Ariel Rubinstein: Lecture Notes in Microeconomic Theory

LECTURE 9

Risk Aversion

Lotteries with Monetary Prizes

We proceed to a discussion of a decision maker satisfying vNM assumptions where the space of prizes Z is a set of real numbers and $a \in Z$ is interpreted as "receiving \$$a$." Note that in Lecture 8 we assumed the set Z is finite; here, in contrast, we apply the expected utility approach to a set that is infinite. For simplicity we will still only consider lotteries with finite support. In other words, in this lecture, a lottery p is a real function on Z such that $p(z) \geq 0$ for all $z \in Z$, and there is a finite set Y such that $\sum_{z \in Y} p(z) = 1$.

We will follow our general methodology and make special assumptions that fit the interpretation of the members of Z as sums of money. Let $[x]$ be the lottery that yields the prize x with certainty. We will say that $\succsim$ satisfies *monotonicity* if $a > b$ implies $[a] \succ [b]$. Thus, if u is a vNM utility function representing a monotonic preference relation, then u is a strictly increasing function.

An axiomatization (not presented here) of vNM preferences on an infinite space Z requires strengthening of the continuity assumption so that if $p \succ q$, then small changes in the prizes, and not just in probabilities, leave the preferences unchanged. From here on we focus the discussion on preference relations over the space of lotteries for which there is a continuous function u (referred to as a vNM utility function), such that the preference relation is represented by the function $Eu(p) = \sum_{z \in Z} p(z)u(z)$. This function assigns to the lottery p the expectation of the random variable that receives the value $u(x)$ with a probability $p(x)$.

The following argument, called the *St. Petersburg Paradox*, is sometimes presented as a justification for assuming that vNM utility functions are bounded. Assume that a decision maker has an unbounded vNM utility function u. Consider playing the following "trick" on him:

1. Assume he possesses wealth x_0.
2. Offer him a lottery that will reduce his wealth to 0 with probability 1/2 and will increase his wealth to x_1 with probability 1/2 so that $u(x_0) < [u(0) + u(x_1)]/2$. By the unboundedness of u, there exists such an x_1.
3. If he loses, you are happy. If he is lucky, a moment before you give him x_1, offer him a lottery that will give him x_2 with probability 1/2 and 0 otherwise, where x_2 is such that $u(x_1) < [u(0) + u(x_2)]/2$.
4. And so on...

Our decision maker will find himself with wealth 0 with probability 1!

First-Order Stochastic Domination

We say that *p first-order stochastically dominates q* (written as pD_1q) if $p \succsim q$ for any $\succsim$ on $L(Z)$ satisfying vNM assumptions as well as monotonicity in money. That is, pD_1q if $Eu(p) \geq Eu(q)$ for all increasing u. Obviously, pD_1q if the entire support of p is to the right of the entire support of q. But, we are interested in a more interesting condition on a pair of lotteries p and q, one that will be not only sufficient, but also necessary for p to first-order stochastically dominate q.

For any lottery p and a number x, define $G(p,x) = \sum_{z \geq x} p(z)$ (the probability that the lottery p yields a prize at least as high as x). Denote by $F(p,x)$ the cumulative distribution function of p, that is, $F(p,x) = Probability\{z|z < x\}$.

Claim:

pD_1q iff for all x, $G(p,x) \geq G(q,x)$ (alternatively, pD_1q iff for all x, $F(p,x) \leq F(q,x)$). (See fig. 9.1.)

Proof:

Let $x_0 < x_1 < x_2 < \ldots < x_K$ be the prizes in the union of the supports of p and q. First, note the following alternative expression for $Eu(p)$:

$$Eu(p) = \sum_{k \geq 0} p(x_k)u(x_k) = u(x_0) + \sum_{k \geq 1} G(p, x_k)(u(x_k) - u(x_{k-1})).$$

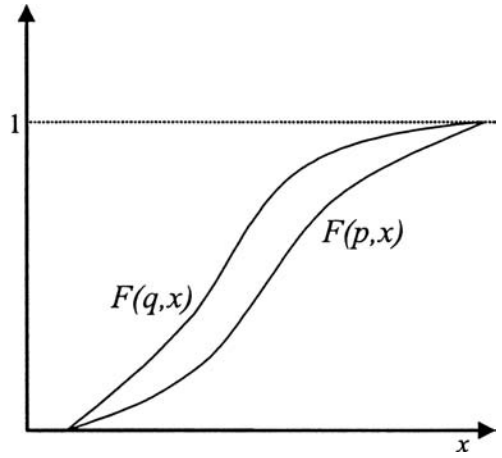


Figure 9.1
p first-order stochastically dominates q.

