LECTURE 5

Demand: Consumer Choice

The Rational Consumer's Choice from a Budget Set

In Lecture 4 we discussed the consumer's preferences. In this lecture we adopt the "rational man" paradigm in discussing consumer choice.

Given a consumer's preference relation $\succsim$ on $X = \Re_+^K$, we can talk about his choice from any set of bundles. However, since we are laying the foundation for "price models," we are interested in the consumer's choice in a particular class of choice problems called budget sets. A *budget set* is a set of bundles that can be represented as $B(p, w) = \{x \in X \mid px \leq w\}$, where p is a vector of positive numbers (interpreted as prices) and w is a positive number (interpreted as the consumer's wealth).

Obviously, any set $B(p, w)$ is compact (it is closed since it is defined by weak inequalities, and bounded since for any $x \in B(p, w)$ and for all k, $0 \leq x_k \leq w/p_k$). It is also convex since if $x, y \in B(p, w)$, then $px \leq w$, $py \leq w$, $x_k \geq 0$, and $y_k \geq 0$ for all k. Thus, for all $\alpha \in [0, 1]$, $p[\alpha x + (1 - \alpha)y] = \alpha px + (1 - \alpha)py \leq w$ and $\alpha x_k + (1 - \alpha)y_k \geq 0$ for all k, that is, $\alpha x + (1 - \alpha)y \in B(p, w)$.

We will refer to the problem of finding the $\succsim$-best bundle in $B(p, w)$ as the *consumer's problem*.

Claim:

If $\succsim$ is a continuous relation, then all consumer problems have a solution.

Proof:

If $\succsim$ is continuous, then it can be represented by a continuous utility function u. By the definition of the term "utility representation," finding an $\succsim$ optimal bundle is equivalent to solving the problem $max_{x \in B(p,w)} u(x)$. Since the budget set is compact and u is continuous, the problem has a solution.

To emphasize that a utility representation is not necessary for the current analysis, let us study a direct proof of the previous claim, avoiding the notion of utility.

Direct Proof:

For any $x \in B(p,w)$ define the set $Inferior(x) = \{y \in B(p,w) | x \succ y\}$. By the continuity of the preferences, every such set is open. Assume there is no solution to the consumer's problem of maximizing $\succsim$ on $B(p,w)$. Then, every $z \in B(p,w)$ is a member of some set $Inferior(x)$, that is, the collection of sets $\{Inferior(x) | \ x \in X\}$ covers $B(p,w)$. A collection of open sets that covers a compact set has a finite subset of sets that covers it. Thus, there is a finite collection $Inferior(x^1), \ldots,$ $Inferior(x^n)$ that covers $B(p,w)$. Letting x^j be the optimal bundle within the finite set $\{x^1, \ldots, x^n\}$, we obtain that x^j is an optimal bundle in $B(p,w)$, a contradiction.

Claim:

If $\succsim$ is convex, then the set of solutions for a choice from $B(p,w)$ (or any other convex set) is convex.

Proof:

If both x and y maximize $\succsim$ given $B(p,w)$, then $\alpha x + (1-\alpha)y \in B(p,w)$ and, by the convexity of the preferences, $\alpha x + (1-\alpha)y \succsim x \succsim z$ for all $z \in B(p,w)$. Thus, $\alpha x + (1-\alpha)y$ is also a solution to the consumer's problem.

Claim:

If $\succsim$ is strictly convex, then every consumer's problem has at most one solution.

Proof:

Assume that both x and y (where $x \neq y$) are solutions to the consumer's problem $B(p, w)$. Then $x \sim y$ (both are solutions to the same maximization problem) and $\alpha x + (1 - \alpha)y \in B(p, w)$ (the budget set is convex). By the strict convexity of $\succsim$, $\alpha x + (1 - \alpha)y \succ x$, which is a contradiction of x being a maximal bundle in $B(p, w)$.

The Consumer's Problem with Differentiable Preferences

When the preferences are differentiable, we are provided with a "useful" condition for characterizing the optimal solution.

