Chapter 18

Estimating the Cost of International Capital
Overview

Translate first, then discount—or vv?

Two procedures
When to do what?

The Single-Country CAPM
From Asset Returns to Portfolio Return
The tangency solution
How the weights affect mean and variance
How to make a portfolio efficient
The Market Portfolio as the Benchmark

The International CAPM
Why do we need an InCAPM?
Why Xrisk pops up in the InCAPM
Do assets have a financial nationality?
Aggregating the Efficiency Conditions
The InCAPM

Wrapping up
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Suppose an Australian firm considers an investment in India. Issues:

► Can we assess NPV in INR, as locals would do?
  - would look simpler — no?
  - ... but works only if AU and IN are both (part of) an integrated market

► How does a multi-country market work, where investors “think in different currencies” depending on where they live?
  - risks and returns depend on the currency they are measured in,
  - ... which violates the ”homogenous expectations” assumption of standard CAPM
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Wrapping up
Two facts need reconciliation:

◊ **CF’s are probably first computed in FC (INR):**
  - cost data in INR; local sales “priced to market”
  - all these prices are probably sticky in terms of INR, not AUD

◊ **Yet commonly used CoCa is in HC (AUD)**

So

– either we translate the cashflows into AUD
– or we shift to an INR CoCa
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◊ “When in Rome, act like the Romans”?
  ▶ set CoCA on basis of INR risk-free rate, and add risk premium computed from INR stock returns (risk, price of risk)
  ▶ compute FCPV
  ▶ if desired, convert PV into HC

... or ...

◊ “My shareholders consume/think in AUD”?
  ▶ translate expected CFs into AUD, including the covariance
  ▶ set CoCA on basis of AUD risk-free rate, and add risk premium computed from AUD stock returns (risk, price of risk)
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▷ compute PV in HC
When to do what?

Does the translation/discounting procedure matter?

- **In practice**: the “local” version looks easier (but only if we cut corners, as we’ll see)
- **In principle**: no if mkts are integrated, yes if mkts aren’t
  - Australians care about the value to them, not about how Indians would value the project. The two are not the same in segmented markets.
  - We can ”see” the Australian’s required return in their own capital market. So we value “à l’Australienne”

Can there be a CAPM-type no-PPP equilibrium?

- presence of (real) exchange risk means expectations cannot be homogenous: (real) expected returns and risks for asset $j$ differ across investors
- But this special type of investors heterogeneity is easily incorporated (InCAPM instead of CAPM)
When to do what?

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When to do what? Overview

<table>
<thead>
<tr>
<th>CoCa model</th>
<th>currency of calculations</th>
</tr>
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<tbody>
<tr>
<td><strong>1. Foreign investments:</strong></td>
<td></td>
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<tr>
<td>• home and host financially integrated</td>
<td>inCAPM</td>
</tr>
<tr>
<td>• home and host financially segmented</td>
<td>inCAPM</td>
</tr>
<tr>
<td>• home country part of larger financial market</td>
<td></td>
</tr>
<tr>
<td>• home country totally isolated</td>
<td>CAPM</td>
</tr>
<tr>
<td><strong>2. domestic investments</strong></td>
<td></td>
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The Cost of International Capital

P. Sercu, *International Finance: Theory into Practice*

**Outline**

- Translate first, then discount—or vv?
  - Two procedures
  - When to do what?

**The Single-Country CAPM**

- From Asset Returns to Portfolio Return
- The tangency solution
- How the weights affect mean and variance
- How to make a portfolio efficient
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**The International CAPM**

- Why do we need an InCAPM?
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Wrapping up
### From Asset Returns to Portfolio Return

Key relation—stated in excess return terms, here—between portfolio return \( \tilde{r}_p \) and asset returns \( \tilde{r}_j \) & weights \( x_j \):

\[
\tilde{r}_p - r = \sum_{j=1}^{N} x_j (\tilde{r}_j - r)
\]

#### Example

<table>
<thead>
<tr>
<th>( j )</th>
<th>( V_{j,0} )</th>
<th>( n_j )</th>
<th>( n_j V_{j,0} )</th>
<th>( x_j )</th>
<th>( V_{j,1} )</th>
<th>( n_j V_{j,1} )</th>
<th>( r_j )</th>
<th>( r_j - r )</th>
<th>( x_j (r_j - r) )</th>
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<tbody>
<tr>
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<td>0.40</td>
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<tr>
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<td></td>
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<td>+100</td>
<td>+0.10</td>
<td></td>
<td>105</td>
<td></td>
<td>+0.050</td>
<td></td>
<td>=0.105</td>
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<td></td>
<td></td>
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From Asset Returns to Portfolio Return