Now, if $G(p, x_k) \geq G(q, x_k)$ for all k, then for all increasing u,

$$Eu(p) = u(x_0) + \sum_{k \geq 1} G(p, x_k)(u(x_k) - u(x_{k-1})) \geq$$
$$u(x_0) + \sum_{k \geq 1} G(q, x_k)(u(x_k) - u(x_{k-1})) = Eu(q).$$

Conversely, if there exists k^* for which $G(p, x_{k^*}) < G(q, x_{k^*})$, then we can find an increasing function u so that $Eu(p) < Eu(q)$, by setting $u(x_{k^*}) - u(x_{k^*-1})$ to be very large and the other increments to be very small.

We have just discussed the simplest example of questions of the type: "Given a set of preference relations on $L(Z)$, for what pairs $p, q \in L(Z)$ is $p \succsim q$ for all $\succsim$ in the set?" In the problem set you will discuss another example of this kind of question.

Risk Aversion

We say that $\succsim$ is *risk averse* if for any lottery p, $[Ep] \succsim p$.

We will see now that for a decision maker with preferences $\succsim$ obeying the vNM axioms, risk aversion is closely related to the concavity of the vNM utility function representing $\succsim$.

First recall some basic properties of concave functions (if you are not familiar with those properties, this will be an excellent opportunity for you to prove them yourself):

1. An increasing and concave function must be continuous (but not necessarily differentiable).
2. The *Jensen Inequality*: If u is concave, then for any finite sequence $(\alpha_k)_{k=1,\dots,K}$ of positive numbers that sum up to 1, $u(\sum_{k=1}^{K} \alpha_k x_k) \geq \sum_{k=1}^{K} \alpha_k u(x_k)$.
3. The *Three Strings Lemma*: For any $a < b < c$ we have

$$[u(c) - u(b)]/(c - b) \leq [u(c) - u(a)]/(c - a) \leq [u(b) - u(a)]/(b - a).$$

4. If u is differentiable, then for any $a < c$, $u'(a) \geq u'(c)$, and thus $u''(x) \leq 0$ for all x.

Claim:

Let $\succsim$ be a preference on $L(Z)$ represented by the vNM utility function u. The preference relation $\succsim$ is risk averse iff u is concave.

Proof:

Assume that u is concave. By the Jensen Inequality, for any lottery p, $u(E(p)) \geq Eu(p)$ and thus $[E(p)] \succsim p$.

Assume that $\succsim$ is risk averse and that u represents $\succsim$. For all $\alpha \in (0, 1)$ and for all $x, y \in Z$, we have by risk aversion $[\alpha x + (1 - \alpha)y] \succsim \alpha x \oplus (1 - \alpha)y$ and thus $u(\alpha x + (1 - \alpha)y) \geq \alpha u(x) + (1 - \alpha)u(y)$, that is, u is concave.

Certainty Equivalence and the Risk Premium

Let $E(p)$ be the expectation of the lottery p, that is, $E(p) = \sum_{z \in Z} p(z)z$. Given a preference relation $\succsim$ over the space $L(Z)$, the *certainty equivalence* of a lottery p, $CE(p)$, is a prize satisfying $[CE(p)] \sim p$. (To justify the existence of $CE(p)$ we need to assume that $\succsim$ is monotonic and continuous in the sense that if $p \succ q$, the inequality is maintained if we change both lotteries' probabilities *and* prizes a "little bit"). The