Claim:

If x^* is an optimal bundle in the consumer problem and k is a consumed commodity (i.e., $x_k^* > 0$), then it must be that $v_k(x^*)/p_k \geq v_j(x^*)/p_j$ for all other j, where $v_k(x^*)$ are the "subjective value numbers" (see the definition of differentiable preferences in Lecture 4).

For the case in which the preferences are represented by a utility function u, we have $v_k(x^*) = \partial u/\partial x_k(x^*)$. In other words, the "value per dollar" at the point x^* of the k-th commodity (which is consumed) must be as large as the "value per dollar" of any other commodity.

Proof:

Assume that x^* is a solution to the consumer's problem $B(p, w)$ and that $x_k^* > 0$ and $v_k(x^*)/p_k < v_j(x^*)/p_j$ (see fig. 5.1). A "move" in the direction of reducing the consumption of the k-th commodity by 1 and increasing the consumption of the j-th commodity by p_k/p_j is an improvement since $v_j(x^*)p_k/p_j - v_k(x^*) > 0$. As $x_k^* > 0$, we can find $\varepsilon > 0$ small enough such that decreasing k's quantity by ε and increasing j's quantity by $\varepsilon p_k/p_j$ is feasible. This brings the consumer to a strictly better bundle, contradicting the assumption that x^* is a solution to the consumer's problem.

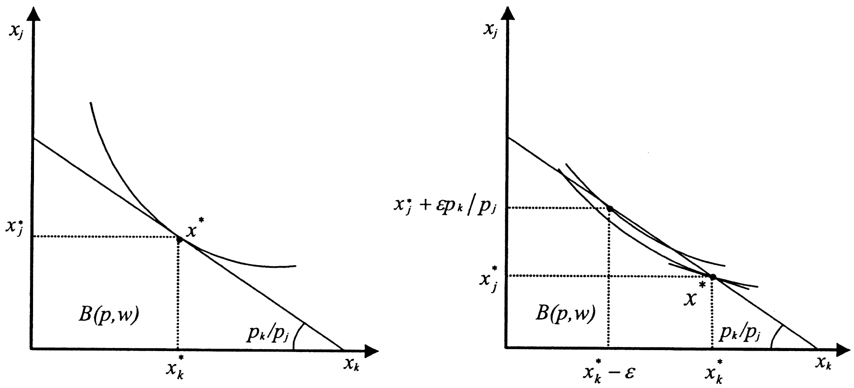


Figure 5.1
(a) x^* is a solution to the consumer problem $B(p, w)$.
(b) x^* is not a solution to the consumer problem $B(p, w)$.

From the above we can derive the "classic" necessary conditions on the consumer's maximization:

Claim:

If x^* is a solution to the consumer's problem $B(p, w)$ and both $x_k^* > 0$ and $x_j^* > 0$, then the ratio $v_k(x^*)/v_j(x^*)$ must be equal to the price ratio p_k/p_j.

In order to establish sufficient conditions for maximization, we require also that the preferences be convex.

Claim:

If $\succsim$ is monotonic, convex, continuous, and differentiable, and if at x^*

- $px^* = w$,
- for all k such that $x_k^* > 0$, and for any commodity l, $v_k(x^*)/p_k \geq v_l(x^*)/p_l$,

then x^* is a solution to the consumer's problem.

Proof:

If x^* is not a solution, then there is a bundle z such that $pz \leq px^*$ and $z \succ x^*$. By continuity and monotonicity, there is a bundle $y \neq z$, with $y_k \leq z_k$ such that $y \succ x^*$ and $py < pz \leq px^*$. By convexity, any small move in the direction $(y - x^*)$ is an improvement and by differentiability, $v(x^*)(y - x^*) > 0$.

