$(1 - \frac{c}{p_R}) U_b = p_R - c - I,$$

which gives the same expression as previously.

**Review Problem 3 (project choice and monitoring).**

(i) The incentive constraints for projects 1 and 2 are

$$(\Delta p) R_b \geq B \quad \text{and} \quad (\Delta p) R_b \geq b,$$

respectively. The cutoff levels of cash on hand for the two projects are given by

$$p_R \left( R - \frac{B}{\Delta p} \right) = I - A_p$$

and

$$q_R \left( R - \frac{b}{\Delta p} \right) = I - A_q.$$  

(ii) • Suppose that $A \geq A_p$. Then, choosing project 2 instead of project 1 yields, to the entrepreneur, 

$$\max\{q_R R_b, p_R R_b + b\} < \max\{p_R R_b, p_R R_b + B\}.$$  

• In contrast, if $A \in (A_q, A_p)$, then the entrepreneur gets 

$$\max\{p_R R_b, p_R R_b + B\} = p_R R_b + B$$

(minus the private cost, $\psi$, of substituting the project), since

$$q_R (R - R_b) = I - A \geq I - A_p = p_R \left( R - \frac{B}{\Delta p} \right),$$

and so

$$R_b < \frac{B}{\Delta p},$$

The issue is moot if

$$\max_{\{A \in [A_q, A_p]\}} \{p_R R_b + B - q_R R_b\} = (p_R - q_R) \frac{B}{\Delta p} + B \leq \psi.$$  

(iii) • $p_R R_m = \chi R_m$ and $(\Delta p) R_m = c$ imply 

$$M = p_R R_m - I_m = c + p_R R_m - I_m = c + \left( \frac{p_R - \chi}{\chi} \right) \frac{c}{\Delta p}.$$  

• The first inequality in the condition stated in the question says that the NPV is higher when monitored in project 1 than when unmonitored in project 2. So for $A < A_p$, the entrepreneur would prefer project 1 monitored to project 2. The second inequality states that pledgeable income is higher under project 2. So we now have four regions:

- $[0, A_q)$: no project,
- $[A_q, A_p)$: project 2,
- $[A_p, \infty)$: project 1 monitored,
- $[A_p, \infty)$: project 1 unmonitored.

(iv) • Incentive constraints

$$p_R q_R R_2 \geq \begin{cases} p_R q_R R_2 + b = B, & (1) \\ p_R q_R R_2 + B, & (2) \\ p_R q_R R_2 + B + b = B + b. & (3) \end{cases}$$

(1) is obviously nonbinding. 

If (3) is binding, then the pledgeable income is 

$$(p_R + q_R) R - [B + b].$$

If (2) is binding, it is 

$$(p_R + q_R) R - p_R B \frac{\Delta p}{\Delta p}.$$  

The new NPV is $(p_R + q_R) R - 2I$.

• In the latter case, the financing condition is

$$(p_R + q_R) R - p_R B \frac{\Delta p}{\Delta p} > I - A$$

or

$$\left[ p_R \left( R - \frac{B}{\Delta p} \right) - [I - A] \right] + q_R R - I \geq 0.$$  

• In the former case, the financing condition is

$$(p_R + q_R) R - (B + b) \geq 2I - A.$$  

**Review Problem 4 (exit strategies).**

(i) Pledgeable income: maximum income that can be promised to investors without destroying incentives. Incentive constraint is

$$(\Delta p) R_b \geq B.$$  

And so

$$p_R R - p_R R_b = p_R R - B \geq I - A$$

for $p_L = 0$. The borrower’s expected utility is 

$$U_b = \text{NPV} = p_R R - I.$$  

(ii) • The incentive constraint is

$$(1 - \lambda) p_R R_b + \lambda \mu R \geq B + [\lambda \mu + (1 - \lambda)] R$$
or

\[(1 - \lambda) (p_1 R_b - r) \geq B.\]

- If \(p_1 R_b < r\), then option (a) is optimal regardless of the existence of an opportunity. There is then no incentive to behave.

- The investors are willing to finance if and only if

\[p_1 R - (1 - \lambda) p_1 R_b - \lambda r \geq I - A,\]

or, using the incentive constraint,

\[p_1 R - \left( r + \frac{B}{1 - \lambda} \right) (1 - \lambda) - \lambda r \geq I - A,\]

or

\[p_1 R - I \geq B - A + r.\] \hspace{1cm} (1)

- (5) is just the NPV.
- From (4), financing is harder to obtain if option (a) (the liquidity option) is available, unless \(r\) is small. But the entrepreneur’s welfare is higher provided the entrepreneur can get financing.