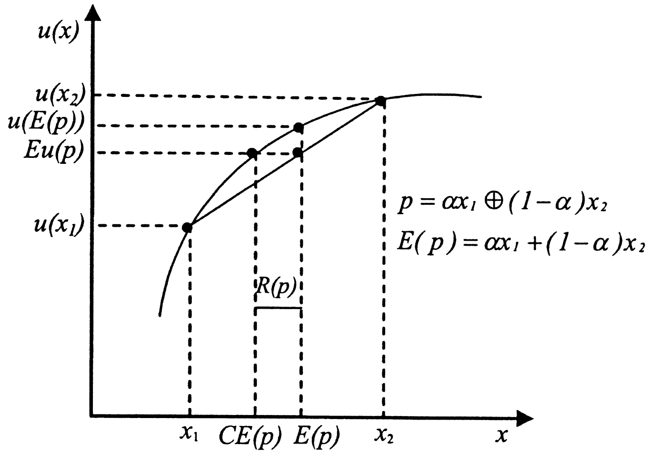


Figure 9.2
CE and risk premium.

risk premium of p is the difference $R(P) = E(p) - CE(p)$. By definition, the preferences are risk averse if and only if $R(p) \geq 0$ for all p. (See fig. 9.2.)

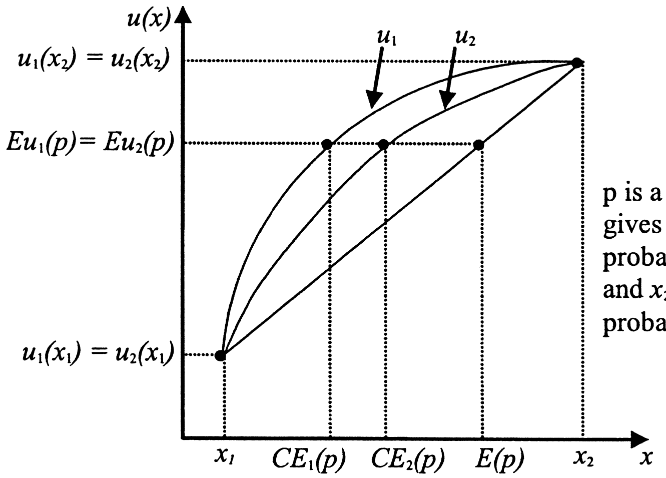
The "More Risk Averse" Relation

We wish to formalize the statement "one decision maker *is more risk averse* than another." To understand the logic of the following definitions let us start with an analogous phrase: What is the meaning of the statement "A is more war averse than B"? One possible meaning is that whenever A is ready to go to war, B is as well. Another possible meaning is that when facing the threat of war, A is ready to agree to a less attractive compromise in order to prevent war than B. The following two definitions are analogous to these two interpretations.

1. The preference relation $\succsim_1$ *is more risk averse than* $\succsim_2$ if for any lottery p and degenerate lottery c, $p \succsim_1 c$ implies that $p \succsim_2 c$.

In case the preferences are monotonic, we have a second definition:

2. The preference relation $\succsim_1$ *is more risk averse than* $\succsim_2$ if $CE_1(p) \leq CE_2(p)$ for all p.



p is a lottery that gives x_1 with probability p_1 and x_2 with probability p_2

Figure 9.3
1 is more risk averse than 2.

In case the preferences satisfy vNM assumptions, we have a third definition:

3. Let u_1 and u_2 be vNM utility functions representing $\succsim_1$ and $\succsim_2$, respectively. The preference relation $\succsim_1$ is more risk averse than $\succsim_2$ if the function φ, defined by $u_1(t) = \varphi(u_2(t))$, is concave.

I find definition (1) particularly attractive since it is meaningful in any space of prizes (not only those in which consequences are numerical) and for a general set of preferences (and not only those satisfying vNM assumptions). (See fig. 9.3.)

Claim:

If both $\succsim_1$ and $\succsim_2$ are preference relations on $L(Z)$ represented by increasing and continuous vNM utility functions, then the three definitions are equivalent.

Proof:

- If (2), then (1).
  Assume (2). Then, if $p \succsim_1 [c]$, it has to be that $[CE_1(p)] \succsim_1 [c]$ and thus $CE_1(p) \geq c$, which implies also that $CE_2(p) \geq c$, that is, $p \succsim_2 [c]$.

