Macroeconomic Theory
A Dynamic General Equilibrium Approach

Michael Wickens

Princeton University Press
Princeton and Oxford
Questions
Questions to Accompany Chapter 2

1) We have assumed that the economy discounts $s$ periods ahead using the geometric (or exponential) discount factor $\beta_s = (1 + \theta)^{-s}$ for $\{s = 0, 1, 2, \ldots\}$. Suppose instead that the economy uses the sequence of hyperbolic discount factors $\beta_s = \{1, \varphi \beta, \varphi \beta^2, \varphi \beta^3, \ldots\}$, where $0 < \varphi < 1$.

(a) Compare the implications for discounting of using geometric and hyperbolic discount factors.

(b) For the centrally planned model,

\[ y_t = c_t + i_t, \]
\[ \Delta k_{t+1} = i_t - \delta k_t, \]
\[ y_t = A k_t^\alpha, \]

where $y_t$ is output, $c_t$ is consumption, $i_t$ is investment, $k_t$ is the capital stock, and the objective is to maximize

\[ V_t = \sum_{s=0}^{\infty} \beta_s U(c_{t+s}), \]

derive the optimal solution of the basic centrally planned model under hyperbolic discounting, and comment on any differences with the solution based on geometric discounting.

2) Assuming geometric discounting $\beta_s$, the utility function $U(c_t) = \ln c_t$, and the production function $y_t = A k_t^\alpha$:

(a) derive the optimal long-run solution, and

(b) analyze the short-run solution

3) In question (2), instead of eliminating the Lagrange multipliers $\lambda_t$ when deriving the optimal solution from the first-order conditions, obtain the joint solution for \{c_t, k_t, \lambda_t\}. How would you interpret $\lambda_t$?

4) Consider the CES production function

\[ y_t = A[\alpha k_t^{1-(1/y)} + (1 - \alpha)n_t^{1-(1/y)}]^{1/(1-(1/y))}. \]

Show that the CES function becomes the Cobb-Douglas function as $\gamma \to 1$.

5) Consider the following centrally planned model with labor:

\[ y_t = c_t + i_t, \]
\[ \Delta k_{t+1} = i_t - \delta k_t, \]
\[ y_t = A[\alpha k_t^{1-(1/y)} + (1 - \alpha)n_t^{1-(1/y)}]^{1/(1-(1/y))}, \]
where the objective is to maximize

\[ V_t = \sum_{s=0}^{\infty} \beta^s [\ln c_{t+s} + \varphi \ln l_{t+s}] \]

and where \( y_t \) is output, \( c_t \) is consumption, \( i_t \) is investment, \( k_t \) is the capital stock, \( n_t \) is employment, and \( l_t \) is leisure \((l_t + n_t = 1)\).

(a) Derive the long-run solutions for consumption, labor, and capital.
(b) What are the implied real interest rate and wage rate?
(c) Comment on the implications of having an elasticity of substitution between capital and labor different from unity.

(6) Comment on the statement: “the saddlepath is a knife-edge solution; once the economy departs from the saddlepath it is unable to return to equilibrium and will instead either explode or collapse.”
Questions to Accompany Chapter 3

(1) Rework the optimal growth solution in terms of the original variables, i.e., without first taking deviations about trend growth.

(2) Consider the Solow–Swan model of growth for the production function \( Y_t = A(e^{\mu t} K_t)^\alpha (e^{\nu t} N_t)^{1-\alpha} \).

   (a) Examine the solutions when technical progress is Hicks-neutral so that \( \mu = \nu \), and when technical progress is labor augmenting so that \( \mu = 0 \).

   (b) Show that the model has constant steady-state growth only when technical progress is labor augmenting.

(2) Compare the solutions for optimal growth when technical progress is Hicks neutral and labor augmenting.

(3) Suppose that technical progress is embodied in new investment so that the production function is \( Y_t = (\sum_{v=0}^{\infty} e^{\eta(t-v)} I_{t-v})^{\alpha} N_t^{1-\alpha} \) and that there is no depreciation of capital. Derive

   (a) the solution to the Solow–Swan model;

   (b) the optimal growth solution where the objective is to maximize

\[
\sum_{s=0}^{\infty} \beta^s \ln c_{t+s}.
\]

(4) Consider the following two-sector endogenous growth economy with two types of capital due to Rebelo (1991): physical \( k_t \) and human \( h_t \). Both types of capital are required to produce goods output \( y_t \) and new human capital \( i^h_t \). The model is

\[
y_t = c_t + i^k_t, \\
\Delta k_{t+1} = i^k_t - \delta k_t, \\
\Delta h_{t+1} = i^h_t - \delta h_t, \\
y_t = A(\phi k_t)^\alpha (\mu h_t)^{1-\alpha}, \\
i^h_t = A[(1-\phi)k_t]^{\varepsilon}[(1-\mu)h_t]^{1-\varepsilon},
\]

where \( i^k_t \) is investment in physical capital, \( \phi \) and \( \mu \) are the shares of physical and human capital used in producing goods and \( \alpha > \varepsilon \). The economy maximizes

\[
V_t = \sum_{s=0}^{\infty} \beta^s \frac{c_{t+s}^{1-\sigma}}{1-\sigma}.
\]
(a) Assuming that each type of capital receives the same rate of return in both activities, derive

(i) the optimal long-run solution,
(ii) the rate of growth of each variable in steady state.

(b) How are these solutions affected when

(i) $\phi = \mu$,
(ii) $\phi = \mu = 1$,
(iii) $\alpha > \epsilon$?

(c) Hence summarize the implications of this model of growth.
Questions to Accompany Chapter 4

(1) The household budget constraint may be expressed in different ways from equation (4.2). Derive the optimal solution for consumption and compare these with the solution based on equation (4.2) for each of the following ways of writing the budget constraint:

(a) \( a_{t+1} = (1 + r)(a_t + x_t - c_t) \), i.e., current assets and income assets not consumed are invested.

(b) \( \Delta a_t + c_t = x_t + r a_{t-1} \), where the dating convention is that \( a_t \) denotes the end of period stock of assets and \( c_t \) and \( x_t \) are consumption and income during period \( t \).

(c) \( W_t = \sum_{s=0}^{\infty} \frac{c_{t+s}}{(1+r)^s} = \sum_{s=0}^{\infty} \frac{x_{t+s}}{(1+r)^s} + r a_t \), where \( W_t \) is household wealth.

(2) The representative household is assumed to choose \( \{c_t, c_{t+1}, \ldots\} \) to maximize

\[
V_t = \sum_{s=0}^{\infty} \beta^s U(c_{t+s}), \quad 0 < \beta = \frac{1}{1+\theta} < 1,
\]

subject to the budget constraint \( \Delta a_{t+1} + c_t = x_t + r a_t \), where \( c_t \) is consumption, \( x_t \) is exogenous income, \( a_t \) is the (net) stock of financial assets at the beginning of period \( t \), and \( r \) is the (constant) real interest rate.

