Errata for *Creating Symmetry: The Artful Mathematics of Wallpaper Patterns*

Page maintained by Frank A. Farris, with thanks to James S. Walker, University of Wisconsin, Eau Claire. Page reference notation: m^n means page m, line n from the top, and m_n means page m, line n from the bottom. Find any new errors? Have questions? Please email Frank Farris using ffarris at scu dot edu .

1_{13}: upper-left quadrant *should read* upper-right quadrant

9_1: formulas (2.3) *should read* formulas (2.2)

12^6: modulo m *should read* modulo 5

13^{11}: $e^{2k\pi/m}$ *should read* $e^{2k\pi i/m}$

13_7: $e^{4\pi/6}$ *should read* $e^{4\pi i/6}$

14^{12}: $e^{2\pi k/m}$ *should read* $e^{2\pi ki/m}$

27_4: $k = -1$ *should read* $j = -1$

27_2: k ranging *should read* j ranging

32_4: $\dfrac{f(t)-f(s)}{2\pi\sin\big((t-s)/2\big)} \le M$ *should read* $\left|\dfrac{f(t)-f(s)}{2\pi\sin\big((t-s)/2\big)}\right| \le M$

33^3: The reasoning used in this estimate is incorrect. A section at the end of this document explains the details and offers a correction, with thanks to Walker.

33^{11}: $a_n = \int_0^{2\pi} f(t)e^{int}\,dt$ *should read* $a_n = \dfrac{1}{2\pi}\int_0^{2\pi} f(t)e^{-int}\,dt$

35_{17}: $\sum_{n<0}^{\infty} a_n \overline{z}^n$ *should read* $\sum_{n>0}^{\infty} a_{-n} \overline{z}^n$

39_7: upper quarter of the picture *should read* upper quarter of the bottom picture

40_7: preserves *should read* preserve

58^8: $n+m<0$ *should read* $n+m>0$

64_9: $f(x+1,y) = f(x,y+1)$ *should read* $f(x,y) = f(x+1,y) = f(x,y+1)$

78^2: $e^{-2\pi/3}$ *should read* $e^{-2\pi i/3}$ (Note: This typo occurs in two places.)

78^2: $e^{2\pi/6}$ *should read* $e^{2\pi i/6}$

85_{16}: $\tau^{-1}\sigma_c$ *should read* $\tau\sigma_c$

95_6: $2(z+\overline{z})$ *should read* $\frac{1}{2}(z+\overline{z})$

109_5: cell *should read* cell.

129_{11}: side mirror *should read* slant mirror

132_1: $\widehat{E}_{n,m}$. *should read* $E_{n,m}$.

138^1: walk through count *should read* walk through the count

141: role of the the *should read* role of the

151_{14}: $a(\overline{s}\,\overline{z})$ *should read* $a(s\overline{z})$

151_{10}: by $\overline{s}$ *should read* by s

Comment: If we follow the directions in the text (repeating wave-by-wave analysis, etc.), then we calculate as follows:

$$\begin{aligned}\sum_{\nu} a_\nu E_{s\overline{\nu}}(z) &= \sum_{\omega} a_{s\overline{\omega}} E_\omega(z) \\ &= \sum_{\nu} a_{s\overline{\nu}} E_\nu(z)\end{aligned}$$

where the first equation is using $\nu = s\overline{\omega}$ and invariance of the lattice under conjugation and multiplication by s (which is what is actually stated in part 2 of Proposition 4). Therefore, we have

$$[g]a(v) = a(s\overline{\nu})e^{2\pi i \mathrm{Re}(t\overline{v})}.$$

152_2: $a(-i\overline{v}_{n,m})$ *should read* $a(i\overline{v}_{n,m})$

Comment: The typo in the last item can cause the reader aggravation, for which the author apologizes. For $s = i$, and $\omega = i$ (assuming a square lattice for the p4g group) we calculate as follows:

$$\begin{aligned}i\overline{v_{n,m}} &= i\,\frac{-i}{\mathrm{Im}(\omega)}\,(ni+m) \\ &= \frac{i}{\mathrm{Im}(\omega)}\,(-mi+n) \\ &= v_{m,n}\end{aligned}$$

166: In the answer for Exercise 56: $\dfrac{5+\overline{\omega}_3}{5+\omega_3}\,\overline{z}$ *should read* $\dfrac{5+\omega_3}{5+\overline{\omega}_3}\,\overline{z}$

184^{18}: The destroys *should read* This destroys

199^{13}: $p = 3$ this *is* *should read* $p = 6$ this *is*

Remark: In this last item, it is the case that $p = 3$ yields $z \to z - 1$, which generates all the integer translates. Nevertheless, to obtain $z \to z+1$, which is the unit translation (as was observed earlier in the paragraph) one needs to use $p = 6$.