Key relation—stated in excess return terms, here—between portfolio return $\tilde{r}_p$ and asset returns $\tilde{r}_j$ & weights $x_j$:

$$\tilde{r}_p - r = \sum_{j=1}^{N} x_j(\tilde{r}_j - r)$$

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<tr>
<th>j</th>
<th>risky:</th>
<th>time–0 data and decisions</th>
<th>time–1 result</th>
<th>(excess) rates of return</th>
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<tbody>
<tr>
<td>j</td>
<td>$v_{j,0}$</td>
<td>$n_j$</td>
<td>$n_jv_{j,0}$</td>
<td>$x_j$</td>
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Two assets—one risk-free, one risky

\[ \tilde{r}_p = x\tilde{r}_s + (1 - x)r_0 = r_0 + x(\tilde{r}_s - r_0) \]

\[ \Rightarrow \begin{cases} 
E(\tilde{r}_p) = r_0 + xE(\tilde{r}_s - r_0), \\
sd(\tilde{r}_p) = |x|sd(\tilde{r}_s) 
\end{cases} \]
Two assets—one risk-free, one risky

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\end{cases} \]
Two imperfectly correlated risky assets

\[
\tilde{r}_p = x_1 \tilde{r}_1 + (1 - x_1) \tilde{r}_2; \Rightarrow
\]

\[
\begin{cases}
E(\tilde{r}_p) = E(\tilde{r}_2) + x [E(\tilde{r}_1) - E(\tilde{r}_2)], \\
sd(\tilde{r}_p) = \sqrt{x_1^2 \text{var}(\tilde{r}_1) + 2x_1(1 - x_1) \text{cov}(\tilde{r}_1, \tilde{r}_2) + (1 - x_1)^2 \text{var}(\tilde{r}_2)}
\end{cases}
\]
Many risky assets and one risk-free

- many risky assets:
  - bound is similar, but most portfolios are now inside the bound
  - only portfolios on the upper half of the bound are efficient

- add a risk-free asset
  - best risky portfolio is the tangency one:

![Graph showing the relationship between expected return and standard deviation for many risky assets and one risk-free asset.](image)
Many risky assets and one risk-free

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  – bound is similar, but most portfolios are now inside the bound
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  – best risky portfolio is the tangency one:
How weights affect mean and variance (1)

From $\tilde{r}_p = r + \sum_{j=1}^{N} x_j (\tilde{r}_j - r)$:

$$E(\tilde{r}_p) = r + \sum_{j=1}^{N} x_j E(\tilde{r}_j - r),$$

$$\text{var}(\tilde{r}_p) = \sum_{j=1}^{N} x_j \sum_{k=1}^{N} x_k \text{cov}(\tilde{r}_j, \tilde{r}_k).$$

Understanding the variance formula:

- portfolio variance is a weighted average of each asset’s covariance with the entire portfolio:

$$\text{cov}(\tilde{r}_p, \tilde{r}_p) = \text{cov}(\sum_{k=1}^{N} x_k \tilde{r}_k, \tilde{r}_p) = \sum_{k=1}^{N} x_k \text{cov}(\tilde{r}_k, \tilde{r}_p).$$

- each of these assets’ portfolio covariances is, in turn, a weighted average of the asset’s covariance with all components of the portfolio:

$$\text{cov}(\tilde{r}_j, \tilde{r}_p) = \text{cov}(\tilde{r}_j, \sum_{k=1}^{N} x_k \tilde{r}_k) = \sum_{k=1}^{N} x_k \text{cov}(\tilde{r}_j, \tilde{r}_k).$$
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$$
\begin{align*}
E(\tilde{r}_p) &= r + \sum_{j=1}^{N} x_j E(\tilde{r}_j - r), \\
\text{var}(\tilde{r}_p) &= \sum_{j=1}^{N} x_j \sum_{k=1}^{N} x_k \text{cov}(\tilde{r}_j, \tilde{r}_k).
\end{align*}
$$

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How weights affect mean and variance (2)

Example:

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<tr>
<th></th>
<th>$E(\tilde{r}_j - r)$</th>
<th>$\text{cov}(\tilde{r}_j, \tilde{r}_1)$</th>
<th>$\text{cov}(\tilde{r}_j, \tilde{r}_2)$</th>
<th>$x_j$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.200</td>
<td>0.16</td>
<td>0.05</td>
<td>0.50</td>
</tr>
<tr>
<td>2</td>
<td>0.122</td>
<td>0.05</td>
<td>0.09</td>
<td>0.40</td>
</tr>
</tbody>
</table>

\[
E(\tilde{r}_p - r) = 0.50 \times 0.200 + 0.40 \times 0.122 = 0.1488
\]

\[
\text{cov}(\tilde{r}_1, \tilde{r}_p) = 0.50 \times 0.160 + 0.40 \times 0.050 = 0.1000
\]

\[
\text{cov}(\tilde{r}_2, \tilde{r}_p) = 0.50 \times 0.050 + 0.40 \times 0.090 = 0.0610
\]

\[
\Rightarrow \text{cov}(\tilde{r}_p, \tilde{r}_p) = 0.50 \times 0.100 + 0.40 \times 0.061 = 0.0744
\]
Marginal effect of $x_j$ on mean and variance