(a) Derive the Euler equation and give an intuitive explanation of it.

(b) Assuming that \( r = \theta \) and using the approximation

\[
\frac{U'(c_{t+1})}{U'(c_t)} = 1 - \sigma \Delta \ln c_{t+1},
\]

show that optimal consumption is constant.

(c) Does this mean that changes in income will have no effect on consumption? Explain.

(3) Derive optimal household consumption when the utility function reflects habit persistence of the following forms:

(a) \( U(c_t) = - \frac{(c_t - Yc_{t-1})^2}{2} + \alpha(c_t - Yc_{t-1}) \);

(b) \( U(c_t) = \frac{(c_t - Yc_{t-1})^{1-\sigma}}{1-\sigma} \).

(4) Suppose that households have savings of \( s_t \) at the start of the period, consume \( c_t \) but have no income. The household budget constraint is \( \Delta s_{t+1} = r(s_t - c_t) \), \( 0 < r < 1 \), where \( r \) is the real interest rate.
(a) If the household’s problem is to maximize discounted utility

\[ V_t = \sum_{s=0}^{\infty} \beta^s \ln c_{t+s}, \]

where \( \beta = 1/(1+r) \):

(i) Show that the solution is \( c_{t+1} = c_t \).

(ii) What is the solution for \( s_t \)?

(b) If the household’s problem is to maximize expected discounted utility

\[ V_t = E_t \sum_{s=0}^{\infty} \beta^s \ln c_{t+s}, \]

(i) Show that the solution is \( 1/c_t = E_t[1/c_{t+1}] \).

(ii) Using a second-order Taylor series expansion about \( c_t \) show that the solution can be written as \( E_t[\Delta c_{t+1}/c_t] = E_t[(\Delta c_{t+1}/c_t)^2] \).

(iii) Hence, comment on the differences between the nonstochastic and the stochastic solutions.

(5) A household lives for two periods, \( t \) and \( t+1 \). The discount factor for period two is \( \beta = 1 \). It receives exogenous income \( x_t \) and \( x_{t+1} \), where \( x_{t+1} \) is distributed \( N(x_t, \sigma^2) \), but has no assets. Find the optimal level of \( c_t \)

(a) if the utility function is quadratic, \( U(c_t) = -\frac{1}{2} c_t^2 + \alpha c_t, \alpha > 0; \) and

(b) if the utility function is exponential, \( U(c_t) = -e^{-\alpha c_t}, \alpha > 0. \)

(c) What is the coefficient of relative risk aversion in case (b)?

(6) Suppose that firms face additional (quadratic) costs associated with the accumulation of capital and labor so that firm profits are

\[ \Pi_t = Ak_t^\alpha n_t^{1-\alpha} - w_t n_t - i_t - \frac{1}{2} \mu (\Delta k_{t+1})^2 - \frac{1}{2} \nu (\Delta n_t)^2, \]

where \( \mu, \nu > 0 \), the real wage \( w_t \) is exogenous, and \( \Delta k_{t+1} = i_t - \delta k_t \). If firms maximize the expected present value of the firm \( E_t[\sum_{s=0}^{\infty}(1+r)^{-s}\Pi_{t+s}] \):

(a) Derive the demand functions for capital and labor in the long run and the short run.

(b) What would be the response of capital and labor demand to

(i) a temporary increase in the real wage in period \( t \), and

(ii) a permanent increase in the real wage from period \( t \)?
Questions to Accompany Chapter 5

(1) Suppose that the government finances expenditures by taxes on consumption at the rate \( \tau \) and by debt. Starting from a position where the budget is balanced and there is no government debt, analyze the consequences of

(a) a temporary increase in government expenditures in period \( t \),
(b) a permanent increase in government expenditures from period \( t \).

(2) Suppose that government expenditures \( g_t \) are all capital expenditures and the stock of government capital \( G_t \) is a factor of production. If the economy is described by

\[
\begin{align*}
y_t &= c_t + i_t + g_t, \\
y_t &= Ak_t^aG_t^{1-a}, \\
\Delta k_{t+1} &= i_t - \delta k_t, \\
\Delta G_{t+1} &= g_t - \delta G_t,
\end{align*}
\]

and the aim is to maximize \( \sum_{s=0}^{\infty} \beta^s \ln c_{t+s} \), obtain the optimal solution.

(3) Suppose that government finances its expenditures through lump-sum taxes \( T_t \) and debt \( b_t \) but there is a cost of collecting taxes given by

\[
\Phi(T_t) = \phi_1 T_t + \frac{1}{2} \phi_2 T_t^2, \quad \Phi'(T_t) \geq 0.
\]

If the national income and government budget constraints are

\[
\begin{align*}
y_t &= c_t + g_t + \Phi(T_t), \\
\Delta b_{t+1} + T_t &= g_t + rb_t,
\end{align*}
\]

where output \( y_t \) is exogenous, and the aim is to maximize \( E_t \sum_{s=0}^{\infty} \beta^s U(c_{t+s}) \) for \( \beta = 1/(1+r) \):

(a) Find the optimal solution for taxes.
(b) Analyze the effects on taxes and debt of

(i) a temporary shock to government expenditures in period \( t \),
(ii) a permanent increase in government expenditures.

(4) Assuming that output growth is zero, inflation and the rate of growth of the money supply are \( \pi \), and that government expenditures on goods and services plus transfers less total taxes equals \( z_t \), analyze the consequences for the sustainability of the fiscal stance.
(5) Consider an economy without capital with proportional taxes on consumption and labor that is described by the following equations:

\[ y_t = An_t^\alpha = c_t + g_t, \]
\[ g_t + rb_t = \tau_c^t c_t + \tau_w^t w_t n_t + \Delta b_{t+1}, \]
\[ U(c_t, l_t) = \frac{c_t^{1-\sigma}}{1-\sigma} + \gamma \ln l_t. \]

(a) State the household budget constraint.
(b) If the economy seeks to maximize \( \sum_{s=0}^{\infty} \beta^s U(c_{t+s}) \), derive the optimal levels of consumption and employment for given \( g_t, b_t, \) and tax rates.
(c) Find the optimal rates of consumption and labor taxes by solving the associated Ramsey problem.
Questions to Accompany Chapter 6

(1)
(a) Consider the following two-period overlapping generations (OLG) model. People consume in both periods but work only in period two. The inter-temporal utility of the representative individual in the first period is

\[ U = \ln c_1 + \beta [\ln c_2 + \alpha \ln (1 - n_2) + \gamma \ln g_2], \]

where \( c_1 \) and \( c_2 \) are consumption in periods one and two, \( n_2 \) is work, and \( g_2 \) is government expenditure in period two. Production in periods one and two are

\[ y_1 = Rk_1 = c_1 + k_2, \]
\[ y_2 = Rk_2 + an_2 = c_2 + g_2. \]

Find the optimal centrally planned solution.