Correcting the convergence estimate

The reasoning used to obtain

$$|f(t) - f_N(t)| \leq M \left| \int_0^{2\pi} \sin\bigl((N + \tfrac{1}{2})(t - s)\bigr)\, ds \right|$$

is invalid. It must be, because ***the inequality is false*** for some functions. Notice that it implies

$$|f(t) - f_N(t)| \leq \frac{2M|\cos\bigl((N + \frac{1}{2})t\bigr)|}{N + \frac{1}{2}}.$$

The right hand side has zeros at $\frac{\pi/2}{N+1/2}$, $\frac{3\pi/2}{N+1/2}$, etc. But, if we take $f(t) = e^{i(N+1)t}$ we have for the left hand side: $|f(t) - f_N(t)| = |e^{i(N+1)t}| = 1$, which has no zeros. That is a contradiction. Despite the incorrect reasoning, the quantity estimated can be shown to approach 0 as N grows without bound. In Weinberg, cited in the book, this is done using the Reimann-Lebesgue Lemma, which the reader is invited to investigate. Walker offers a direct way to repair the difficulty:

Remark The end of the proof of **Theorem 3** can be repaired in the following way. Choose $\epsilon > 0$, a very small positive quantity. Let $\delta > 0$ be a small positive number, less than π, to be specified later. Now, for each such δ, we have

$$\begin{aligned}
|f(t) - f_N(t)| &= \left| \int_{-\pi}^{-\delta} (f(t) - f(s)) D_N(t-s)\, ds + \int_{-\delta}^{\delta} (f(t) - f(s)) D_N(t-s)\, ds \right. \\
&\quad \left. + \int_{\delta}^{\pi} (f(t) - f(s)) D_N(t-s)\, ds \right| \\
&\leq \left| \int_{-\pi}^{-\delta} \frac{(f(t) - f(s)}{2\pi \sin\bigl((t-s)/2\bigr)} \sin\bigl((N + \tfrac{1}{2})(t-s))\bigr)\, ds \right| + \int_{-\delta}^{\delta} \left| \frac{f(t) - f(s)}{2\pi \sin\bigl((t-s)/2\bigr)} \right| ds \\
&\quad + \left| \int_{\delta}^{\pi} \frac{(f(t) - f(s)}{2\pi \sin\bigl((t-s)/2\bigr)} \sin\bigl((N + \tfrac{1}{2})(t-s))\bigr)\, ds \right|.
\end{aligned}$$

Now, the middle terms on the right side satisfies

$$\int_{-\delta}^{\delta} \left| \frac{f(t) - f(s)}{2\pi \sin\bigl((t-s)/2\bigr)} \right| ds \leq 2M\delta < \frac{\epsilon}{3}$$

by choosing δ sufficiently close to zero. Once such a δ is chosen, then integration by parts shows that the other two terms are both equal to multiples of $1/(N + \frac{1}{2})$. Hence, we can choose N sufficiently large that each of them is no larger than $\epsilon/3$. Thus,

$$|f(t) - f_N(t)| < \frac{\epsilon}{3} + \frac{\epsilon}{3} + \frac{\epsilon}{3} = \epsilon$$

for N sufficiently large. Since ϵ can be taken arbitrarily close to 0 that proves $\lim_{N\to\infty} f_N(t) = f(t)$, and we are done.

The interested reader can learn more from the cited book by Weinberger, from Tolstov[1] or from Walker[2]. These references prove convergence for the stronger case of continuous functions with piecewise continuous derivatives, as needed for the polygonal examples in the exercises.

[1]G. Tolstov, *Fourier Series,* Dover, 1965.
[2]J.S. Walker, *Fourier Analysis,* Oxford, 1988.