Three-asset example:

- Asset 1’s marginal contribution to portfolio expected excess return is its own expected excess return:

$$E(\tilde{r}_p - r) = x_1 E(\tilde{r}_1 - r) + x_2 E(\tilde{r}_2 - r);$$

$$\Rightarrow \frac{\partial E(\tilde{r}_p - r)}{\partial x_1} = E(\tilde{r}_1 - r).$$

- Asset 1’s marginal contribution to portfolio variance is (twice) its covariance with the portfolio return:

$$\text{var}(\tilde{r}_p) = x_1^2 \text{var}(\tilde{r}_1) + 2x_1x_2 \text{cov}(\tilde{r}_1, \tilde{r}_2) + x_2^2 \text{var}(\tilde{r}_2);$$

$$\Rightarrow \frac{\partial \text{var}(\tilde{r}_p)}{\partial x_1} = 2\text{cov}(\tilde{r}_1, \tilde{r}_p).$$

Proof:

$$\frac{\partial \text{var}(\tilde{r}_p)}{\partial x_1} = 2x_1 \text{var}(\tilde{r}_1) + 2x_2 \text{cov}(\tilde{r}_1, \tilde{r}_2),$$

$$= 2[x_1 \text{cov}(\tilde{r}_1, \tilde{r}_1) + x_2 \text{cov}(\tilde{r}_1, \tilde{r}_2)],$$

$$= 2[\text{cov}(\tilde{r}_1, \tilde{r}_1) + \text{cov}(\tilde{r}_1, x_2 \tilde{r}_2)],$$

$$= 2\text{cov}(\tilde{r}_1, x_1 \tilde{r}_1 + x_2 \tilde{r}_2).$$
Marginal effect of $x_j$ on mean and variance

Three-asset example:

- Asset 1’s marginal contribution to portfolio expected excess return is its own expected excess return:

$$
\mathbb{E}(\tilde{r}_p - r) = x_1 \mathbb{E}(\tilde{r}_1 - r) + x_2 \mathbb{E}(\tilde{r}_2 - r);
$$

$$\Rightarrow \frac{\partial \mathbb{E}(\tilde{r}_p - r)}{\partial x_1} = \mathbb{E}(\tilde{r}_1 - r).
$$

- Asset 1’s marginal contribution to portfolio variance is (twice) its covariance with the portfolio return:

$$
\text{var}(\tilde{r}_p) = x_1^2 \text{var}(\tilde{r}_1) + 2x_1x_2 \text{cov}(\tilde{r}_1, \tilde{r}_2) + x_2^2 \text{var}(\tilde{r}_2);
$$

$$\Rightarrow \frac{\partial \text{var}(\tilde{r}_p)}{\partial x_1} = 2 \text{cov}(\tilde{r}_1, \tilde{r}_p).
$$

Proof: 
$$
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$$

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$$

$$= 2[\text{cov}(\tilde{r}_1, x_1 \tilde{r}_1) + \text{cov}(\tilde{r}_1, x_2 \tilde{r}_2)],
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- Asset 1’s marginal contribution to portfolio expected excess return is its own expected excess return:

$$E( \tilde{r}_p - r) = x_1 E( \tilde{r}_1 - r) + x_2 E( \tilde{r}_2 - r);$$

$$\Rightarrow \frac{\partial E( \tilde{r}_p - r)}{\partial x_1} = E( \tilde{r}_1 - r).$$

- Asset 1’s marginal contribution to portfolio variance is (twice) its covariance with the portfolio return:

$$\text{var}( \tilde{r}_p) = x_1^2 \text{var}( \tilde{r}_1) + 2x_1x_2 \text{cov}( \tilde{r}_1, \tilde{r}_2) + x_2^2 \text{var}( \tilde{r}_2);$$

$$\Rightarrow \frac{\partial \text{var}( \tilde{r}_p)}{\partial x_1} = 2 \text{cov}( \tilde{r}_1, \tilde{r}_p).$$

Proof: $$\frac{\partial \text{var}( \tilde{r}_p)}{\partial x_1} = 2x_1 \text{var}( \tilde{r}_1) + 2x_2 \text{cov}( \tilde{r}_1, \tilde{r}_2),$$

$$= 2[ x_1 \text{cov}( \tilde{r}_1, \tilde{r}_1) + x_2 \text{cov}( \tilde{r}_1, \tilde{r}_2)],$$

$$= 2[ \text{cov}( \tilde{r}_1, x_1 \tilde{r}_1) + \text{cov}( \tilde{r}_1, x_2 \tilde{r}_2)],$$

$$= 2\text{cov}( \tilde{r}_1, x_1 \tilde{r}_1 + x_2 \tilde{r}_2).$$
**Effect of $w$ on $var(\tilde{r}_p)$: example**