(b) Now suppose that government taxes both labor and capital in period two so that the private sector's intertemporal budget constraint is

\[ c_1 + \frac{c_2}{R_2} = Rk_1 + \frac{(1 - \tau_2)an_2}{R_2} \]

and the government budget constraint is

\[ g_2 = \tau_2 an_2 + (R - R_2)k_2, \]

where \( R_2 \) is the after-tax return to capital and \( \tau_2 \) is the rate of tax of labor in period two. Find the solution when taxes are preannounced. Why is this solution not time consistent?

(c) Assume now that the government optimizes taxes in period two taking \( k_2 \) as given. Show that the optimal labor tax in period two is zero, and find the optimal rate of capital taxation in period two.

(d) Taking the solution in (c) as given, solve the consumer's problem in period one.

(e) Compare \( U \) in this case with case (b).

(2) Consider the following two-period OLG model in which each generation has the same number of people, \( N \). The young generation receives an endowment of \( x_1 \) when young and \( x_2 = (1 + \phi)x_1 \) when old, where \( \phi \) can be positive or negative. The endowments of the young generation grow over time at the rate \( \gamma \). Each unit of saving (by the young) is invested and produces \( 1 + \mu \) units of output (\( \mu > 0 \)) when they are old. Each of the young generation maximizes
\[ \ln c_{1,t} + \ln \left(1 + \frac{1}{1 + r}\right) \ln c_{2,t+1}, \] where \( c_{1,t} \) is consumption when young and \( c_{2,t+1} \) is consumption when old.

(a) Derive the consumption and savings of the young generation and the consumption of the old generation.

(b) How do changes in \( \phi \), \( \gamma \), and \( \mu \) affect these solutions?

(c) If \( \phi = \mu \) how does this affect the solution?
Questions to Accompany Chapter 7

(1) An open economy has the balance of payments identity

\[ x_t - Qx_t^m + r^* f_t = \Delta f_{t+1}, \]

where \( x_t \) is exports, \( x_t^m \) is imports, \( f_t \) is the net holding of foreign assets, \( Q \) is the terms of trade, and \( r^* \) is the world rate of interest. Total output \( y_t \) is either consumed at home \( c_t^h \) or is exported, thus

\[ y_t = c_t^h + x_t. \]

Total domestic consumption is \( c_t; y_t \) and \( x_t \) are exogenous.

(a) Derive the Euler equation that maximizes \( \sum_{s=0}^{\infty} \beta^s \ln c_{t+s} \) with respect to \( \{c_t, c_{t+1}, \ldots; f_{t+1}, f_{t+2}, \ldots\} \), where \( \beta = 1/(1 + \theta) \).

(b) Explain how and why the relative magnitudes of \( r^* \) and \( \theta \) affect the steady-state solutions of \( c_t \) and \( f_t \).

(c) Explain how this solution differs from that of the corresponding closed economy.

(d) Hence comment on whether there are any benefits of trade in this model.

(e) Obtain the solution for the current account.

(f) What are the effects on the current account and net asset position of a permanent increase in \( x_t \)?

(2) Consider two countries which consume home and foreign goods \( c_{H,t} \) and \( c_{F,t} \). Each period the home country maximizes

\[ U_t = \frac{c_{H,t}^{(\sigma-1)/\sigma} + c_{F,t}^{(\sigma-1)/\sigma}}{\sigma/(\sigma-1)} \]

and has an endowment of \( y_t \) units of the home produced good. The foreign country is identical and its variables are denoted with an asterisk. Every unit of a good that is transported abroad has a real resource cost equal to \( \tau \) so that, in effect, only a proportion \( 1 - \tau \) arrives at its destination. In the home economy \( P_{H,t} \) is the price of the home good and \( P_{F,t} \) is the price of the foreign good. \( P_{H,t}^* \) and \( P_{F,t}^* \) are the corresponding prices in the foreign country where all prices are measured in terms of a common unit of world currency.

(a) If goods markets are competitive, what is the relation between the four prices and how are the terms of trade in each country related?

(b) Derive the relative demands for home and foreign goods in each country.

(c) How is this affected if \( y_t = y_t^*? \)
(d) For $y_t = y_t^*$ obtain the implications of transport costs for

(i) the terms of trade, and

(ii) the openness of the economy (i.e., the relative expenditures on home and foreign goods)?

(3) Suppose the model in question (2) is modified so that there are two periods and intertemporal utility is

$$V_t = U(c_t) + \beta U(c_{t+1}),$$

where $c_t = \left[ c_H^{\alpha/(\sigma-1)} + c_F^{\alpha/(\sigma-1)} \right]^{\sigma/(\sigma-1)}$. Endowments in the two periods are $y_t$ and $y_{t+1}$. Foreign prices $P_{H,t}^*$ and $P_{F,t}^*$ and the world interest rate are assumed given. The first-period budget constraint is

$$P_{H,t}y_t + B = P_{H,t}c_{H,t} + P_{F,t}c_{F,t} = P_t,$$

where the general price level is $P_t$ and $B$ is borrowing from abroad at the foreign real interest rate $r^*$. The second period budget constraint is

$$P_{H,t+1}y_{t+1} - (1 + r^*)B = P_{H,t+1}c_{H,t+1} + P_{F,t+1}c_{F,t+1} = P_{t+1}c_{t+1}.$$

(a) Derive the optimal solution for the home economy, including the domestic price level $P$.

(b) Show that if the foreign good is always imported, its home price is $P_t = P_{F,t}^*/(1 - \tau)$.

(c) Derive the domestic real interest rate $r$ and examine whether real interest parity exists.

(d) How does $c_t$ depend on $r$?

(4) Suppose the “world” is compromised of two identical countries. Each country consumes home and foreign goods and maximizes

$$V_t = \sum_{s=0}^{\infty} \beta^s \left( c_H^{\alpha(1-s) + c_F^{1-s}} \right)^{1-\sigma}$$

subject to its budget constraint. Expressed in terms of home’s prices, the home country budget constraint is

$$P_{H,t}c_{H,t} + S_tP_{F,t}c_{F,t} + \Delta B_{t+1} = P_{H,t}y_H + r_tB_t,$$

where $c_{H,t}$ is consumption of home produced goods, $c_{F,t}$ is consumption of foreign produced goods, $P_{H,t}$ is the price of the home country’s output, which is denoted $y_{H,t}$ and is exogenous, $P_{F,t}$ is the price of the foreign country’s output, $B_t$ is the home country’s borrowing from abroad expressed in domestic currency which is at the world rate of interest $r_t$, and $S_t$ is the nominal exchange rate.
(a) Using an asterisk to denote the foreign country equivalent variable (e.g., $c^*_H, t_{is}$ is the foreign country’s consumption of domestic output), what are the national income and balance of payments identities for the home country?

(b) Derive the optimal solution for the home country, including its general price level $P_t$, taking the foreign country—its output, exports, and prices—and the exchange rate as given.