- If (3) then (2).
  By definition, $Eu_i(p) = u_i(CE_i(p))$. Thus, $CE_i(p) = u_i^{-1}(Eu_i(p))$. If $\varphi = u_1 u_2^{-1}$ is concave, then by the Jensen Inequality:

$$u_1(CE_2(p)) = u_1(u_2^{-1}(Eu_2(p)) = \varphi(\sum_k p(x_k)u_2(x_k)) \geq$$

$$(\sum_k p(x_k)\varphi u_2(x_k)) = \sum_k p(x_k)u_1(x_k) = E(u_1(p)) = u_1(CE_1(p)).$$

  Thus, $CE_2(p) \geq CE_1(p)$.
- If (1), then (3).
  Consider three numbers $u_2(x) < u_2(y) < u_2(z)$ in the range of u_2 and let $\lambda \in (0,1)$ satisfy $u_2(y) = \lambda u_2(x) + (1-\lambda)u_2(z)$. Let us see that $u_1(y) \geq \lambda u_1(x) + (1-\lambda)u_1(z)$.
  If $u_1(y) < \lambda u_1(x) + (1-\lambda)u_1(z)$, then for some $w > y$ close enough to y, we have both $w \prec_1 \lambda x \oplus (1-\lambda)z$ and $w \succ_2 \lambda x \oplus (1-\lambda)z$, which contradicts (1). Thus, $y \succsim_1 \lambda x \oplus (1-\lambda)z$ and $u_1(y) \geq \lambda u_1(x) + (1-\lambda)u_1(z)$, from which it follows that $\varphi(u_2(y)) \geq \lambda\varphi(u_2(x)) + (1-\lambda)\varphi(u_2(z))$. Thus, φ is concave.

The Coefficient of Absolute Risk Aversion

The following is another definition of the relation "more risk averse" applied to the case in which vNM utility functions are differentiable:

4. Let u_1 and u_2 be differentiable vNM utility functions representing $\succsim_1$ and $\succsim_2$, respectively. The preference relation $\succsim_1$ *is more risk averse than* $\succsim_2$ if $r_2(x) \leq r_1(x)$ for all x, where $r_i(x) = -u_i''(x)/u_i'(x)$.

The number $r(x) = -u''(x)/u'(x)$ is called the *coefficient of absolute risk aversion* of u at x. We will see that a higher coefficient of absolute risk aversion means a more risk-averse decision maker.

To see that (3) and (4) are equivalent, note the following chain of equivalences:

- Definition (3) (that is, $u_1 u_2^{-1}$ is concave) is satisfied iff
- the function $d/dt[u_1(u_2^{-1}(t))]$ is nonincreasing in t iff
- $u_1'(u_2^{-1}(t))/u_2'(u_2^{-1}(t))$ is nonincreasing in t iff (since $(\varphi^{-1})'(t) = 1/\varphi'(\varphi^{-1}(t))$)
- $u_1'(x)/u_2'(x)$ is nonincreasing in x (since $u_2^{-1}(t)$ is increasing in t) iff

- $log[(u'_1/u'_2)(x)] = logu'_1(x) - logu'_2(x)$ is nonincreasing in x iff
- the derivative of $logu'_1(x) - logu'_2(x)$ is nonpositive iff
- $r_2(x) - r_1(x) \leq 0$ for all x where $r_i(x) = -u''_i(x)/u'_i(x)$ iff
- definition (4) is satisfied.

For a better understating of the coefficient of absolute risk aversion, it is useful to look at the preferences on the restricted domain of lotteries of the type $(x_1, x_2) = px_1 \oplus (1-p)x_2$, where the probability p is fixed. Denote by u a differentiable vNM utility function that represents a risk-averse preference.

Let $x_2 = \psi(x_1)$ be the function describing the indifference curve through (t, t), the point representing $[t]$. It follows from risk aversion that all lotteries with expectation t, that is, all lotteries on the line $\{(x_1, x_2) | \, px_1 + (1-p)x_2 = t\}$, are not above the indifference curve through (t, t). Thus, $\psi'(x_1) = -p/(1-p)$.

By definition of u as a vNM utility function representing the preferences over the space of lotteries, we have $pu(x_1) + (1-p)u(\psi(x_1)) = u(t)$. Taking the derivative with respect to x_1, we obtain $pu'(x_1) + (1-p)u'(\psi(x_1))\psi'(x_1) = 0$. Taking the derivative with respect to x_1 once again, we obtain

$$pu''(x_1) + (1-p)u''(\psi(x_1))[\psi'(x_1)]^2 + (1-p)u'(\psi(x_1))\psi''(x_1) = 0.$$

At $x_1 = t$ we have

$$pu''(t) + u''(t)p^2/(1-p) + (1-p)u'(t)\psi''(t) = 0.$$

Therefore,

$$\psi''(t) = -u''(t)/u'(t)[p/(1-p)^2] = r(t)[p/(1-p)^2].$$

That is, the second derivative of the indifference curve through the certain lottery t is $r(t)[p/(1-p)^2]$.