- The incentive constraint is

\[\lambda \left[ q_1 \mu r \right] + (1 - \lambda) p_1 R_b \geq B + q_1 (\lambda \mu + 1 - \lambda) r.\]

- The pledgeable income is

\[p_1 R - \lambda q_1 r - (1 - \lambda) p_1 R_b,\]

and must exceed total net outlay by investors \((I + c - A)\). To obtain the condition in question (iv), replace \((1 - \lambda) p_1 R_b\) using the incentive constraint.

- The entrepreneur’s utility is then

\[U_b = p_1 R - I + \lambda q_1 (\mu - 1) r - c.\]

- When \(q_1 = q_1 = 1\), we obtain the same answers as in question (iii), as one should.

- In the case of a perfect signal \((q_1 = 1, q_1 = 0)\), the financing condition is then

\[p_1 R - I \geq B - A - \lambda (\mu - 1) r + c.\]

**Review Problem 5 (property rights institutions and international finance).** (i) Let \(\theta^*\) denote the fraction of claims held by domestic residents. The government maximizes total domestic surplus:

\[\max_t \{ B_0 (t) - \theta^* t \}\]

or

\[B_0^* (t^*) = \theta^* = \frac{S_D}{I^* - A}.\]

The financing constraint becomes

\[(1 - t^*) p_0 I = I - A;\]

hence,

\[t^* = \frac{A}{1 - (1 - t^*) p_0}.\]

(ii) An increase in \(S_D\) raises \(\theta^*\) and \(I^*\), and lowers \(t^*\).

(iii) It would be optimal to commit to \(t = 0\).

(iv) The government would fully tax foreigners and not tax domestic residents. Hence,

\[I^* - A = S_D.\]

There is no tax on domestic investors, who obtain a rate of return exceeding 0.

**Review Problem 6 (inside liquidity).** (i) The investors’ breakeven constraint is

\[[1 + \lambda \rho X] I - A = [(1 - \lambda) + \lambda X] \rho_0 I.\]

Hence,

\[I = \frac{A}{[1 + \lambda \rho X] - [(1 - \lambda) + \lambda X] \rho_0}.\]

(ii) The NPV is

\[U_b = [(1 - \lambda + \lambda X) \rho_1 - (1 + \lambda X) \rho_1] I = \frac{\rho_1 - c(x)}{c(x) - \rho_0} A,\]

where

\[c(x) = \frac{1 + \lambda \rho X}{1 - \lambda + \lambda X}\]

is the average cost per unit of preserved investment. Minimizing \(c(x)\) yields \(x = 1\) if and only if

\[(1 - \lambda) \rho \leq 1.\]

(iii) There is a shortage of liquidity equal to \((\rho - \rho_0) I\).

(iv) The date-1 value of the average share in the index is \((1 - \lambda) \rho_0 I\) (assuming that the investors’ stake in the distressed firms has been diluted). And so, if \(\rho - \rho_0 > (1 - \lambda) \rho_0\), the index does not bring enough liquidity to the distressed firms.

The solution is a liquidity pool (e.g., a system of credit lines with a bank: see Chapter 15).

**Review Problem 7 (monitoring).** (i) The large monitor chooses \(x\) so as to maximize

\[\max_{x} \left[ x p_1 + (1 - x) p_L \right] \alpha R - c(x).\]

And so

\[c'(x) = (\Delta p) \alpha R.\]
The NPV is maximal when $x$ solves
\[
\max \{ xp_H R + (1 - x)(p_L R + B) - c(x) \}
\]
or
\[
c'(x) = (\Delta p) R - B,
\]
corresponding to
\[
\alpha = 1 - \frac{B/\Delta p}{R} < 1 - \frac{R_b}{R}.
\]
• Explanation: when the large monitor holds all outside shares, there is no externality of monitoring on the small investors and a negative externality on the borrower.

(ii) • Suppose $\alpha$ shares are tendered. Then
\[
x^*(\alpha) = (c')^{-1}(\Delta p) R, \quad \text{an increasing function of } \alpha.
\]
Therefore,
\[
P(\alpha) = [x^*(\alpha) p_H + (1 - x^*(\alpha)) p_L] R.
\]
$\alpha(P)$ is the inverse function and is increasing.

A higher price is consistent with more shares being tendered, as this generates more monitoring and thus a higher value per share.