Let $\mu = v_k(x^*)/p_k$ for all k with $x_k^* > 0$. Now,

$$0 > p(y - x^*) = \sum p_k(y_k - x_k^*) \geq \sum v_k(x^*)(y_k - x_k^*)/\mu$$

(since for a good with $x_k^* > 0$ we have $p_k = v_k(x^*)/\mu$, and for a good k with $x_k^* = 0$, $(y_k - x_k^*) \geq 0$ and $p_k \geq v_k(x^*)/\mu$.) Thus, $0 \geq v(x^*)(y - x^*)$, a contradiction.

The Demand Function

We have arrived at an important stage on the way to developing a market model in which we derive demand from preferences. Assume that the consumer's preferences are such that for any $B(p, w)$, the consumer's problem has a unique solution. Let us denote this solution by $x(p, w)$. The function $x(p, w)$ is called the *demand function*. The domain of the demand function is $\Re_{++}^{K+1}$ whereas its range is $\Re_{+}^{K}$.

Example:

Consider a consumer in a world with two commodities having the following lexicographic preference relation, attaching the first priority to the sum of the quantities of the goods and the second priority to the quantity of commodity 1:
$x \succsim y$ if $x_1 + x_2 > y_1 + y_2$ or both $x_1 + x_2 = y_1 + y_2$ and $x_1 \geq y_1$.

This preference relation is strictly convex but not continuous. It induces the following noncontinuous demand function:

$$x((p_1, p_2), w) = \begin{cases} (0, w/p_2) & \textit{if} \quad p_2 < p_1 \\ (w/p_1, 0) & \textit{if} \quad p_2 \geq p_1 \end{cases}.$$

We now turn to studying some properties of the demand function.

Claim:

$x(p, w) = x(\lambda p, \lambda w)$ (i.e., the demand function is *homogeneous of degree zero*).

Proof:

This follows (with no assumptions about the preference relations) from the basic equality $B(\lambda p, \lambda w) = B(p, w)$ and the assumption that the behavior of the consumer is "a choice from a set."

Note that this claim is sometimes interpreted as implying that "uniform inflation does not matter." This is an incorrect interpretation. We assumed, rather than concluded, that choice is made from a set independently of the way that the choice set is framed. Inflation can affect choice since behavior may be sensitive to the nominal prices and wealth even if the budget set is unchanged.

Claim (Walras's law):

If the preferences are monotonic, then any solution x to the consumer's problem $B(p, w)$ is located on its budget curve (and thus, $px(p, w) = w$).

Proof:

If not, then $px < w$. There is an $\varepsilon > 0$ such that $p(x_1 + \varepsilon, \dots, x_K + \varepsilon) < w$. By monotonicity, $(x_1 + \varepsilon, \dots, x_K + \varepsilon) \succ x$, thus contradicting the assumption that x is optimal in $B(p, w)$.

Claim:

If $\succsim$ is a continuous preference, then the demand function is continuous in prices (and also in w, see problem set).

Proof:

Once again, we could use the fact that the preferences have a continuous utility representation and apply a standard "maximum theorem." (If the function $f(x, a)$ is continuous, then the function $h(a) = argmax_x f(x, a)$ is continuous.) However, I prefer to present a proof that does not use the notion of a utility function:

Assume not. Then, there is a sequence of price vectors p^n converging to p^* such that $x(p^*, w) = x^*$, and $x(p^n, w)$ does not converge to x^*. Thus, we can assume that (p^n) is a sequence converging to p^* such that for all n distance $(x(p^n, w), x^*) > \varepsilon$ for some positive ε.

All numbers p^n_k are greater than some positive number m. Therefore, all vectors $x(p^n, w)$ belong to some compact set (the hypercube of bundles with no quantity above w/m) and thus, without loss of generality, we can assume that $x(p^n, w) \to y^*$ for some $y^* \neq x^*$.