Note that on this restricted space of lotteries, $\succsim_1$ is more risk averse than $\succsim_2$ in the sense of definition (1) iff the indifference curve of $\succsim_1$ through (t, t), denoted by ψ_1, is never below the indifference curve of $\succsim_2$ through (t, t), denoted by ψ_2. Combined with $\psi'_1(t) = \psi'_2(t)$, we obtain that $\psi''_1(t) \geq \psi''_2(t)$ and thus $r_2(t) \leq r_1(t)$. (See fig. 9.4.)

The Doctrine of Consequentialism

Conduct the following “thought experiment”:

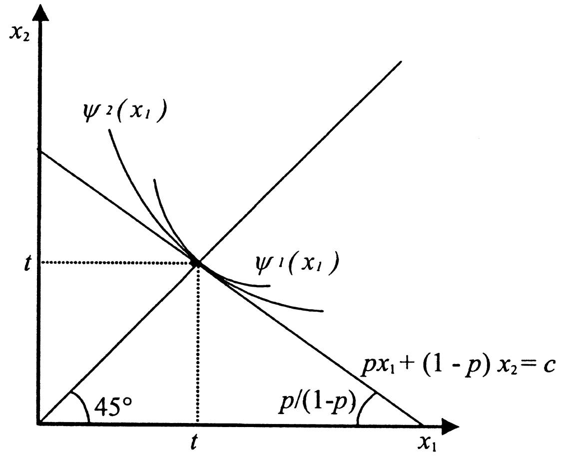


Figure 9.4
1 is more risk averse than 2.

You have $2000 in your bank account. You have to choose between

1. a sure loss of $500
   and
2. a lottery in which you lose $1000 with probability 1/2 and lose 0 with probability 1/2.

What is your choice?

Now assume that you have $1000 in your account and that you have to choose between

3. a certain gain of $500
   and
4. a lottery in which you win $1000 with probability 1/2 and win 0 with probability 1/2.

What is your choice?

In the first case, most people preferred the lottery to the certain prize (chose (2)), while in the second case most people preferred the sure prize (chose (3)). Such a preference does not conflict with expected utility theory if we interpret a prize to reflect a "monetary change." However, if we assume that the decision maker takes the final wealth levels to be his prizes, we have a problem: in terms of final wealth levels, both choices can be presented as being between a sure prize of $1500 and a lottery that yields $2000 or $1000 with probability 1/2 each.

Nevertheless, in the economic literature it is usually assumed that a decision maker's preferences over wealth changes are induced from his preferences with regard to "final wealth levels." Formally, when starting with wealth w, denote by $\succsim_w$ the decision maker's preferences over lotteries in which the prizes are interpreted as "changes" in wealth. By the *doctrine of consequentialism* all relations $\succsim_w$ are derived from the same preference relation, $\succsim$, defined over the "final wealth levels" by $p \succsim_w q$ iff $w + p \succsim w + q$ (where $w + p$ is the lottery that awards a prize $w + x$ with probability $p(x)$). If $\succsim$ is represented by a vNM utility function u, this doctrine implies that for all w, the function $v_w(x) = u(w + x)$ is a vNM utility function representing the preferences $\succsim_w$.

Invariance to Wealth

We say that the preference relation $\succsim$ exhibits *invariance to wealth* (in the literature it is often called *constant absolute risk aversion*) if the induced preference relation $\succsim_w$ is independent of w, that is, $(w + L_1) \succsim (w + L_2)$ is true or false independent of w.

We will see that if u is a continuous vNM utility function representing preferences $\succsim$, which exhibit risk aversion and invariance to wealth, then u must be exponential.

Let us first confine ourselves to the $\Delta -$ *grid* prize space, $Z = \{x \mid x = n\Delta$ for some integer $n\}$. This domain has a special meaning when we take Δ to be the smallest (indivisible) unit of money.