• The large shareholder’s profit for a given $P$ is
\[
\max_x \{ xp_H (1 - x)p_L | \alpha(P) R - c(x) - P \alpha(P) \} = -c(x^*(\alpha(P))), \quad \text{in equilibrium.}
\]

• Thus there is no monitoring, and the borrower cannot raise funds.

(iii) • The large shareholder needs to be able to dilute (see Chapter 11). Burkart et al. (1998) look at takeover bids with such dilution and show that the upward-sloping supply curve arises on the equilibrium path (and not only off the path as above).

• Possible explanations: overpayment by empire builders; informed trade; benefits from control (gain access to production technology, below-market transfer prices to large shareholder’s subsidiary, etc.).

**Review Problem 8 (biotechnology research agreements).** In the case of continuation, the incentive compatibility constraint is
\[
(p_H + \tau) R_b(\tau) \geq (p_L + \tau) R_b(\tau) + B
\]
and so
\[
R_b(\tau) \geq R_b = \frac{B}{\Delta p}.
\]

(i) **Contractible research activity.** Let us first assume that the contract can specify $e = 1$, and so there is no moral hazard at date 0.

The optimal contract maximizes the entrepreneur’s utility $U_b$ (also equal to the NPV, since the entrepreneur chooses the contract so as leave no surplus to the financier) subject to the financier’s breakeven constraint and the incentive constraint: Program I:
\[
U_b = \max_{\{x(\tau), y(\tau), R_b(\tau)\}} \{ E[x(\tau)][(p_H + \tau) R - J] + y(\tau) C_1 + [1 - y(\tau)] \beta C_1 - I \}
\]
s.t.
\[
E[x(\tau)]((p_H + \tau)[R - R_b(\tau)]) - J + [1 - y(\tau)] \beta C_1 \geq I - A,
\]
\[
R_b(\tau) \geq B/\Delta p \quad \text{for all } \tau.
\]

Let $\mu$ denote the shadow price of the investors’ breakeven constraint, $\theta(\tau)$ the shadow price of the incentive constraint, and $L$ the Lagrangian:
\[
\frac{\partial L}{\partial x(\tau)} = (p_H + \tau) R - J + \mu[(p_H + \tau)[R - R_b(\tau)] - J],
\]
\[
\frac{1}{C_1} \frac{\partial L}{\partial y(\tau)} = (1 - \beta) - \mu \beta,
\]
\[
\frac{\partial L}{\partial R_b(\tau)} = -\mu x(\tau)(p_H + \tau) + \theta(\tau).
\]


• **High-payment region.** When $A > A_H$ (financially unconstrained entrepreneur), then $\mu = 0$. The continuation rule is the first-best, efficient continuation rule,
\[
x(\tau) = 1 \quad \text{if and only if } (p_H + \tau) R - J \iff \tau \geq \tau^{FB},
\]
and there is no reversion,
\[
y(\tau) = 1 \quad \text{for all } \tau.
\]

Because of risk neutrality, there is some indeterminacy as to the level of $R_b(\tau)$. One can, for example,
take it to be constant and equal to some \( R_b \) (over \([\tau^B, \hat{\tau}]\)). Furthermore, \( R_b \) decreases as \( A \) decreases and is equal to \( R_b \) when \( A = A_l \).

For \( A < A_h, \mu > 0 \) and \( R_b(\tau) = R_b \) for all \( \tau \). There exists a cutoff \( \tau^* \) such that

\[
\chi(\tau) = 1 \quad \text{if and only if} \quad \tau \geq \tau^*
\]

and

\[
(p_H + \tau^*)(R - \frac{B}{\Delta p}) < J < [p_H + \tau^*]R
\]

(the biotech entrepreneur accepts a less frequent continuation so as to please the pharmaceutical company; note that, at the cutoff value, the latter still incurs a loss).

• **Termination region.** When \( A_m < A < A_h \),

\[
\tau^* > \tau^B, \quad \gamma(\tau) = 1 \quad \text{and} \quad R_b = R_b.
\]

The cutoff \( \tau^* \) increases as \( A \) decreases, while \( \mu \) increases. The pharmaceutical company is less and less keen on refinancing as \( A \) decreases, but does not need to be granted inefficient reversion rights.

For \( A = A_m, \mu = (1 - \beta)/\beta \).