(c) What is the corresponding solution for the foreign country?

(d) For the home country analyze the effects on consumption, the current account and the net foreign asset position of shocks to home output, foreign exports and the exchange rate.

(e) Compare the effects on the sustainability of the current account of temporary and permanent shocks to these variables.

(f) How would your answers differ if domestic and foreign goods were perfect substitutes?

(5) For the model described in question (2), suppose that there is world central planner who maximizes the sum of individual country welfares:

$$W_t = \sum_{s=0}^{\infty} \beta^s \left[ \frac{(c^*_H, t_{is}c^1_{F, t_{is}})^{1-\sigma}}{1 - \sigma} + \frac{[(c^*_H, t_{is})^{\alpha}(c^*_F, t_{is})^{1-\alpha}]^{1-\sigma}}{1 - \sigma} \right].$$

(a) What are the constraints in this problem?

(b) Derive the optimal world solution subject to these constraints where outputs and the exchange rate are exogenous.

(c) Comment on any differences with the solutions in question (2).

(d) What are the effects of shocks to home and foreign outputs and to the exchange rate?
Questions to Accompany Chapter 8

(1) Suppose that the nominal household budget constraint is
\[ \Delta B_t + \Delta M_{t+1} + P_t c_t = P_t x_t + (1 + r_t) B_t, \]
where \( c_t \) is consumption, \( x_t \) is exogenous income, \( B_t \) is nominal bond holding, \( M_t \) is nominal money balances, \( P_t \) is the general price level, \( m_t = M_t / P_t \), and \( r \) is the real rate of return.

(a) Derive the real budget constraint.

(b) Comment on whether this implies that money is not be super-neutral in the whole economy.

(c) If households maximize
\[ V_t = \sum_{s=0}^{\infty} \beta^s U(c_{t+s}, m_{t+s}), \]
where the utility function is
\[ U(c_t, m_t) = \frac{[c_t^\alpha m_t^{1-\alpha} / \alpha^\alpha (1 - \alpha)^{1-\alpha}]^{1-\sigma}}{1 - \sigma}, \]
obtain the long-run demand for money.

(2) Consider an economy in which money is the only financial asset, and suppose that households hold money solely in order to smooth consumption expenditures. The nominal household budget constraint for this economy is
\[ P_t c_t + \Delta M_{t+1} = P_t y_t, \]
where \( c_t \) is consumption, \( y_t \) is exogenous income, \( P_t \) is the price level, and \( M_t \) is nominal money balances.

(a) If households maximize \( \sum_{s=0}^{\infty} \beta^s U(c_{t+s}, y_t) \), derive the optimal solution for consumption.

(b) Compare this solution with the special case where \( \beta = 1 \) and inflation is zero.

(c) Suppose that \( x_t \) is expected to remain constant except in period \( t+1 \) when it is expected to increase temporarily. Examine the effect on money holdings and consumption.

(d) Hence comment on the role of real balances in determining consumption.
(3) Suppose that in the economy described in question (1) there is a government which consumes a random amount $g_t = g + e_t$, where $e_t$ is an independently and identically distributed random shock with zero mean. The government imposes a lump-sum tax of $T_t = g$ and balances its budget with the use of money. $y_t$ is now determined by the national income identity $y_t = c_t + g_t$.

(a) If households maximize $E_t \sum_{s=0}^{\infty} \beta^s \ln c_{t+s}$, derive the optimal solutions for consumption and money holding.

(b) Comment on how money shocks affect consumption and money holding.

(c) Is money super-neutral in this economy?

(4)

(a) Comment briefly on the “classical dichotomy” in macroeconomics that nominal shocks have no long-run effect on real variables.

(b) Consider the nominal household budget constraint

$$P_{t+1} k_{t+1} + \Delta M_{t+1} + P_t c_t + P_t T_t = w_t n_t + (1 + r_t) P_t k_t,$$

where $c$ is consumption, $k$ is the capital stock, $n$ is employment, $M$ is nominal money balances, $T$ is real taxes, $P$ is the general price level, $w$ is the real wage rate, and $r$ is the real rate of return on capital.

(i) Derive the real household budget constraint.

(ii) What does this reveal about the effect of inflation on real variables such as consumption?

(c) Suppose that the government’s budget constraint tax is $T_t = m_t - (1 + \pi_t) m_{t+1}$, where $T < 0$ and $m$ is real money.

(i) Interpret $T$.

(ii) Combine the household and government budget constraints to eliminate $m$ and obtain the consolidated constraint.

(d) If the economy’s production function $F(k_t, n_t)$ is homogeneous of degree one and factors are paid their marginal products so that $F_{k,t} = \delta = r_t$ and $F_{n,t} = w_t$,

(i) show that $F(k_t, n_t) = (r_t + \delta) k_t + w_t n_t$,

(ii) and that in the long-run $c = rk + wn$.

(e) Discuss the implications of these results for the “classical dichotomy.”

(5) Suppose that some goods $c_{1,t}$ must be paid for only with money $M_t$ and the rest $c_{2,t}$ are bought on credit $L_t$ using a one-period loan to be repaid at the start of next period at the risk-free nominal rate of interest $R$. The prices of these goods are $P_{1t}$ and $P_{2t}$. If households maximize $\sum_{s=0}^{\infty} (1 + R)^{-s} U(c_{t+s})$, where

$$U(c_t) = \frac{c_{1,t}^{\alpha} c_{2,t}^{1-\alpha}}{\alpha^\alpha (1 - \alpha)^{1-\alpha}},$$

and income $y_t$ is exogenous:
(a) Find the optimal solutions for $c_{1,t}, c_{2,t}, M_t,$ and $L_t$.
(b) Explain the interest sensitivities of money and credit.
(c) Suppose that households had the freedom to choose to use either money or credit for all purchases. How would this affect the solution?

(6) Consider the following demand for money function which has been used to study hyperinflation:

$$m_t - p_t = -\alpha(E_t p_{t+1} - p_t), \quad \alpha > 0,$$

where $M_t = \text{nominal money}, m_t = \ln M_t, P_t = \text{price level},$ and $p_t = \ln P_t$.

(a) Contrast this with the usual demand function for money, and comment on why it might be a suitable formulation for studying hyperinflation?
(b) Derive the equilibrium values of $p_t$ and the rate of inflation if the supply of money is given by

$$\Delta m_t = \mu + \varepsilon_t,$$

where $\mu > 0$ and $E_t[\varepsilon_{t+1}] = 0$.
(c) What will be the equilibrium values of $p_t$ if

(i) the stock of money is expected to deviate temporarily in period $t + 1$ from this money supply rule and take the value $m^*_t$;

(ii) the rate of growth of money is expected to deviate permanently from the rule and from period $t + 1$ grow at the rate $\nu$?

(7) Consider the effect of inflation on front-end loading.