By continuity of u, for any wealth level x there is a number q such that $(1 - q)(x - \Delta) \oplus q(x + \Delta) \sim x$. By the invariance to wealth, q is independent of x. Thus, we have $u(x + \Delta) - u(x) = ((1 - q)/q)[u(x) - u(x - \Delta)]$ for all x. This means that the increments in the function u, when x is increased by Δ, constitute a geometric sequence with a factor of $(1 - q)/q$ (where q might depend on Δ). We conclude that the function u, defined on the $\Delta -$ *grid*, must be an affine transformation of $((1 - q)/q)^{x/\Delta}$.

Let us now return to the case of $Z = \Re$ and look at the preferences over the restricted space of all lotteries of the type $(x_1, x_2) = px_1 \oplus (1 - p)x_2$ for some arbitrary fixed probability number $p \in (0, 1)$. A necessary condition for $\succsim$ to exhibit constant absolute risk aversion is that the indifference curve through (t, t) is the same as that through $(0, 0)$, shifted in the direction of (t, t). In other words, denoting the indifference through (t, t) by $x_2 = \psi_t(x_1)$, we have $\psi_t(x_1) = \psi_0(x_1 - t) + t$.

Assuming that the function u is differentiable, we derive that $\psi_t''(t) = \psi_0''(0)$. We have already seen that $\psi_t''(t) = -[p/(1-p)^2]$ $[u_i''(t)/u_i'(t)]$ and thus there exists a constant α such that $-u''(t)/u'(t) = \alpha$ for all t. This implies that $[log u'(t)]' = -\alpha$ for all t and $log u'(t) = -\alpha t + \beta$ for some β. It follows that $u'(t) = e^{-\alpha t + \beta}$. If $\alpha = 0$, the function $u(t)$ must be linear (implying risk neutrality). If $\alpha \neq 0$, it must be that $u(t) = ce^{-\alpha t} + d$ for some c and d.

To conclude, if u is a vNM continuous utility function representing preferences that are monotonic and exhibit both risk aversion and invariance to wealth, then u is an affine transformation of either the function t or a function $-e^{-\alpha t}$ (with $\alpha > 0$).

Critique of the Doctrine of Consequentialism

Denote by $1/2(-D) \oplus 1/2(+G)$ the lottery in which there is an equal probability of gaining \$$G$ and losing \$$D$. Consider a risk-averse decision maker who likes money, obeys expected utility theory, and adheres to the doctrine of consequentialism. Matthew Rabin noted that if such a decision maker turns down the lottery $L = 1/2(-10) \oplus 1/2(+11)$, at any wealth level between \$0 and \$5000 (a quite plausible assumption), then at the wealth level \$4000 he must reject the lottery $1/2(-100) \oplus 1/2(+71000)$ (a quite ridiculous conclusion).

The intuition for this observation is quite simple. Let Δ be the marginal utility of one dollar at the wealth level w. If L is rejected at w, then it must be that the marginal utility level at $w + 21$ is not more than $(21/22)\Delta$. To see this, note that the marginal utility at $w + 21$ is (by the concavity of u) not greater than $[u(w + 21) - u(w + 10)]/11$. Since L is rejected in w, $u(w + 10) \geq [u(w) + u(w + 21)]/2$ and thus the marginal utility at $w + 21$ is not greater than

$$\begin{aligned}&\{u(w + 21) - [u(w + 21) + u(w + 0)]/2\}/11\\ &= [u(w + 21) - u(w + 0)]/22 \leq (21/22)\Delta.\end{aligned}$$

Thus, the sequence of marginal utilities within the domain of wealth levels in which L is rejected falls at a geometric rate. This implies that for the lottery $1/2(-D) \oplus 1/2(+G)$ to be accepted even for a relatively low D, one would need a huge G.

What conclusions should we draw from this observation? In my opinion, in contrast to what some scholars claim, this is not a refutation of expected utility theory. Rabin's argument relies on the doctrine of consequentialism, which is not a part of expected utility theory. Expected utility theory is invariant to the interpretation of the prizes. Independently of the theory of decision making under uncertainty that we use, the set of prizes should be the set of consequences in the mind of the decision maker. Thus, it is equally reasonable to assume the consequences are "wealth changes" or "final wealth levels."