Since $p^n x(p^n, w) \leq w$ for all n, it must be that $p^* y^* \leq w$, that is, $y^* \in B(p^*, w)$. Since x^* is the unique solution for $B(p^*, w)$, we have $x^* \succ y^*$. By the continuity of the preferences, there are neighborhoods of x^* and y^* in which the strict preference is preserved. For sufficiently large n, $x(p^n, w)$ is in that neighborhood of y^*. Choose a bundle z^* in the neighborhood of x^* so that $p^* z^* < w$. For all sufficiently large n, $p^n z^* < w$; however, $z^* \succ x(p^n, w)$, which is a contradiction.

Rationalizable Demand Functions

As in the general discussion of choice, we will now examine whether choice procedures are consistent with the rational man model. We can think of various possible definitions of rationalization.

One approach is to look for a preference relation (without imposing any restrictions that fit the context of the consumer) such that the chosen element from any budget set is the unique bundle maximizing the preference relation in that budget set. Thus, we say that the preferences $\succsim$ *fully rationalize* the demand function x if for any (p, w) the bundle $x(p, w)$ is the unique $\succsim$ maximal bundle within $B(p, w)$.

Alternatively, we could say that "being rationalizable" means that there are preferences such that the consumer's behavior is consistent with maximizing those preferences, that is, for any (p, w) the bundle $x(p, w)$ is a $\succsim$ maximal bundle (not necessarily unique) within

$B(p, w)$. This definition is "empty" since any demand function is consistent with maximizing the "total indifference" preference. This is why we usually say that the preferences $\succsim$ *rationalize* the demand function x if they are *monotonic* and for any (p, w), the bundle $x(p, w)$ is a $\succsim$ maximal bundle within $B(p, w)$.

Of course, if behavior satisfies homogeneity of degree zero and Walras's law, it is still not necessarily rationalizable in any of those senses:

Example 1:

Consider the demand function of a consumer who spends all his wealth on the "more expensive" good:

$$x((p_1, p_2), w) = \begin{cases} (0, w/p_2) & \textit{if} \quad p_2 \geq p_1 \\ (w/p_1, 0) & \textit{if} \quad p_2 < p_1 \end{cases} .$$

This demand function is not entirely inconceivable, and yet it is not rationalizable. To see this, assume that it is fully rationalizable or rationalizable by $\succsim$. Consider the two budget sets $B((1, 2), 1)$ and $B((2, 1), 1)$. Since $x((1, 2), 1) = (0, 1/2)$ and $(1/2, 0)$ is an internal bundle in $B((1, 2), 1)$, by any of the two definitions of rationalizability, it must be that $(0, 1/2) \succ (1/2, 0)$. Similarly, $x((2, 1), 1) = (1/2, 0)$ and $(0, 1/2)$ is an internal bundle in $B((2, 1), 1)$. Thus, $(0, 1/2) \prec (1/2, 0)$, a contradiction.

Example 2:

A consumer chooses a bundle $(z, z, \ldots, z)$, where z satisfies $z\Sigma p_k = w$.

This behavior is fully rationalized by any preferences according to which the consumer strictly prefers any bundle on the main diagonal over any bundle that is not (because, for example, he cares primarily about purchasing equal quantities from all sellers of the K goods), while on the main diagonal his preferences are according to "the more the better". These preferences rationalize his behavior in the first sense but are not monotonic.

This demand function is also fully rationalized by the monotonic preferences represented by the utility function $u(x_1, \ldots, x_K) = min\{x_1, \ldots, x_K\}$.

Example 3:

Consider a consumer who spends α_k of his wealth on commodity k (where $\alpha_k \geq 0$ and $\Sigma_{k=1}^{K}\alpha_k = 1$). This rule of behavior is not formulated as a maximization of some preference relation. It can however be fully rationalized by the preference relation represented by the Cobb-Douglas utility function $u(x) = \Pi_{k=1}^{K} x_k^{\alpha_k}$. A solution x^* to the consumer's problem $B(p, w)$ must satisfy $x_k^* > 0$ for all k (notice that $u(x) = 0$ when $x_k = 0$ for some k). Given the differentiability of the preferences, a necessary condition for the optimality of x^* is that $v_k(x^*)/p_k = v_l(x^*)/p_l$ for all k and l where $v_k(x^*) = du/dx_k(x^*) = \alpha_k u(x^*)/x_k^*$ for all k. It follows that $p_k x_k^*/p_l x_l^* = \alpha_k/\alpha_l$ for all k and l and thus $x_k^* = \alpha_k w/p_k$ for all k.