(a) Assume an economy with zero inflation. Households take out a loan for $n$ periods. Each period they pay interest on the loan at the nominal interest rate $R$ (also the real interest rate $r$). The loan is not repaid until the end. What is the present value of the cost of this loan using the discount rate $R$?

(b) Suppose instead that the economy has an inflation rate of $\pi > 0$. Households now pay each period at the nominal interest rate $R = r + \pi$. Calculate the present value of the cost of the loan using the new discount rate $R$.

(c) Suppose that each period households take out a loan to pay the additional interest charges due to inflation and repay these loans in the final period together with the original loan. Recalculate the present value of the cost of the loan.

(d) Compare the expenditure flows made by the households each period including the final period for these three cases, and hence comment on the real effects of inflation.

(e) Banks are often reluctant to make loans to pay extra interest charges. What is the likely effect of this on the real cost of inflation?
Questions to Accompany Chapter 9

(1) Consider an economy that produces a single good in which households maximize

\[ U_t = \sum_{s=0}^{\infty} \beta^s \left[ \ln c_{t+s} + \phi \ln l_{t+s} + \gamma \ln \frac{M_{t+s}}{P_{t+s}} \right] \]

subject to the nominal budget constraint

\[ P_t c_t + \Delta B_{t+1} + \Delta M_{t+1} = P_t d_t + W_t n_t + r B_t, \]

where \( c \) is consumption, \( l \) is leisure, \( n \) is employment \((n + l = 1)\), \( W \) is the nominal wage rate, \( d \) is total real firm net revenues distributed as dividends, \( B \) is nominal bond holdings, \( R \) is the nominal interest rate, \( M \) is nominal money balances, and \( P \) is the price level. Firms maximize the present value of nominal net revenues

\[ \Pi_t = \sum_{s=0}^{\infty} (1 + r)^{-s} P_{t+s} d_{t+s}, \]

where \( d_t = y_t - w_t n_t \), the real wage is \( w_t = W_t / P_t \), and the production function is \( y_t = A_t n_t^\alpha \).

(a) Derive the optimal solution on the assumption that prices are perfectly flexible.

(b) Suppose that, following a shock, for example, to the money supply or to technological progress \( A_t \), firms are able to adjust their price with probability \( \rho \) and otherwise price retains its previous value. Examine the consequences for the dynamic behavior of the economy.

(c) Suppose instead that wages adjust to shocks with probability \( \rho \). Reexamine the consequences for the dynamic behavior of the economy.

(2) Rework the analysis of price determination under imperfect competition with \( N = 2 \) and with total household consumption obtained by aggregating using the aggregator

\[ c_t = \frac{c_t(1)^\phi c_t(2)^{1-\phi}}{\phi \phi (1 - \phi)^{1-\phi}}. \]

(3) Rework the analysis of price determination with intermediate goods with \( N = 2 \) and with the final output related to intermediate inputs through

\[ y_t = \frac{y_t(1)^\phi y_t(2)^{1-\phi}}{\phi \phi (1 - \phi)^{1-\phi}}. \]
(4) Consider the model for pricing with intermediate inputs. The demand for a firm’s output was shown to be

\[ y_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\phi} y_t. \]

Its profits can be written as

\[ \Pi_t(i) = P_t(i)y_t(i) - C_t(i), \]

where its total cost is

\[ C_t(i) = \frac{\phi - 1}{\phi} P_t(i)y_t(i). \]

(a) Find the optimal price \( P_t(i)^* \) if the firm maximizes profits period by period while taking \( y_t \) and \( P_t \) as given.

(b) If instead the firm chooses a price which it plans to keep constant for all future periods and hence maximizes \( \sum_{s=0}^{\infty} (1 + r)^{-s}\Pi_{t+s}(i) \), show that the resulting optimal price can be written as

\[ P_t(i)^\# = \frac{\phi(1 + r)}{(\phi - 1)r} \sum_{s=0}^{\infty} (1 + r)^{-s} MC_{t+s}(i), \]

where \( MC_t(i) \) is marginal cost.

(c) What is this price if expressed in terms of \( P_t(i)^* \)?

(d) Hence comment on the effect on today’s price of anticipated future shocks to demand and costs.

(5) Suppose in question (4) that the firm seeks to maximize

\[ \sum_{s=0}^{\infty} (1 + r)^{-s}\Pi_{t+s}(i) \]

but may reset its price each period with probability \( \rho \).

(a) What is the optimal price in period \( t \) expressed in terms of \( P_t(i)^* \)?

(b) Comment on the differences between this price and \( P_t(i)^\# \).

(6) Consider an economy with two sectors \( i = 1, 2 \). Each sector sets its price for two periods but does so in alternate periods. The general price level in the economy is the average of sector prices: \( p_t = \frac{1}{2}(p_{1,t} + p_{2,t}) \). In the period the price is reset it is determined by the average of the current and the expected future optimal price: \( p_t^\# = \frac{1}{2}(p_t^* + E_t p_{t+1}^*), \) hence \( p_t = \frac{1}{2}(p_t^* + p_{t-1}^\#) \). The optimal price is assumed to be determined by \( p_t^* - p_t = \phi(w_t - p_t) \), where \( w_t \) is the wage rate.

(a) Derive the general price level if wages are generated by \( \Delta w_t = e_t \), where \( e_t \) is a zero-mean i.i.d. process.

(b) How does the price level respond in period \( t \) to an anticipated shock in wages in period \( t + 1 \)?
(1) Suppose that a consumer’s initial wealth is given by $W_0$, and the consumer has the option of investing in a risky asset which has a rate of return $r$ and a risk-free asset which has a sure rate of return $f$. If the consumer maximizes the expected value of a strictly increasing, concave utility function $U(W)$ by choosing to hold the risky versus the risk-free asset, and if the variance of the return on the risky asset is $V(r)$:

(i) Find an expression for the risk premium $\rho$ that makes the consumer indifferent between holding the risky or the risk-free asset.

(ii) What is the risk-premium if $E(r) = f$?

(b) Suppose that the consumer’s utility function is the hyperbolic absolute risk aversion (HARA) function

$$U(W) = \frac{1 - \sigma}{\sigma} \left[ \frac{\alpha W}{1 - \sigma} + \beta \right]^\sigma, \quad \alpha > 0, \beta > 0, \sigma > 0.$$ 

(i) Explain how absolute risk aversion differs from relative risk aversion.

(ii) Discuss how the magnitude of the risk premium varies as a function of the parameters $\alpha$, $\beta$, and $\sigma$.

(2) Suppose there exists a representative risk averse consumer who derives utility from current and future consumption as

$$U = U(c_t) + \beta E_t U(c_{t+1}),$$

where $0 < \beta < 1$ is the consumer's subjective discount factor, and the single-period utility function has the form

$$U(c_t) = \frac{c_t^{1-\sigma} - 1}{1 - \sigma}, \quad \sigma \geq 0.$$ 

The consumer receives an exogenous income of $y_t$ in period $t$, and $y_{t+1}$ in period $t + 1$. While $y_t$ is known at date $t$, $y_{t+1}$ is random. The consumer can save by purchasing shares in a stock or by holding a risk-free one-period bond. The ex-dividend price of the stock is given by $P_t$ in period $t$. The stock pays a random stream of dividends from period $t + 1$ given by $\{D_{t+1+i}\}_{i=0}^\infty$. The bond sells for $P^b_t$ in period $t$ and has a payoff of one in period $t + 1$.