• **Termination-and-rights reversion.** When \( A_l < A < A_m \),

\[
R_b = R_b.
\]

Reversion rights are used in order to secure financing. In the absence of date-0 moral hazard there is some indeterminacy as to the state of nature in which reversion occurs (only the expressed amount of reversion is determined). However, with (an arbitrarily small amount of) date-0 moral hazard, MLRP implies (see below) that it is strictly optimal to set

\[
\gamma(\tau) = \begin{cases} 
1 & \text{for } \tau \geq \tau^{**}, \\
0 & \text{for } \tau < \tau^{**},
\end{cases}
\]

for some \( \tau^{**} \). Let us therefore focus on such a cutoff rule.

As \( A \) decreases, \( \tau^{**} \) increases (reversion becomes more frequent).

• **No-financing region.** \( A < A_1 \).

(ii) **Noncontractible research activity.** When the initial contract cannot specify the nature of the research activity, there is moral hazard. From Assumption 1, the contract must ensure that the entrepreneur selects \( e = 1 \). The optimal contract is then obtained by solving the following program.

**Program II.** This equals Program I plus the *ex ante* incentive compatibility constraint:

\[
\int_\tau^\infty [\chi(\tau)(p_H + \tau)R_b(\tau) + \gamma(\tau)C_1]dF(\tau) > \int_\tau^\infty [\chi(\tau)(p_H + \tau)R_b(\tau) + \gamma(\tau)C_0]dG(\tau).
\]

Let \( L_r \) denote the Lagrangian of the new program, and \( \lambda \) the Kuhn–Tucker multiplier of the *ex ante* (IC) constraint. The first-order conditions are

\[
\frac{\partial L_r}{\partial \gamma(\tau)} = \begin{cases} 
[(p_H + \tau)R - J] + \mu[(p_H + \tau)/(R - R_b(\tau)) - J] + \lambda[(p_H + \tau)R_b(\tau)]\frac{1 - g(\tau)}{f(\tau)} & \text{if } \tau \in \tau^B, \\
-\mu x(\tau)(p_H + \tau) + \lambda x(\tau)(p_H + \tau)\frac{1 - g(\tau)}{f(\tau)} & \text{if } \tau \notin \tau^B.
\end{cases}
\]

where all Kuhn–Tucker multipliers, \( \mu, \lambda, \) and \( \theta(\tau) \), are nonnegative.

As in part (ii), optimization over probabilities yields corner solutions: for each \( \tau, x(\tau), y(\tau) \in \{0, 1\} \); also \( R_b(\tau) \in \{B/\Delta p, R\} \). Furthermore, from MLRP, both \( 1 - g(\tau)/f(\tau) \) and \( C_1 - C_0 g(\tau)/f(\tau) \) increasing in \( \tau \). An optimal contract can be described as follows: (i) \( x(\tau) = 1 \) if and only if \( \tau \in [\tau^*, \hat{\tau}] \), otherwise \( x(\tau) = 0 \); (ii) there exists \( \tau^B \in [\tau^*, \hat{\tau}] \) such that \( R_b(\tau) = R \) over \( (\tau^B, \hat{\tau}] \) and \( R_b(\tau) = B/\Delta p \) otherwise; and (iii) \( y(\tau) = 1 \) if and only if \( \tau \in [\tau^{**}, \hat{\tau}] \), otherwise \( y(\tau) = 0 \).

Depending on the values of \( \mu \) and \( \lambda \), four cases can be distinguished. We omit the analysis for \( \mu = \lambda = 0 \) and \( \mu > 0 = \lambda \), because the optimal contract then takes the same form as in the corresponding cases in part (i); if \( \mu = 0 \) the borrower’s reward \( R_b(\tau) \) is chosen in \( \{B/\Delta p, R\} \) here.

To facilitate discussion, let \( \hat{\tau} \in (\tau, \hat{\tau}) \) satisfy \( f(\hat{\tau}) = g(\hat{\tau}) \). From MLRP and

\[
\int_\tau^{\hat{\tau}} [f(\tau) - g(\tau)]d\tau = 0,
\]
we know that $\hat{\tau}$ exists if $f(\tau)/g(\tau)$ is continuous. For simplicity we will assume so. Again from MLRP we have for all $\tau \geq \hat{\tau}$, $f(\tau) \geq g(\tau)$, and for all $\tau \leq \hat{\tau}$, $f(\tau) \leq g(\tau)$.

- $\mu = 0 < \lambda$, this is likely to be the case when, for example, $A$ is large so that the investors’ participation is not an issue, but $C_1$ is small relative to $C_0$ and so ex ante (IC) poses a problem.