### Example

<table>
<thead>
<tr>
<th></th>
<th>$E(\tilde{r}_j - r)$</th>
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\[
\Delta \text{var} = \frac{.037524 - .036800}{.01} = \frac{.000724}{.01} = 0.0724 \approx 2 \times \text{cov}(\tilde{r}_1, \tilde{r}_p).
\]
Effect of \( w \) on \( \text{var}(\tilde{r}_p) \): example

### Example

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**portfolio 1:** \( x_1 = .40, x_2 = .40, x_0 = .20 \)

\[
\begin{align*}
\text{cov}(\tilde{r}_1, \tilde{r}_p) & = 0. \ldots \times 0. \ldots + 0. \ldots \times 0. \ldots = 0.036000, \\
\text{cov}(\tilde{r}_2, \tilde{r}_p) & = 0. \ldots \times 0. \ldots + 0. \ldots \times 0. \ldots = 0.056000, \\
\text{cov}(\tilde{r}_p, \tilde{r}_p) & = 0. \ldots \times 0. \ldots + 0. \ldots \times 0. \ldots = 0.036800,
\end{align*}
\]

**portfolio 2:** \( x_1 = .41, x_2 = .40, x_0 = .19 \)

\[
\begin{align*}
\text{cov}(\tilde{r}_1, \tilde{r}_p) & = 0. \ldots \times 0. \ldots + 0. \ldots \times 0. \ldots = 0.036400, \\
\text{cov}(\tilde{r}_2, \tilde{r}_p) & = 0. \ldots \times 0. \ldots + 0. \ldots \times 0. \ldots = 0.056500, \\
\text{cov}(\tilde{r}_p, \tilde{r}_p) & = 0. \ldots \times 0. \ldots + 0. \ldots \times 0. \ldots = 0.037524,
\end{align*}
\]

\[
\frac{\Delta \text{var}}{\Delta x_1} = \frac{0.037524 - 0.036800}{0.01} = \frac{0.000724}{0.01} = 0.0724 \approx 2 \times \text{cov}(\tilde{r}_1, \tilde{r}_p).
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Effect of $w$ on $\text{var}(\tilde{r}_p)$: example

**Example**

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**portfolio 1:** $x_1 = 0.40, x_2 = 0.40, x_0 = 0.20$

\[
\text{cov}(\tilde{r}_1, \tilde{r}_p) = 0.\ldots \times 0.\ldots + 0.\ldots \times 0.\ldots = 0.036 \, 000,
\]

\[
\text{cov}(\tilde{r}_2, \tilde{r}_p) = 0.\ldots \times 0.\ldots + 0.\ldots \times 0.\ldots = 0.056 \, 000,
\]

\[
\text{cov}(\tilde{r}_p, \tilde{r}_p) = 0.\ldots \times 0.\ldots + 0.\ldots \times 0.\ldots = 0.036 \, 800,
\]

**portfolio 2:** $x_1 = 0.41, x_2 = 0.40, x_0 = 0.19$

\[
\text{cov}(\tilde{r}_1, \tilde{r}_p) = 0.\ldots \times 0.\ldots + 0.\ldots \times 0.\ldots = 0.036 \, 400,
\]

\[
\text{cov}(\tilde{r}_2, \tilde{r}_p) = 0.\ldots \times 0.\ldots + 0.\ldots \times 0.\ldots = 0.056 \, 500,
\]

\[
\text{cov}(\tilde{r}_p, \tilde{r}_p) = 0.\ldots \times 0.\ldots + 0.\ldots \times 0.\ldots = 0.037 \, 524,
\]

\[
\frac{\Delta \text{var}}{\Delta x_1} = \frac{0.037 \, 524 - 0.036 \, 800}{0.01} = \frac{0.000 \, 724}{0.01} = 0.0724 \approx 2 \times \text{cov}(\tilde{r}_1, \tilde{r}_p).
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Effect of $w$ on $\text{var}(\tilde{r}_p)$: example

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**portfolio 1:** $x_1 = .40, x_2 = .40, x_0 = .20$

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\begin{align*}
\text{cov}(\tilde{r}_1, \tilde{r}_p) &= 0. \ldots \times 0. \ldots + 0. \ldots \times 0. \ldots = 0.036000, \\
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\end{align*}
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\text{cov}(\tilde{r}_2, \tilde{r}_p) &= 0. \ldots \times 0. \ldots + 0. \ldots \times 0. \ldots = 0.056500, \\
\text{cov}(\tilde{r}_p, \tilde{r}_p) &= 0. \ldots \times 0. \ldots + 0. \ldots \times 0. \ldots = 0.037524,
\end{align*}
\]

\[
\Delta \text{var} = \frac{0.037524 - 0.036800}{0.01} = \frac{0.000724}{0.01} = 0.0724 \approx 2 \times \text{cov}(\tilde{r}_1, \tilde{r}_p).
\]
How to make a portfolio efficient