(a) Find an expression for the stock price that must hold at the consumer’s optimum. Interpret this expression.
(b) Find an expression for the bond price that must hold at the consumer's optimum. Interpret this expression.

(c) Derive an expression for the risk premium on the stock that must hold at the consumer's optimum. Interpret this expression.

(3)

(a) What is the significance of an asset having the same payoff in all states of the world?

(b) Consider a situation involving three assets and two states. Suppose that one asset is a risk-free bond with a return of 20%, a second asset has a price of 100 and payoffs of 80 and 200 in the two states, and a third asset has payoffs of 100 and 0 in the two states. Find

(i) the prices of the implied contingent claims in the two states,
(ii) the price of the third asset.

Additionally, answer the following question.

(iii) What is the risk premium associated with the second asset?

(4) Consider the following two-period problem for a household in which there is one state of the world in the first period and two states in the second period. Income in the first period is 6; in the second period it is 5 in state one which occurs with probability 0.8, and is 10 in state two. If instantaneous utility is \(\ln c_t\) and the rate of time discount is 0.2 derive

(a) the optimal levels of consumption in each period,
(b) the state prices,
(c) the stochastic discount factors,
(d) the risk-free “rate of return” (i.e., rate of change) to income in period two,
(e) the “risk premium” for income in period two.

(5) Investors form a portfolio consisting of two risky assets which has the return \(r^p = xr_1 + (1 - x)r_2\), where \(r_1\) denote the returns on two risky assets and \(x\) denotes the fraction invested in asset 1. The returns on these assets are normally distributed as \(r_1 \sim N(\mu_1, \sigma_1^2)\) and \(\text{Cov}(r_1, r_2) = \rho \sigma_1 \sigma_2\).

(a) Find the value of \(x\) that minimizes the variance of the portfolio return \(r^p\).

(b) Derive the mean-variance portfolio frontier under the assumption that the risky assets are

(i) uncorrelated,
(ii) perfectly negatively correlated,
(iii) perfectly positively correlated.
(c) If \( E(r_1) = 30\% \) and \( E(r_2) = 20\% \), the variance-covariance matrix for the two risky assets is

\[
\Sigma = \begin{bmatrix}
0.0081 & 0 \\
0 & 0.0025 \\
\end{bmatrix}.
\]

What is the optimal portfolio?

(d) Suppose that an investor A chooses a portfolio consisting of 75% in asset 1 and 25% in asset 2 while another investor B chooses a different portfolio with 50% in asset 1 and 50% in asset 2.

(i) Treating each investor's portfolio as the "market portfolio," what betas will each investor calculate for asset 1?

(ii) Consequently, which of the following is true and why?

- * Investor A will require a higher rate of return on asset 1 than investor B.
- * They will both require the same rate of return.
- * Investor B will require a higher rate of return on asset 1 than will investor A.
Questions to Accompany Chapter 11

(1) A household with the utility function

\[ U(c_t) = \frac{(c_t - y c_{t-1})^{1-\gamma}}{1-\gamma} \]

which maximizes \( E_t \sum_{s=0}^{\infty} \beta^s U(c_{t+s}) \) can either invest in equity with a return of \( r_t \) or a risk-free one-period bond with return \( r_f^t \). Derive

(a) the optimal consumption plan,
(b) the equity premium, and
(c) the optimal consumption plan if there is only a risk-free asset.

In addition, comment on your findings.

(2)

(a) A household with the utility function \( U(c_t) = \ln c_t \), which maximizes \( E_t \sum_{s=0}^{\infty} \beta^s U(c_{t+s}) \), can either invest in a one-period domestic risk-free bond with nominal return \( R_t \), or a one-period foreign currency bond with nominal return (in foreign currency) of \( R_t^* \). If the nominal exchange rate (the domestic price of foreign exchange) is \( S_t \) derive

(i) the optimal consumption plan,
(ii) the foreign exchange risk premium.

(b) Suppose that foreign households have an identical utility function but a different discount factor \( \beta^* \), what is their consumption plan and their risk premium.

(c) Is the market complete? If not:

(i) What would make it complete?
(ii) How would this affect the two risk premia?

(3)

(a) If in question (1) there are only two-period bonds, what is

(i) the optimal consumption plan,
(ii) the risk premium for the domestic household?

(b) How would your answers change if instead that there is a one-period domestic risk-free bond but only a two-period foreign bond?
(4) Let $S_t$ denote the current price in dollars of one unit of foreign currency; $F_{t,T}$ is the delivery price agreed to in a forward contract; $r$ the domestic interest rate with continuous compounding; $r^*$ the foreign interest rate with continuous compounding.

(a) Consider the following two portfolios:

(i) one long forward contract plus an amount of cash equal to $F_{t,T}e^{-r(T-t)}$;
(ii) an amount $e^{-r^*(T-t)}$ of the foreign currency.

Find the value of the forward exchange rate $F_{t,T}$.

(b) Suppose that the foreign interest rate exceeds the domestic interest rate at date $t$, $r^* > r$. What is the relation between the forward and spot exchange rates?

(c) A one-year long forward contract on a non-dividend-paying stock is entered into when the stock price is $40$ and the risk-free interest rate is 10% per annum with continuous compounding.

What are the forward price and the initial value of the forward contract?

(d) Six months later, the price of the stock is $45$ and the risk-free interest rate is still 10%. What are the forward price and the value of the forward contract?

(5) Consider a Vasicek model with two independent latent factors $z_{1t}$ and $z_{2t}$. The price of an $n$-period bond and the log discount factor may be written as

$$p_{n,t} = -[A_n + B_{1n}z_{1t} + B_{2n}z_{2t}],$$
$$m_{t+1} = -[z_{1t} + z_{2t} + \lambda e_{t+1}],$$

and the factors are generated by

$$z_{i,t+1} - \mu_i = \phi_i(z_{it} - \mu_i) + \epsilon_{i,t+1}, \quad i = 1, 2.$$

(a) Derive the no-arbitrage condition for an $n$-period bond and its risk premium.

(c) Express the yield on an $n$-period bond and its risk premium in terms of the yields on one- and two-period bonds.

(d) Derive an expression for the $k$-period ahead forward rate on an $n$-period bond.