Example 4:

Let $K = 2$. Consider the behavior of a consumer who allocates his wealth between commodities 1 and 2 in the proportion p_2/p_1 (the cheaper the good, the higher the share of the wealth devoted to it). Thus, $x_1 p_1/x_2 p_2 = p_2/p_1$ and $x_i(p, w) = (p_j/(p_i + p_j))w/p_i$. This demand function satisfies Walras's law as well as homogeneity of degree zero.

To see that this demand function is fully rationalizable, note that $x_i/x_j = p_j^2/p_i^2$ (for all i and j) and thus $p_1/p_2 = \sqrt{x_2}/\sqrt{x_1}$. The quasi-concave function $\sqrt{x_1} + \sqrt{x_2}$ satisfies the condition that the ratio of its partial derivatives is equal to $\sqrt{x_2}/\sqrt{x_1}$. Thus, for any (p, w), the bundle $x(p, w)$ is the solution to the maximization of $\sqrt{x_1} + \sqrt{x_2}$ in $B(p, w)$.

The Weak and Strong Axioms of Revealed Preferences

We now look for general conditions that will guarantee that a demand function $x(p, w)$ can be fully rationalized (a similar discussion would apply to the other definition of rationalizability that requires that $x(p, w)$ maximizes a monotonic preference relation). Of course, one does not necessarily need these general conditions to determine whether a demand function is rationalizable. Guessing is often an excellent strategy.

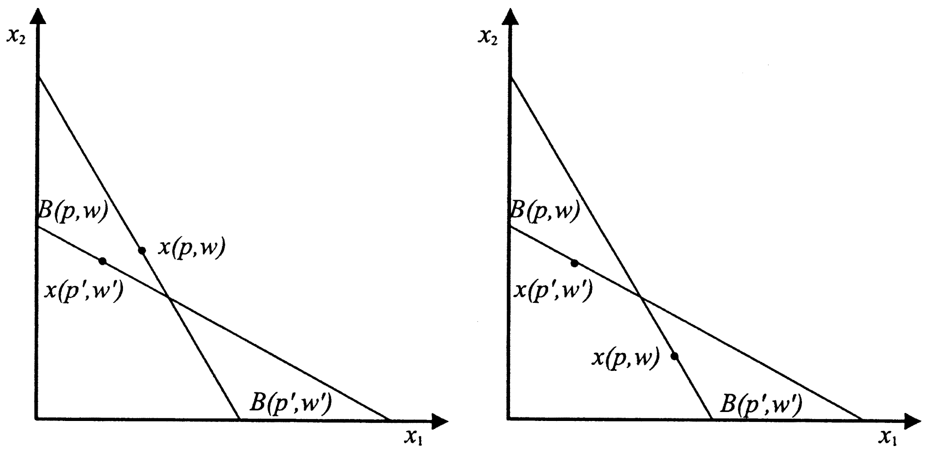


Figure 5.2
(a) Satisfies the weak axiom. (b) Does not satisy the weak axiom.

In the general discussion of choice functions, we saw that the weak axiom (WA) was a necessary and sufficient condition for a choice function to be derived from some preference relation. In the proof, we constructed a preference relation out of the choices of the decision maker from sets containing two elements. We showed (by looking into his behavior at the choice set $\{a, b, c\}$) that WA implies that it is impossible for a to be revealed as better than b, b revealed as better than c, and c revealed as better than a. However, in the context of a consumer, finite sets are not within the scope of the choice function.