◊ **Micro economics:** budget allocation problem has as efficiency condition that

\[
\frac{\text{marginal utility}}{\text{marginal cost}} = \text{same across all goods } j
\]

◊ **Mean-Variance:** the “good” side is not utility but expected return; and the bad side is variance. So the efficiency condition is

\[
\frac{\text{j’s contribution to } E(\tilde{r}_p)}{\text{j’s contribution to } \text{var}(\tilde{r}_p)} = \text{same across all assets } j
\]

\[
\frac{E(\tilde{r}_j - r)}{\text{cov}(\tilde{r}_j, \tilde{r}_p)} = \lambda, \text{ for all risky assets } j=1, \ldots, N
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\]
(In)Efficiency and Risk Aversion: examples

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<td>Asset 2</td>
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portfolio with weights \( x_1 = .40, x_2 = .40, x_0 = .20 \) is not efficient:

\[
\begin{align*}
\text{cov}(\tilde{r}_1, \tilde{r}_p) & = 0. \ldots \times 0. \ldots + \ldots \times 0. \ldots = 0.036 \\
\text{cov}(\tilde{r}_2, \tilde{r}_p) & = 0. \ldots \times 0. \ldots + \ldots \times 0. \ldots = 0.056 \\
\end{align*}
\]

\[
\frac{0. \ldots}{0. \ldots} = 2.555 \neq \frac{0. \ldots}{0. \ldots} = 2.643
\]

portfolio with weights \( x_1 = .40, x_2 = .60, x_0 = .00 \) is efficient:

\[
\begin{align*}
\text{cov}(\tilde{r}_1, \tilde{r}_p) & = 0. \ldots \times 0. \ldots + \ldots \times 0. \ldots = 0.046 \\
\text{cov}(\tilde{r}_2, \tilde{r}_p) & = 0. \ldots \times 0. \ldots + \ldots \times 0. \ldots = 0.074 \\
\end{align*}
\]

\[
\frac{0. \ldots}{0. \ldots} = 2 = \frac{0. \ldots}{0. \ldots}
\]

portfolio with weights \( x_1 = .20, x_2 = .30, x_0 = .50 \) is efficient:

\[
\begin{align*}
\text{cov}(\tilde{r}_1, \tilde{r}_p) & = 0. \ldots \times 0. \ldots + \ldots \times 0. \ldots = 0.023 \\
\text{cov}(\tilde{r}_2, \tilde{r}_p) & = 0. \ldots \times 0. \ldots + \ldots \times 0. \ldots = 0.037 \\
\end{align*}
\]

\[
\frac{0. \ldots}{0. \ldots} = 4 = \frac{0. \ldots}{0. \ldots}
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**portfolio with weights** $x_1 = .40, x_2 = .40, x_0 = .20$ **is not efficient:**

\[
\begin{align*}
\text{cov}(\tilde{r}_1, \tilde{r}_p) & = 0.036 \\
\text{cov}(\tilde{r}_2, \tilde{r}_p) & = 0.056,
\end{align*}
\]

\[
\frac{0.036}{0.056} = 2.555 \neq \frac{0.046}{0.074} = 2.643
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\]

\[
\frac{0.046}{0.074} = 2 = \frac{0.046}{0.074}
\]

**portfolio with weights** $x_1 = .20, x_2 = .30, x_0 = .50$ **is efficient:**

\[
\begin{align*}
\text{cov}(\tilde{r}_1, \tilde{r}_p) & = 0.023 \\
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\end{align*}
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\text{cov}(\tilde{r}_1, \tilde{r}_p) &= 0.092 \times 0.04 + 0.092 \times 0.05 = 0.036, \\
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\end{align*}
\]

\[
\frac{0.023}{0.037} = 4 = \frac{0.037}{0.037}
\]
The Market Portfolio as the Benchmark

◊ Homogeneous expectations and opportunities:

▷ Peter’s tangency portfolio is the same as Paul’s and Mary’s
▷ So everybody holds a combination of the tangency portfolio and riskfree assets
▷ So the aggregate portfolio of all investors is a combination of the tangency portfolio and riskfree assets, with a weight that reflects the average risk-aversion
▷ In a closed economy, this aggregate portfolio must be the portfolio of all existing shares (“the market portfolio”).