Since, at $\tau = \hat{\tau}$, $f(\hat{\tau}) > g(\hat{\tau})$ and $(p_H + \hat{\tau})R > J$, we must have $x(\hat{\tau}) = 1$. This in turn implies that

$$\frac{\partial L}{\partial R_b(\tau)} \bigg|_{\tau = \hat{\tau}} = \left\{ \lambda(p_H + \hat{\tau}) \left[ 1 - \frac{g(\hat{\tau})}{f(\hat{\tau})} \right] + \theta(\hat{\tau}) \right\} f(\hat{\tau}) > 0,$$ and therefore $\tau^B < \hat{\tau}$. Also we must have $\tau^B \geq \hat{\tau}$. To boost ex ante incentives, the entrepreneur is provided with a high stake (the whole $R$) in the specific project when the signal is very favorable ($\tau > \tau^B \geq \hat{\tau}$).

To determine $\tau^*$, it must lie between $\tau^B$ and $\hat{\tau}$, when they are not equal. The tradeoff here is between NPV and ex ante incentives: either $\tau^B < \hat{\tau}$ and $\tau^B < \tau^* < \hat{\tau}$, from NPV concerns the specific project should continue more often ($\tau^B < \tau^*$), but reducing $\tau^*$ harms ex ante incentives; or $\tau^B > \hat{\tau}$ and $\tau^B > \tau^* > \hat{\tau}$, and then increasing NPV calls for a higher $\tau^*$, but this again has an adverse effect on ex ante incentives. Note that as $\lambda$ gets larger, i.e., as ex ante (IC) becomes more stringent, $\tau^*$ moves toward $\hat{\tau}$.

To determine $\tau^{**}$, whether reversion is used ($\tau^{**} > \tau$) is determined by the sign of the FOC at $\tau$, 

$$\frac{\partial L}{\partial y(\tau)} \bigg|_{\tau = \tau^*} = \left\{ 1 - \beta \right\} C_1 + \lambda \left[ C_1 - C_0 \frac{g(\tau)}{f(\tau)} \right] f(\tau).$$

If it is positive (which is possible because $C_1 - C_0 g(\tau)/f(\tau) < 0$), then reversion is employed in order to boost incentives. This is more likely to be the case as $\lambda$ gets larger, i.e., as ex ante (IC) gets more stringent.

- $\mu$ and $\lambda > 0$, both (IR) and ex ante (IC) are binding.

As above, for the derivative with respect to $R_b(\tau)$, if $\tau^B < \hat{\tau}$ it must be the case that

$$\frac{\partial L}{\partial R_b(\tau)} \bigg|_{\tau = \hat{\tau}} = \left\{ -\mu(p_H + \hat{\tau}) + \lambda(p_H + \hat{\tau}) \left[ 1 - \frac{g(\hat{\tau})}{f(\hat{\tau})} \right] \right\} f(\hat{\tau}) > 0.$$

Again we must have $\tau^B > \hat{\tau}$. And, as suggested by the intuition, this range shrinks or expands ($\tau^B$ increases or decreases) when (IR) or ex ante (IC) becomes more stringent, respectively.

For $\tau^*$, the optimal value lies between $\min\{\tau^B, \hat{\tau}\}$ and $\max\{\hat{\tau}, \tau^P\}$, where $\tau^P > \tau^B$ and is defined by $(p_H + \tau^P)(R - B/\Delta p) \equiv J$. When $A$ decreases, $\mu$ is larger, the concern over pledgeable income becomes more important, and the optimal threshold moves toward $\tau^P$. On the other hand, when $C_0$ increases and ex ante (IC) becomes more important, $\tau^*$ moves toward $\hat{\tau}$.

For $\tau^{**}$, the strictly negative term $-\mu \beta C_1$ in the FOC shows that a binding (IR) induces the use of reversion in order to boost pledgeable income. For the reversion at any $\tau$, $y(\tau) = 0$, increases the investor’s return by $\beta C_1$, and this contributes to the project value with a coefficient $\mu$. On the other hand, from ex ante incentive concern, $y(\tau) = 1$ only for those $\tau$ high enough so that $C_1 - C_0 g(\tau)/f(\tau) > 0$. There may not exist any $\tau$ satisfying this condition. In this case, both (IR) and ex ante (IC) require $\tau^{**}$ to increase. The two forces work together and in opposition to the NPV concern (the term $(1 - \beta)C_1$) in determining the optimal threshold. But if such $\tau$ exist (and this will be an interval $[\tau^C, \hat{\tau}]$), where $\tau^C$ satisfies $C_1 - C_0 g(\tau^C)/f(\tau^C) = 0$, then the optimal contract should reflect the incentive value of assigning $y(\tau) = 1$ over the range $[\tau^C, \hat{\tau}]$. The NPV and ex ante (IC) considerations both demand less reversion, which goes against the concern over pledgeable income (reflected by $-\mu \beta C_1$).