In the same spirit, adjusting to the context of the consumer, we might try to define $x \succ y$ if there is (p, w) so that both x and y are in $B(p, w)$ and $x = x(p, w)$. In the context of the consumer model the Weak Axiom is read: if $px(p', w') \leq w$ and $x(p, w) \neq x(p', w')$, then $p'x(p, w) > w'$. WA guarantees that it is impossible that both $x \succ y$ and $y \succ x$. However, the defined binary relation is not necessarily complete: there can be two bundles x and y such that for any $B(p, w)$ containing both bundles, $x(p, w)$ is neither x nor y. Furthermore, in the general discussion, we guaranteed transitivity by looking at the union of a set in which a was revealed to be better than b and a set in which b was revealed to be as good as c. However, when the sets are budget sets, their union is not necessarily a budget set. (See fig. 5.2.)

Apparently WA is not a sufficient condition for extending the binary relation $\succ$, as defined above, into a complete and transitive relation (an example with three goods from Hicks 1956 is discussed in Mas-Colell et al. 1995). A necessary and sufficient condition for a demand function x satisfying Walras's law and homogeneity of degree zero to be rationalized is the following:

Strong Axiom of Revealed Preference (SA):

If $(x^n)_{n=1,\ldots,N}$ is a sequence of bundles and $(B(p^n, w^n))_{n=1,\ldots,N}$ is a sequence of budget sets so that for all $n \leq N-1$, $x^n \neq x^{n+1}$ and x^n is chosen from $B(p^n, w^n)$ which also contains x^{n+1}, then $x^1 \notin B(p^N, w^N)$.

The Strong Axiom is basically equivalent to the assumption that the relation $\succ$ derived from revealed behavior is transitive. But $\succ$ is not necessarily a complete relation, and thus we are left with the question of whether $\succ$ can be extended into preferences. Proving that this is possible is beyond the scope of this course. In any case, the SA is "cumbersome," and using it to determine whether a certain demand function is rationalizable may not be a trivial task.

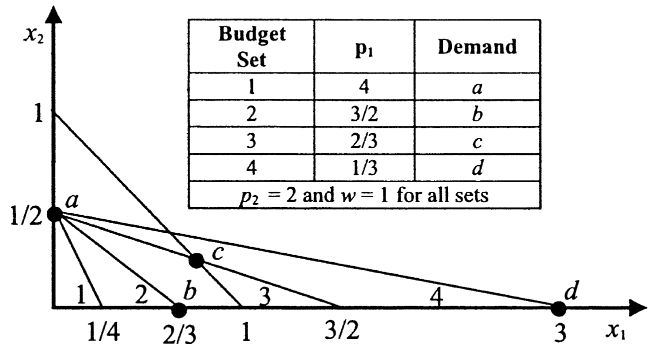
Decreasing Demand

The consumer model discussed so far constitutes the standard framework for deriving demand. Our intuition tells us that demand for a good falls when its price increases. However, this does not follow from the standard assumptions about the rational consumer's behavior which we have discussed so far. The following is an example of a preference relation that induces demand that is nondecreasing in the price of one of the commodities:

An Example in Which Demand for a Good May Increase with Price

Consider the preferences represented by the following utility function:

$$u(x_1, x_2) = \begin{cases} x_1 + x_2 & \textit{if} \quad x_1 + x_2 < 1 \\ x_1 + 4x_2 & \textit{if} \quad x_1 + x_2 \geq 1 \end{cases} .$$



Budget Set	p_1	Demand
1	4	*a*
2	3/2	*b*
3	2/3	*c*
4	1/3	*d*
$p_2 = 2$ and $w = 1$ for all sets		

Figure 5.3

These preferences might reflect reasoning of the following type: "In the bundle x there are $x_1 + x_2$ units of vitamin A and $x_1 + 4x_2$ units of vitamin B. My first priority is to get enough vitamin A. However, once I satisfy my need for 1 unit of vitamin A, I move on to my second priority, which is to consume as much as possible of vitamin B." (See fig 5.3.)