◊ Bottom line: the market portfolio must be efficient too; so

\[
\forall j : \frac{\mathbb{E}(\tilde{r}_j - r)}{\text{cov}(\tilde{r}_j, \tilde{r}_m)} = \lambda_m, \\
\Rightarrow \mathbb{E}(\tilde{r}_j - r) = \lambda_m \text{cov}(\tilde{r}_j, \tilde{r}_m). \quad (1)
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Identifying $\lambda_m$—the CAPM

Piece of cake. If $\forall j$: $\lambda_m = \frac{E(\tilde{r}_j - r)}{\text{cov}(\tilde{r}_j, \tilde{r}_m)}$, then for any portfolio $p$ (with weights $x_j$):

$$\lambda_m = \frac{\sum_{j=1}^{n} x_j E(\tilde{r}_j - r)}{\sum_{j=1}^{n} x_j \text{cov}(\tilde{r}_j, \tilde{r}_m)} = \frac{E(\tilde{r}_p - r)}{\text{cov}(\tilde{r}_p, \tilde{r}_m)}$$

Now pick the market portfolio as our $p$. Then

$$\lambda_m = \frac{E(\tilde{r}_m - r)}{\text{cov}(\tilde{r}_m, \tilde{r}_m)} = \frac{E(\tilde{r}_m - r)}{\text{var}(\tilde{r}_m)}.$$ 

Substitute:

$$E(\tilde{r}_j - r) = \lambda_m \text{cov}(\tilde{r}_j, \tilde{r}_m),$$

$$= \frac{E(\tilde{r}_m - r)}{\text{var}(\tilde{r}_m)} \text{cov}(\tilde{r}_j, \tilde{r}_m),$$

$$= \beta_j E(\tilde{r}_m - r),$$

with $\beta_j$ as in $\tilde{r}_j = \alpha_j + \beta_j \tilde{r}_m + \epsilon_j$, the “market model”. $\beta_j \tilde{r}_m$ is the undiversifiable risk or market risk, $\epsilon_j$ the firm-specific or idiosyncratic or diversifiable risk.
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$$

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$$E(\tilde{r}_j - r) = \lambda_m \text{cov}(\tilde{r}_j, \tilde{r}_m),$$

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Now pick the market portfolio as our $p$. Then

$$\lambda_m = \frac{E(\tilde{r}_m - r)}{\text{cov}(\tilde{r}_m, \tilde{r}_m)} = \frac{E(\tilde{r}_m - r)}{\text{var}(\tilde{r}_m)}.$$

Substitute:

$$E(\tilde{r}_j - r) = \lambda_m \text{cov}(\tilde{r}_j, \tilde{r}_m),$$

$$= \frac{E(\tilde{r}_m - r)}{\text{var}(\tilde{r}_m)} \text{cov}(\tilde{r}_j, \tilde{r}_m),$$

$$= \beta_j E(\tilde{r}_m - r),$$

with $\beta_j$ as in $\tilde{r}_j = \alpha_j + \beta_j \tilde{r}_m + \epsilon_j$, the “market model”. $\beta_j \tilde{r}_m$ is the undiversifiable risk or market risk, $\epsilon_j$ the firm-specific or idiosyncratic or diversifiable risk.
The CAPM for dummies: just using words

- The beta is a measure of the asset’s relative risk—that is, the asset’s market covariance risk $\text{cov}(\tilde{r}_j, \tilde{r}_m)$, rescaled by the portfolio’s total risk, $\text{var}(\tilde{r}_m)$.

Beta can be estimated from the market-model regression.

- A risky asset with beta equal to zero should have an expected return that is equal to the risk-free rate, even if the asset’s return is uncertain.

The reason is that the marginal contribution to the total market risk is zero.

- If an asset’s beta or relative risk is non-zero, the asset’s expected return should contain a risk premium. The additional return that can be expected per unit of beta is the market’s expected excess return above the risk-free rate.
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The CAPM for yuppies: APT interpretation

◊ You can always avoid a stock’s idiosyncratic risk:
  ▶ form j’s “shadow portfolio” \( \hat{j} \): invest a weight \( \beta_j \) in \( m \), and 
  \( (1 - \beta_j) \) riskfree
  ▶ now compare the returns on \( j \) and on \( \hat{j} \):

\[
\tilde{r}_j = \alpha_j + \beta_j \tilde{r}_m + \epsilon_j, \\
\tilde{r}_\hat{j} = (1 - \beta_j) r + \beta_j \tilde{r}_m.
\]

◊ You cannot seriously expect to be rewarded for a risk
  that you can easily avoid.
  So \( j \) and \( \hat{j} \) have the same expected return:

\[
\begin{align*}
E(\tilde{r}_j) &\equiv E(\tilde{r}_\hat{j}), \\
&= (1 - \beta_j) r + \beta_j E(\tilde{r}_m), \\
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CAPM: diversifiable risk gets no reward.
The CAPM for yuppies: APT interpretation

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How and when to use the single-country CAPM

Finding beta?
- single-asset betas are noisy; many prefer industry betas. See e.g. http://pages.stern.nyu.edu/adamodar/New_Home_Page/datafile/Betas.htm
- intervaling effect: monthly-return beta estimates are higher than daily-return ones
- correct for thin trading: Scholes-Williams-Fowler-Rorke; Dimson;
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- Take long-term estimates: 20 yrs is not enough
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Outline

The Cost of International Capital

P. Sercu, *International Finance: Theory into Practice*

Translate first, then discount—or vv?