(e) Suppose that the second factor is an observable variable, how would this affect your answers to parts (a) and (b)?
Questions to Accompany Chapter 12

(1) The Buiter and Miller (1981) model of the exchange rate may be described by the following equations:

\[ y_t = \alpha(s_t + p_t^* - p_t) - \beta(R_t - \Delta p_{t+1} - r_t) + g_t + y y_t^*, \]

\[ m_t - p_t = y_t - \lambda R_t, \]

\[ \Delta p_{t+1} = \theta(y_t - y_n^t) + \pi^#, \]

\[ \Delta s_{t+1} = R_t - R_t^*, \]

where \( y \) is output, \( y^n \) is full employment output, \( g \) is government expenditure, \( s \) is the log exchange rate, \( R \) is the nominal real interest rate, \( r \) is the real interest rate, \( m \) is log nominal money, \( p \) is the log price level, \( \pi^# \) is target inflation, and an asterisk denotes the foreign equivalent.

(a) Derive the long-run and short-run solutions for output, the price level and the exchange rate.

(b) Hence comment on the effects of monetary and fiscal policy.

(c) Suppose that the foreign country is identical and the two countries comprise the “world” economy. Denoting the corresponding world variable as \( \bar{x}_t = x_t + x_t^* \) and the country differential by \( \tilde{x}_t = x_t - x_t^* \),

(i) derive the solutions for the world economy and for the differences between the economies;

(ii) analyze the effects of monetary and fiscal policy on the world economy.

(2) Suppose that the “world” is compromised of two identical countries. Each country consumes home and foreign goods and maximizes

\[ V_t = \sum_{s=0}^{\infty} \beta^s \left[ \left( \frac{c_{H,t+s}^{\alpha} c_{F,t+s}^{1-\alpha}}{1-\sigma} \right)^{1-\sigma} + y \ln \frac{M_{t+s}}{P_{t+s}} \right] \]

subject to its budget constraint and uncovered interest parity. Expressed in terms of home’s prices, the home country budget constraint is

\[ P_{H,t} c_{H,t} + S_t P_{F,t} c_{F,t} + \Delta M_{t+1} + \Delta B_{t+1} = P_{H,t} y_{H,t} + r_t B_t, \]

where \( c_{H,t} \) is consumption of home produced goods, \( c_{F,t} \) is consumption of foreign produced goods, \( P_{H,t} \) is the price of the home country’s output, which is denoted \( y_{H,t} \), \( P_{F,t} \) is the price of the foreign country’s output, \( P_t \) is the domestic level, \( M_t \) is domestic currency, and \( B_t \) is the home country’s borrowing from abroad expressed in domestic currency and is at the world rate of interest \( r_t \) and \( S_t \) is the nominal exchange rate. Output and the supply of money are exogenous.
(a) Using an asterisk to denote the foreign country equivalent variable (e.g., $c_{h,t}^*$ is the foreign country’s consumption of domestic output and $M_t^*$ is the foreign country’s currency), derive the optimal solution for both countries and for the world interest rate and the exchange rate.

(b) Analyze the effects of shocks to home and foreign outputs and money supplies.
Questions to Accompany Chapter 13

(1) Consider the following characterizations of the IS–LM and DGE models.

**IS–LM:**

\[ y = c(y, r) + i(y, r) + g, \]
\[ m - p = L(y, r). \]

**DGE:**

\[ \Delta c = -\frac{1}{\sigma} (r - \theta) = 0, \]
\[ y = c + i + g, \]
\[ y = f(k), \]
\[ \Delta k = i, \]
\[ f_k = r, \]

where \( y \) is output, \( c \) is consumption, \( i \) is investment, \( k \) is the capital stock, \( g \) is government expenditure, \( r \) is the real interest rate, \( m \) is log nominal money, and \( p \) is the log price level.

(a) Comment on each equation and on the underlying approaches to macroeconomics.

(b) Discuss the usefulness of each model for analyzing the short and the long run.

(c) Comment on the implications for the effectiveness of monetary and fiscal policy in the two models.

(2)

(a) How might a country’s international monetary arrangements affect its conduct of monetary policy?

(b) What other factors might influence the way it carries out its monetary policy?

(3) The Lucas–Sargent proposition is that systematic monetary policy is ineffective. Examine this hypothesis using the following model of the economy due to Bull and Frydman (1983):

\[ y_t = \alpha_1 + \alpha_2 (p_{t-1} - E_{t-1}p_t) + u_t, \]
\[ d_t = \beta (m_t - p_t) + v_t, \]
\[ \Delta p_t = \theta (p^*_{t-1} - p_{t-1}), \]
where $y$ is output, $d$ is aggregate demand, $p$ is the log price level, $p^*$ is the market clearing price, $m$ is log nominal money, and $u$ and $v$ are zero-mean mutually and serially independent shocks.

(a) Derive the solution for output and prices.

(b) If $m_t = \mu + m_{t-1} + e_t$, where $e_t$ is a zero-mean serially independent shock, comment on the effectiveness of

(i) an unanticipated shock to money in period $t$,
(ii) a temporary anticipated shock to money in period $t$,
(iii) a permanent anticipated shock to money in period $t$.

(c) Hence comment on the Lucas–Sargent proposition.

(4) Consider the following model of the economy:

\[
\begin{align*}
y_t &= -\beta(R_t - E_t \pi_{t+1} - r), \\
\pi_t &= E_t \pi_{t+1} + \alpha y_t + e_t, \\
R_t &= \gamma(E_t \pi_{t+1} - \pi^*),
\end{align*}
\]

where $e_t$ is a zero-mean serially independent shock.

(a) What are the long-run and short-run solutions for $\pi_t$ and $y_t$?

(b) How would the behavior of inflation, output and monetary policy be affected by

(i) a temporary shock $e_t$,
(ii) an expected shock $e_{t+1}$?

(c) Suppose that the output equation is modified to

\[
y_t = -\beta(R_t - E_t \pi_{t+1} - r) - \theta e_t,
\]

where $e_t$ can be interpreted as a supply shock. How would the behavior of inflation, output, and monetary policy be affected by a supply shock?

(5) Suppose that a monetary authority is a strict inflation targeter attempting to minimize $E(\pi_t - \pi_t^*)^2$ subject to the following model of the economy:

\[
\pi_t = aR_t + z_t + e_t,
\]

where $a = \alpha + \varepsilon_t$, $E(z_t) = z + \varepsilon_{zt}$, and $\varepsilon_{at}$ and $\varepsilon_{zt}$ are random measurement errors of $\alpha$ and $z$, respectively; $\varepsilon_{at}$, $\varepsilon_{zt}$, and $e_t$ are mutually and independently distributed random variables with zero means and variances $\sigma^2_z$, $\sigma^2_{a}$, and $\sigma^2_e$.

(a) What is the optimal monetary policy

(i) in the absence of measurement errors,
(ii) in the presence of measurement errors?