Consider $x((p_1, 2), 1)$. Changing p_1 is like rotating the budget lines around the pivot bundle $(0, 1/2)$. At a high price p_1 (as long as $p_1 > 2$), the consumer demands $(0, 1/2)$. If the price is reduced to within the range $2 > p_1 > 1$, the consumer chooses the bundle $(1/p_1, 0)$. So far, the demand for the first commodity indeed increased when its price fell. However, in the range $1 > p_1 > 1/2$ we encounter an anomaly: the consumer buys as much as possible from the second good subject to the "constraint" that the sum of the goods is at least 1, i.e., $x((p_1, 2), 1) = (1/(2 - p_1), (1 - p_1)/(2 - p_1))$.

The above preference relation is monotonic but not continuous. However, we can construct a close continuous preference that leads to demand that is increasing in p_1 in a similar domain. Let $\alpha_\delta(t)$ be a continuous and increasing function on $[1 - \delta, 1 + \delta]$ where $\delta > 0$, so that $\alpha_\delta(t) = 0$ for all $t \leq 1 - \delta$ and $\alpha_\delta(t) = 1$ for all $t \geq 1 + \delta$. The utility function

$$u_\delta(x) = (\alpha_\delta(x_1 + x_2)(x_1 + 4x_2)) + (1 - \alpha_\delta(x_1 + x_2)(x_1 + x_2))$$

is continuous and monotonic. For δ close to 0, the function $u_\delta = u$ except in a narrow area around the set of bundles for which $x_1 + x_2 = 1$.

Now, take two prices, $H > 1$ and $L < 1$, such that a consumer with utility function u consumes more of the first commodity when facing the budget set $((H, 2), 1)$ than when facing the budget set $((L, 2), 1)$ (that is, $1/H > 1/(2 - L)$). When δ is close enough to 0, the demand induced from u_δ at $B((H, 2), 1)$ is $(1/H, 0)$. Choose ϵ such that $1/(2 - L) + \epsilon < 1/H$. For δ close enough to 0, the bundle in the budget set of $B((L, 2), 1)$ with $x_1 = 1/(2 - L) + \epsilon$ is preferred (according to u_δ) over any other bundle in $B((L, 2), 1)$ with a higher quantity of x_1. Thus, for small enough δ, the induced demand for the first commodity at the lower price is at most $1/(2 - L) + \epsilon$, and is thus lower than the demand at the higher price.

"The Law of Demand"

We are interested in comparing demand in different environments. We have just seen that the classic assumptions about the consumer do not allow us to draw a clear conclusion regarding the relation between a consumer's demand when facing $B(p, w)$ and his demand when facing $B(p + (0, \ldots, \varepsilon, \ldots, 0), w)$.

A clear conclusion can be drawn when we compare the consumer's demand when he faces the budget set $B(p, w)$ to his demand when facing $B(p', x(p, w)p')$. In this comparison we imagine the price vector changing from p to an arbitrary p' and wealth changing in such a way that the consumer has exactly the resources allowing him to consume the same bundle he consumed at (p, w). (See fig. 5.4.)

Claim:

Let x be a demand function satisfying Walras's law and WA. If $w' = p'x(p, w)$, then either $x(p', w') = x(p, w)$ or $[p' - p][x(p', w') - x(p, w)] < 0$.

Proof:

Assume that $x(p', w') \neq x(p, w)$. Then,
$[p' - p][x(p', w') - x(p, w)]$
$= p'x(p', w') - p'x(p, w) - px(p', w') + px(p, w)$
$= w' - w' - px(p', w') + w = w - px(p', w')$

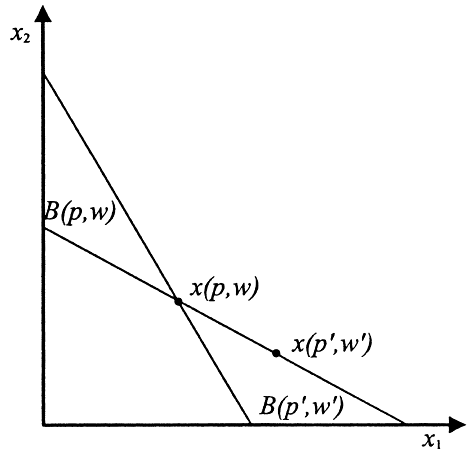


Figure 5.4
A compensated price change from (p, w) to (p', w').