The Single-Country CAPM

Two procedures

When to do what?

The International CAPM

From Asset Returns to Portfolio Return

The tangency solution

How the weights affect mean and variance

How to make a portfolio efficient

The Market Portfolio as the Benchmark

The International CAPM

Why do we need an InCAPM?

Why does it contain Xrisk?

Do assets have a financial nationality?

Aggregating the Efficiency Conditions

The InCAPM

Wrapping up
Why do we need an InCAPM?

◊ **Standard country-by-country CAPM?**

▷ Summing until the national level—e.g. Canada—the country’s aggregate portfolio should be efficient in CAD terms,

▷ ... but it can no longer be equated to the stocks issued issued by local companies:
  - Canadian investors hold many foreign stocks
  - Many Canadian stocks are held abroad

◊ **Aggregating to the world level solves this problem, but**

▷ we might run into problems with “homogeneous opportunities”

▷ we do run into problems with “homogeneous expectations”
  - Canadian T-bill is risky to Americans, not to Canadians
  - US T-bill: vv—and these are just the most obvious examples

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- We need to translate FC returns into the investor’s HC
- The FC T-bill even has $\tilde{s} (=\Delta S/S)$ as its sole source of risk

◊ **An analytical problem:**

- Demand equations (or efficiency conditions) are in different currencies—say CAD and USD
- ... so how can we aggregate and find the link between aggregate demand and the world market portfolio $w$?
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How to translate returns and their moments?

◊ **Exact formula:**

\[
1 + r_j = \frac{\tilde{V}_{j,t+1}}{V_{j,t}} = \frac{\tilde{V}_{j,t+1}\tilde{S}_{t+1}}{V_{j,t}S_{t}} = (1 + \tilde{r}_j^*)(1 + \tilde{s}),
\]

\[
\Rightarrow 1 + r_j^* = \frac{1 + \tilde{r}_j}{1 + \tilde{s}}.
\]

◊ **Quadratic approximation**

\[
r_j^* = \tilde{r}_j - \tilde{s} - \tilde{r}_j\tilde{s} + \tilde{s}^2
\]

<table>
<thead>
<tr>
<th>input data:</th>
<th>true $r^*$</th>
<th>linear approx</th>
<th>quadr approx</th>
</tr>
</thead>
<tbody>
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<td>$r$</td>
<td>$s$</td>
<td>$(r - s)/(1 + s)$</td>
<td>$r - s$</td>
</tr>
<tr>
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<td>0.000</td>
<td>0.1000</td>
<td>0.1000</td>
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<tr>
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**Example**

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\[ r_j^* = \tilde{r}_j - \tilde{s} - \tilde{r}_j\tilde{s} + \tilde{s}^2 \]

**Approximation to mean & variance:**

\[
E(r_p^*) = E(\tilde{r}_p) - E(\tilde{s}) - E(\tilde{r}_p\tilde{s}) + E(\tilde{s}^2)
\]

\[
= E(\tilde{r}_p) - E(\tilde{s}) - [E(\tilde{r}_p)E(\tilde{s}) + \text{cov}(\tilde{r}_p, \tilde{s})] + [E(\tilde{s})^2 + \text{var}(\tilde{s})],
\]

\[
\xrightarrow{d} E(\tilde{r}_p) - E(\tilde{s}) - \text{cov}(\tilde{r}_p, \tilde{s}) + \text{var}(\tilde{s}).
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\[
\text{var}(r_p^*) \xrightarrow{d} \text{var}(\tilde{r}_p - \tilde{s}) = \text{var}(\tilde{r}_p) - 2\text{cov}(\tilde{r}_p, \tilde{s}) + \text{var}(\tilde{s}).
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Example: do Americans-in-Paris like USD exposure?

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<th>Example 1: Positive covariance</th>
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- same distribution for \( W_{US} \) ... and same distribution for \( S \);
- but the large-cov case has ...
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\xrightarrow{\text{d}} &\quad E(\tilde{r}_p) - E(\tilde{s}) - \text{cov}(\tilde{r}_p, \tilde{s}) + \text{var}(\tilde{s}).
\end{align*}
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\[
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\text{var}(r_p^*) &\xrightarrow{\text{d}} \quad \text{var}(\tilde{r}_p - \tilde{s}) = \text{var}(\tilde{r}_p) - 2\text{cov}(\tilde{r}_p, \tilde{s}) + \text{var}(\tilde{s}).
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Do assets have a financial nationality?

Consider

\[ \tilde{r}_j = \delta_j + \gamma \tilde{s}_{CAD/USD} + \epsilon_j, \]

with \( \gamma \) = relative exposure (or elasticity of \( V \) wrt \( S \)).