(b) What are the broader implications of these results for monetary policy?
(6) A highly stylized model of an open economy is
\[ p_t = \alpha p_{t-1} + \theta (s_t - p_t), \]
\[ s_t = R_t + R_{t+1}, \]
where \( p_t \) is the price level, \( s_t \) is the exchange rate, and \( R_t \) is the nominal interest rate. Suppose that monetary policy aims to choose \( R_t \) and \( R_{t+1} \) to minimize
\[ L = (p_t - p^*)^2 + \beta (p_{t+1} - p^*)^2 + \gamma (R_t - R^*)^2, \]
where \( p_{t-1} = R_{t+2} = 0. \)

(a) Find the time consistent solutions for \( R_t \) and \( R_{t+1} \). (Hint: first find \( R_{t+1} \) taking \( p_t \) and \( R_t \) as given.)
(b) Find the optimal solution by optimizing simultaneously with respect to \( R_t \) and \( R_{t+1} \).
(c) Compare the two solutions and the significance of \( \gamma \).

(7) Consider the following model of Broadbent and Barro (1997):
\[ y_t = \alpha (p_t - E_{t-1} p_t) + \epsilon_t, \]
\[ d_t = -\beta r_t + \delta_t, \]
\[ m_t = y_t + p_t - \lambda R_t, \]
\[ r_t = R_t - E_t \Delta p_{t+1}, \]
\[ y_t = d_t, \]
where \( \epsilon_t \) and \( \delta_t \) are zero-mean mutually and serially correlated shocks.

(a) Derive the solution to the model
(i) under money supply targeting,
(ii) inflation targeting.
(b) Derive the optimal money supply rule if monetary policy minimizes
\[ E_t \sum_{s=0}^{\infty} (p_{t+s} - E_{t+s-1} p_{t+s})^2 \]
subject to the model of the economy.
(c) What does this policy imply for inflation and the nominal interest rate?
(d) Derive the optimal interest rate rule and compare the behavior of inflation under the two types of monetary policy.
(e) How would these optimal policies differ if monetary policy was based on targeting inflation instead of the price level?
(8) Consider the following model of an open economy:

\[\pi_t = \mu + \beta E_{t+1} \pi_{t+1} + \gamma x_t + e_{\pi t},\]

\[x_t = -\alpha (R_t - E_t \pi_{t+1} - \theta) + \phi (s_t + p^*_t - p_t) + e_{xt},\]

\[\Delta s_{t+1} = R_t - R^*_t + e_{st},\]

where \(e_{\pi t}, e_{xt},\) and \(e_{st}\) are zero-mean mutually and serially independent shocks to inflation, output and the exchange rate.

(a) Derive the solution to the model.

(b) What is the optimal monetary policy if the monetary authority is a strict inflation targeter with the objective of minimizing

\[E_t \sum_{i=0}^{\infty} \beta^i (\pi_{t+i} - \pi^*_t)^2?\]

(c) How would optimal monetary policy respond to an anticipated future increase in the world interest rate \(R^*_t?)?
(1) Consider a variant on the basic real-business-cycle model. The economy is assumed to maximize
\[
E_t \sum_{s=0}^{\infty} \beta^s \frac{c_t^{1-\sigma}}{1-\sigma}
\]
subject to
\[
\begin{align*}
\gamma_t &= c_t + i_t, \\
\gamma_t &= A_t k_t^\alpha, \\
\Delta k_{t+1} &= i_t - \delta k_t, \\
\ln A_t &= \rho \ln A_{t-1} + \varepsilon_t,
\end{align*}
\]
where \(\gamma_t\) is output, \(c_t\) is consumption, \(i_t\) is investment, \(k_t\) is the capital stock, \(A_t\) is technical progress, and \(\varepsilon_t \sim \text{i.i.d.}(0, \omega^2)\).

(a) Derive the optimal solution.

(b) Log-linearize this solution about its steady state and show that the resulting model can be expressed as a first-order VAR in \(\ln c_t - \ln \bar{c}\) and \(\ln k_t - \ln \bar{k}\), where \(\ln \bar{c}\) and \(\ln \bar{k}\) are the steady-state values of \(\ln c_t\) and \(\ln k_t\).

(c) If, in practice, output, consumption, and capital are nonstationary \(I(1)\) variables:

(i) Comment on why this model is not a useful specification.

(ii) Suggest a simple respecification of the model which would improve its usefulness.

(d) In practice, we find that output, consumption, and capital also have independent sources of random variation.

(i) Why is this not compatible with this model?

(ii) Suggest possible ways in which the model might be respecified to achieve this.
(2) Consider the real-business-cycle model defined in terms of the same variables as in question (1) with the addition of employment, \( n_t \):

\[
U_t = E_t \sum_{s=0}^{\infty} \beta^s \left( c_{t+s} - \gamma y_{t+s} \right)^{1-\sigma} - 1 \left( 1 - \sigma \right),
\]

\[
y_t = c_t + i_t,
\]

\[
y_t = A_t k_t^n n_t^{1-\alpha},
\]

\[
\Delta k_{t+1} = i_t - \delta k_t,
\]

\[
\ln A_t = \rho \ln A_{t-1} + \epsilon_t,
\]

where \( e_t \sim i.i.d. (0, \omega^2) \).

(a) Derive the optimal solution.

(b) Log-linearize this solution about its steady state and express the resulting model as a VAR.

(c) What is the implied dynamic behavior of the real wage and the real interest rate?

(3) Consider two countries which produce an identical good and have identical preferences and technology. In the home country households in each country maximize

\[
\mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \left( c_{t+s} - \gamma y_{t+s} \right)^{1-\sigma} - 1 \left( 1 - \sigma \right),
\]

subject to their budget constraint

\[
c_t + \Delta B_{t+1} = d_t + w_t n_t + r B_t,
\]

where \( c \) is consumption and \( l \) is leisure, \( n = 1 - l \) is work, \( w \) is the wage rate, \( d \) is dividend earnings, and \( B \) is a one-period bond paying interest \( r \). Firms maximize the present-value of the firm

\[
\mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \left( c_{t+s} y_{t+s} \right)^{1-\sigma} - 1 \left( 1 - \sigma \right),
\]

where net revenues are distributed as dividends so that \( d_t = y_t - i_t - w_t n_t \) and \( y \) is output, \( i \) is investment, and capital \( k \) satisfies \( \Delta k_{t+1} = i_t - \delta k_t \). The production function is \( y_t = A_t k_t^n n_t^{1-\alpha} \), where \( A_t \) is technical progress which satisfies \( \ln A_t = \rho \ln A_{t-1} + \epsilon_t \), where \( \epsilon_t \) is a zero-mean i.i.d. variable. Government spends \( g \) and pays for this with taxes on output so that its budget constraint is \( g_t = \tau_t y_t \). The foreign country is the same but its variables are denoted with an asterisk.

(a) Derive the optimal solution assuming that each country is small and takes the other as given.

(b) Show how domestic technology shocks affect the behavior of the solution.
(c) In the world economy the sum of domestic and foreign bonds is zero. Hence

(i) derive the balance of payments for each country assuming interest parity;

(ii) analyze the effects of foreign technology shocks on the domestic economy.
References