(by Walras's law and the assumption that $w' = p'x(p, w)$), and by WA the right-hand side of the equation is less than 0.

Bibliographic Notes

Recommended readings: Kreps 1990, 37–45, Mas-Colell et al. 1995, Chapter 2, A–D, 3, D,J.

The material in this lecture, up to the discussion of differentiability, is fairly standard and closely parallels that found in Arrow and Hahn (1971) and Varian (1984).

Problem Set 5

Problem 1. (*Easy*)
Calculate the demand function for a consumer with the utility function $\sum_k \alpha_k \ln(x_k)$.

Problem 2. (*Easy*)
Verify that when preferences are continuous, the demand function $x(p, w)$ is continuous in prices and in wealth (and not only in p).

Problem 3. (*Easy*)
Show that if a consumer has a homothetic preference relation, then his demand function is homogeneous of degree one in w.

Problem 4. (*Easy*)
Consider a consumer in a world with $K = 2$, who has a preference relation that is quasi-linear in the first commodity. How does the demand for the first commodity change with w?

Problem 5. (*Moderately Difficult*)
Let $\succsim$ be a continuous preference relation (not necessarily strictly convex) and w a number. Consider the set $G = \{(p, x) \in \Re^K \times \Re^K | \ x$ is optimal in $B(p, w)\}$. (For some price vectors there could be more than one $(p, x) \in G$.) Calculate G for the case of $K = 2$ and preferences represented by $x_1 + x_2$. Show that (in general) G is a closed set.

Problem 6. (*Moderately difficult*)
Determine whether the following behavior patterns are consistent with the consumer model (assume $K = 2$):

a. The consumer's demand function is $x(p, w) = (2w/(2p_1 + p_2), w/(2p_1 + p_2))$.
b. He consumes up to the quantity 1 of commodity 1 and spends his excess wealth on commodity 2.

c. The consumer chooses the bundle (x_1, x_2) which satisfies $x_1/x_2 = p_1/p_2$ and costs w. (Does the utility function $u(x) = x_1^2 + x_2^2$ rationalize the consumer's behavior?)

Problem 7. (*Moderately difficult*)

In this question, we consider a consumer who behaves differently from the classic consumer we talked about in the lecture. Once again we consider a world with K commodities. The consumer's choice will be from budget sets. The consumer has in mind a preference relation that satisfies continuity, monotonicity, and strict convexity; for simplicity, assume it is represented by a utility function u.

The consumer maximizes utility up to utility level u^0. If the budget set allows him to obtain this level of utility, he chooses the bundle in the budget set with the highest quantity of commodity 1 subject to the constraint that his utility is at least u^0.

a. Formulate the consumer's problem.
b. Show that the consumer's procedure yields a unique bundle.
c. Is this demand procedure rationalizable?
d. Does the demand function satisfy Walras's law?
e. Show that in the domain of (p, w) for which there is a feasible bundle yielding utility of at least u^0 the consumer's demand function for commodity 1 is decreasing in p_1 and increasing in w.
f. Is the demand function continuous?

Problem 8. (*Moderately difficult*)

A common practice in economics is to view aggregate demand as being derived from the behavior of a "representative consumer." Give two examples of "well-behaved" consumer preference relations that can induce average behavior that is not consistent with maximization by a "representative consumer." (That is, construct two "consumers," 1 and 2, who choose the bundles x^1 and x^2 out of the budget set A and the bundles y^1 and y^2 out of the budget set B so that the choice of the bundle $(x^1 + x^2)/2$ from A and of the bundle $(y^1 + y^2)/2$ from B is inconsistent with the model of the rational consumer.)