Note that in both cases, and when $\hat{\tau} > \tau^P$, it is possible to have optimal $\tau^* > \tau^P$. This, however, is not renegotiation-proof and therefore we need to add a binding renegotiation-proofness constraint $\tau^* = \tau^P$. This constraint imposes a restriction on the entrepreneur’s ability to rely on $x(\tau)$ to curb ex ante incentives, and therefore the optimal contract
will have reversion and a high payment ($τ^b$) to boost \textit{ex ante} incentives.

\textbf{Review Problem 9 (conflict of interest in multitasking).} (i) The derivations are in Section 3.2. Use the incentive constraint

$$(\Delta p)R_b \geq B$$

to infer that the pledgeable income is

$$P_1 \equiv p_R \left( R - \frac{B}{\Delta p} \right).$$

(ii) The inequality

$$p_R(1 - p_R)R > I$$

says that, \textit{in the absence of agency costs}, pursuing the two research strategies is profitable (with probability $1 - p_R$ the first strategy fails; the expected payoff of the second strategy is then $p_R R$).

Let $R_b$ denote the entrepreneur’s reward if the final profit is $R$ (she receives 0 if the profit is 0). The incentive constraints are

$$[1 - (1 - p_R)^2]R_b \geq [p_R + (1 - p_R)p_L]R_b + B$$

and

$$[1 - (1 - p_R)^2]R_b \geq [1 - (1 - p_R)^2]R_b + 2B.$$

The first constraint can be rewritten as

$$(1 - p_R)(\Delta p)R_b \geq B.$$

It is easy to check that this latter condition implies that the second incentive constraint is satisfied.

So the nonpledgeable income is

$$[1 - (1 - p_R)^2] \frac{B}{(1 - p_R)(\Delta p)}.$$

The two research strategies can be funded if and only if the pledgeable income $P_2$ exceeds the net investment cost $2I - A$:

$$P_2 \equiv [1 - (1 - p_R)^2] \left[ R - \frac{B}{(1 - p_R)\Delta p} \right] \geq 2I - A.$$

Suppose that $P_1 = I - A$ (or just above) and so the project can be funded with a single research strategy. Then

$$P_2 - (2I - A) = [P_2 - (2I - A)] - [P_1 - (I - A)]$$

$$= [p_R(1 - p_R)R - I] - \frac{p_R B}{1 - p_R \Delta p}.$$

The first term on the right-hand side represents the increase in the NPV while the second term stands for the increase in the agency cost. There is no way to obtain funding for the two research strategies if the increase in NPV is small.

(iii) With two agents, each pursuing a research strategy, the individual incentive constraints can be written as

$$(\Delta p)(1 - \frac{1}{2}p_R)R_b \geq B.$$

The nonpledgeable income per agent is

$$p_R(1 - \frac{1}{2}p_R)R_b = p_R \left( \frac{B}{\Delta p} \right).$$

Financing is feasible if

$$\hat{P}_2 = [1 - (1 - p_R)^2]R - 2p_R \frac{B}{\Delta p}$$

$$= P_1 + p_R(1 - p_R)R - \frac{p_R B}{\Delta p}$$

$$\geq 2I - 2A$$

(since the new entrepreneur can be asked to contribute $A$).

Note that

$$\hat{P}_2 - (2I - 2A) = (p_R(1 - p_R)R - I) - \left( \frac{p_R B}{\Delta p} - A \right).$$

The right-hand side measures the increase in the NPV of the entrepreneur who owns the research strategies. It represents the difference between

- the increase in expected profit, net of the investment cost,
  $$p_R(1 - p_R)R - I,$$
- and the rent to be left to the new entrepreneur,
  $$\frac{p_R B}{\Delta p} - A.$$

Note that, unless $A = 0$, it may be possible that

$$\hat{P}_2 > 2I - 2A$$

(and the NPV is then strictly positive) and

$$P_2 < 2I - A,$$

since

$$\hat{P}_2 - (2I - 2A) > P_2 - (2I - A).$$