I treat Rabin's argument as further evidence of the empirically problematic nature of the doctrine of consequentialism according to which the decision maker makes *all* decisions having in mind a preference relation over *the same* set of final consequences. It also demonstrates how carefully we should tread when trying to estimate real life agents' utility functions. The practice of estimating an economic agent's risk aversion parameters for small lotteries might lead to misleading conclusions if such estimates are used to characterize the decision maker's preferences regarding lotteries over large sums.

Bibliographic Notes

Recommended readings: Kreps 1990, 81–98; Mas-Colell et al. 1995, Chapter 6, C–D.

The measures of risk aversion are taken from Arrow (1970) and Pratt (1964). For the psychological literature discussed here, see Kahneman and Tversky (1979) and Kahneman and Tversky (2000).

The St. Petersburg Paradox was suggested by Daniel Bernoulli in 1738 (see Bernoulli 1954). The notion of stochastic domination was introduced into the economic literature by Rothschild and Stiglitz (1970). Rabin's argument is based on Rabin (2000).

Problem Set 9

Problem 1. (*Standard.* Based on Rothschild and Stiglitz 1970.)
We say that *p second-order stochastically dominates q* and denote it by pD_2q if $p \succsim q$ for all preferences $\succsim$ satisfying the vNM assumptions, monotonicity *and* risk aversion.

a. Explain why pD_1q implies pD_2q.
b. Let p and ε be lotteries. Define $p + \varepsilon$ to be the lottery that yields the prize t with the probability $\Sigma_{\alpha+\beta=t} p(\alpha)\varepsilon(\beta)$. Interpret $p + \varepsilon$. Show that if ε is a lottery with expectation 0, then for all p, $pD_2(p + \varepsilon)$.
c. (More difficult) Show that pD_2q if and only if for all $t < K$, $\Sigma_{k=0}^{t} [G(p, x_{k+1}) - G(q, x_{k+1})][x_{k+1} - x_k] \geq 0$ where $x_0 < \ldots < x_K$ are all the prizes in the support of either p or q and $G(p, x) = \Sigma_{z \geq x} p(z)$.

Problem 2. (*Standard.* Based on Slovic and Lichtenstein 1968.)
Consider a phenomenon called *preference reversal*. Let $L_1 = 8/9[\$4] \oplus 1/9[\$0]$ and $L_2 = 1/9[\$40] \oplus 8/9[\$0]$.

a. What is the maximal amount you are willing to pay for L_1? For L_2?
b. What lottery do you prefer?
c. Discuss the “typical” answer that ranks L_1 as superior to L_2 but attaches a lower value to L_1 (see Slovic, Tversky and Kahneman 1990).

Problem 3. (*Standard*)
Consider a consumer’s preference over K-tuples of K uncertain assets. Denote the random return on the kth asset by Z_k. Assume that the random variables $(Z_1, \ldots, Z_K)$ are independent and take positive values with probability 1. If the consumer buys the combination of assets $(x_1, \ldots, x_K)$ and if the vector of realized returns is $(z_1, \ldots, z_K)$, then the consumer’s total wealth is $\sum_{k=1}^{K} z_k x_k$. Assume that the consumer satisfies vNM assumptions, that is, there is a function v (over the sum of his returns) so that he maximizes the expected value of v. Assume that v is increasing and concave. The consumer preferences over the space of the lotteries induce preferences on the space of investments. Show that the induced preferences are monotonic and convex.

Problem 4. (*Standard.* Based on Rubinstein 2002.)
Adam lives in the Garden of Eden and eats only apples. Time in the garden is discrete ($t = 1, 2, \ldots$) and apples are eaten only in discrete units. Adam possesses preferences over the set of streams of apple consumption. Assume that Adam:

a. Likes to eat up to 2 apples a day and cannot bear to eat 3 apples a day.
b. Is impatient. He will be delighted to increase his consumption at day t from 0 to 1 or from 1 to 2 apples at the expense of an apple he is promised a day later.
c. At any period in which he does not have an apple, he prefers to get one apple immediately in exchange for two apples tomorrow.
d. Cares only about his consumption in the first 120 years of his life.

Show that if (poor) Adam is offered a stream of 2 apples starting in period 19 for the rest of his life (assuming he does not expect to live more than 120 years), he would be willing to exchange that offer for one apple given right away.