Contents

A Preliminaries 1

1 Introduction for teachers 3
- Purpose and intended audience, 3
- Topics in the book, 6
- Why pluralism?, 13
- Feedback, 18
- Acknowledgments, 19

2 Introduction for students 20
- Who should study logic?, 20
- Formalism and certification, 25
- Language and levels, 34
- Semantics and syntactics, 39
- Historical perspective, 49
- Pluralism, 57
- Jarden’s example (optional), 63

3 Informal set theory 65
- Sets and their members, 68
- Russell’s paradox, 77
- Subsets, 79
- Functions, 84
- The Axiom of Choice (optional), 92
- Operations on sets, 94
- Venn diagrams, 102
- Syllogisms (optional), 111
- Infinite sets (postponable), 116

4 Topologies and interiors (postponable) 126
- Topologies, 127
- Interiors, 133
- Generated topologies and finite topologies (optional), 139

5 English and informal classical logic 146
- Language and bias, 146
- Parts of speech, 150
- Semantic values, 151
- Disjunction (or), 152
- Conjunction (and), 155
- Negation (not), 156
- Material implication, 161
- Cotenability, fusion, and constants
Contents

(postponable), 170  •  Methods of proof, 174  •  Working backwards, 177  •  Quantifiers, 183  •  Induction, 195  •  Induction examples (optional), 199

6  Definition of a formal language 206
  •  The alphabet, 206  •  The grammar, 210  •  Removing parentheses, 215  •  Defined symbols, 219  •  Prefix notation (optional), 220  •  Variable sharing, 221  •  Formula schemes, 222  •  Order preserving or reversing subformulas (postponable), 228

B  Semantics 233

7  Definitions for semantics 235
  •  Interpretations, 235  •  Functional interpretations, 237  •  Tautology and truth preservation, 240

8  Numerically valued interpretations 245
  •  The two-valued interpretation, 245  •  Fuzzy interpretations, 251  •  Two integer-valued interpretations, 258  •  More about comparative logic, 262  •  More about Sugihara’s interpretation, 263

9  Set-valued interpretations 269
  •  Powerset interpretations, 269  •  Hexagon interpretation (optional), 272  •  The crystal interpretation, 273  •  Church’s diamond (optional), 277

10 Topological semantics (postponable) 281
  •  Topological interpretations, 281  •  Examples, 282  •  Common tautologies, 285  •  Nonredundancy of symbols, 286  •  Variable sharing, 289  •  Adequacy of finite topologies (optional), 290  •  Disjunction property (optional), 293
11 More advanced topics in semantics  295
  • Common tautologies, 295  • Images of interpretations, 301  • Dugundji formulas, 307

C  Basic syntactics  311

12 Inference systems  313

13 Basic implication  318
  • Assumptions of basic implication, 319  • A few easy derivations, 320  • Lemmaless expansions, 326  • Detachmental corollaries, 330  • Iterated implication (postponable), 332

14 Basic logic  336
  • Further assumptions, 336  • Basic positive logic, 339
  • Basic negation, 341  • Substitution principles, 343

D  One-formula extensions  349

15 Contraction  351
  • Weak contraction, 351  • Contraction, 355

16 Expansion and positive paradox  357
  • Expansion and mingle, 357  • Positive paradox (strong expansion), 359  • Further consequences of positive paradox, 362

17 Explosion  365

18 Fusion  369

19 Not-elimination  372
  • Not-elimination and contrapositives, 372  • Interchangeability results, 373  • Miscellaneous consequences of not-elimination, 375
20 Relativity

E  Soundness and major logics  381

21 Soundness  383

22 Constructive axioms: avoiding not-elimination  385
  • Constructive implication, 386  • Herbrand-Tarski Deduction Principle, 387  • Basic logic revisited, 393  • Soundness, 397  • Nonconstructive axioms and classical logic, 399  • Glivenko’s Principle, 402

23 Relevant axioms: avoiding expansion  405
  • Some syntactic results, 405  • Relevant deduction principle (optional), 407  • Soundness, 408  • Mingle: slightly irrelevant, 411  • Positive paradox and classical logic, 415

24 Fuzzy axioms: avoiding contraction  417
  • Axioms, 417  • Meredith’s chain proof, 419  • Additional notations, 421  • Wajsberg logic, 422  • Deduction principle for Wajsberg logic, 426

25 Classical logic  430
  • Axioms, 430  • Soundness results, 431  • Independence of axioms, 431

26 Abelian logic  437

F  Advanced results  441

27 Harrop’s principle for constructive logic  443
  • Meyer’s valuation, 443  • Harrop’s principle, 448  • The disjunction property, 451  • Admissibility, 451  • Results in other logics, 452
Contents

28 Multiple worlds for implications 454
  • Multiple worlds, 454  • Implication models, 458  •
  Soundness, 460  • Canonical models, 461  • Completeness, 464

29 Completeness via maximality 466
  • Maximal unproving sets, 466  • Classical logic, 470
  • Wajsberg logic, 477  • Constructive logic, 479  •
  Non-finitely-axiomatizable logics, 485

References 487

Symbol list 493

Index 495