Relative exposures of various assets

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Q: is gold, or oil, a dollar investment “because it is quoted in USD”?
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<td>Canadian importer</td>
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</tr>
</tbody>
</table>

Q: is gold, or oil, a dollar investment “because it is quoted in USD”?
Do assets have a financial nationality?

Consider

\[ \tilde{r}_j = \delta_j + \gamma \tilde{s}_{\text{CAD/USD}} + \epsilon_j, \]

with \( \gamma \) = relative exposure (or elasticity of \( V \) wrt \( S \)).

Relative exposures of various assets

<table>
<thead>
<tr>
<th>asset type</th>
<th>gamma</th>
<th>effect of rising USD</th>
</tr>
</thead>
<tbody>
<tr>
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- The picture:

- Summary

  - Exposures are not either 0 (CDN) or 1 (US), but spread around these values.
  - Lots of overlap in the middle: internationally competing firms have little nationality.
Do assets have a financial nationality?

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◊ Summary

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Aggregating the Efficiency Conditions

✧ The original conditions—a Babylonian chaos:

CDN’s \( p \):
\[
E(\tilde{r}_j - r) = \lambda \text{cov}(\tilde{r}_j, \tilde{r}_p),
\]

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E(\tilde{r}_j - r) = \lambda \text{cov}(\tilde{r}_j, \tilde{r}_w) + \kappa \text{cov}(\tilde{r}_j, s),\]

Proof:
\[
W_{ca}E(\tilde{r}_j - r) = \lambda \text{cov}(\tilde{r}_j, W_{ca} \tilde{r}_p)
\]
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(W_{ca} + W_{us})E(\tilde{r}_j - r) = \lambda \text{cov}(\tilde{r}_j, W_{ca} \tilde{r}_p + W_{us} \tilde{r}^*_p) + W_{us} (1 - \lambda) \text{cov}(\tilde{r}_j, s)
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"\kappa"
The Cost of International Capital

P. Sercu, *International Finance: Theory into Practice*

Translate first, then discount—or vv?

The Single-Country CAPM

Why do we need an InCAPM?
Why does it contain Xrisk?
Do assets have a financial nationality?

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A 2-country InCAPM

- **Need to identify two prices of risk:** $\lambda, \kappa$
  - So we need two benchmarks
  - We wake the world market, and the USD T-bill

- **Let’s cheat a bit** and assume that $\text{cov}(\tilde{r}_w, \tilde{s}) = 0$
  - general: $E(\tilde{r}_j - r) = \lambda \text{cov}(\tilde{r}_j, \tilde{r}_w) + \kappa \text{cov}(\tilde{r}_j, s)$
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    $$\Rightarrow \lambda = \frac{E(\tilde{r}_w - r)}{\text{var}(\tilde{r}_w)}$$
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The Solnik-Sercu-Adler-Dumas model

\[ N \text{ countries} \]

\[ E(r_j - r) = \beta_j; \ldots E(r_w - r) + \gamma_{j,1}; \ldots E(s_{1} + r_{1}^{*} - r) + \gamma_{j,2}; \ldots E(s_{2} + r_{2}^{*} - r) + \ldots \gamma_{j,n}; \ldots E(s_{n} + r_{n}^{*} - r), \]

\[ r_j = \alpha_{j,w}; \xi \ldots r_w + \gamma_{j,s_{1}}; \ldots s_{2} + \gamma_{j,s_{2}}; \ldots s_{2} + \ldots \gamma_{j,s_{n}}; \ldots s_{n} + \epsilon_{j,w}; \xi. \]

\[ \text{Restricted versions (1): trim the list of countries} \]

because \( \gamma \)'s are hard to estimate and Tbill premia small.

- small countries’ exchange rates have a small risk premium

\[ \kappa_l = \frac{W_l}{\sum_{k} W_k} (1 - \lambda) < 1 - \lambda \]

- unconnected countries (no \( X \), \( M \) links; no competitors): cov must be close to zero anyway

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The Cost of International Capital

P. Sercu,
*International Finance: Theory into Practice*

Outline

Translate first, then discount—or vv?
Two procedures
When to do what?

The Single-Country CAPM
From Asset Returns to Portfolio Return
The tangency solution
How the weights affect mean and variance
How to make a portfolio efficient
The Market Portfolio as the Benchmark

The International CAPM
Why do we need an InCAPM?
Why Xrisk pops up in the InCAPM
Do assets have a financial nationality?
Aggregating the Efficiency Conditions
The InCAPM

Wrapping up
Practical implications

◊ When do we use what model?
  ▶ Is host a segmented market? immaterial to the foreign investor!
  ▶ Is this a domestic or a foreign investment? immaterial!
  ▶ Is home a segmented market?
    – if yes: CAPM
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◊ When do we use what currency?
  ▶ home and host integrated? use either HC or FC
  ▶ home and host not integrated: use HC (home)

◊ Estimated risks and returns
  ▶ Market: use long-term, lowish estimates
  ▶ beta: use your priors too, or even exclusively
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