Chapter 8

Currency Options (1): Concepts and Uses
Overview

Introduction
  Puts and Calls
  Some Jargon: IV, I-A-OTM, TV
  Rational Exercising

Institutional Aspects

Using Options (1): Arbitrage
  Lower Bounds
  (European) Put-Call Parity

Using Options (2): hedging
  Advantages

Using Options (3): Speculation

What have we learned?
  Summary
  Are Options too Expensive?
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Are Options too Expensive?
A Young person’s Guide to FX Options

- **Options**: the holder has the right to buy (call option) or sell (put option), at an agreed-upon expiry moment $T$, an agreed-upon quantity of a specified asset ("underlying") at an agreed-upon price (strike or exercise price), from/to the writer of the option.

- **Exercising (killing) the option**: using the right, that is, buying (or selling) at the strike, at $T$ or (for an American-style:) possibly also early, i.e. before $T$.

- **Premium**: the price paid (by the holder, to writer) for the option, irrespective of exercising. Usually paid upfront, rarely at $T$ (forward-style), sometimes partly via mark2market and partly final (futures-style).
A Young person’s Guide to FX Options

Options (1): Concepts and Uses

P. Sercu, *International Finance: Theory into Practice*

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Using Options (1): Arbitrage

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What have we learned?

Options: the holder has the right to \( \begin{cases} \text{buy} & \text{(call option)} \\ \text{sell} & \text{(put option)} \end{cases} \)

at (European-style option)

up until (American style option)

an agreed-upon expiry moment \( T \), an agreed-upon quantity of a specified asset (“underlying”) at an agreed-upon price (strike or exercise price), from/to the writer of the option.

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What have we learned?

Options: the holder has the right to
\[
\begin{cases}
\text{buy} & \text{(call option)} \\
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\end{cases}
\]
at \quad \text{(European-style option)}
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What have we learned?

◦ Options: the holder has the right to \{ \begin{align*}
& \text{buy (call option)} \\
& \text{sell (put option)}
\end{align*} \}
up until an agreed-upon expiry moment \( T \), an agreed-upon quantity of a specified asset (“underlying”) at an agreed-upon price (strike or exercise price), from/to the writer of the option.

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\text{buy} & \text{(call option)} \\
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\end{cases} \)

  - at \( \text{(European-style option)} \)
  - up until \( \text{(American style option)} \)

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A Young person’s Guide to FX Options (2)

- **Intrinsic value or value dead**: what the option would be worth if the exercise decision would have to be taken now.

- **In / at / out of the money (ITM, ATM, OTM)**: the strike relative to the current price is such that immediate exercise would yield a positive / zero / negative cashflow.

- **ITM** means the intrinsic value is positive.

- **Time value**: premium - intrinsic value. Positive if the market thinks that it’s better to postpone exercising. An ATM/OTM option’s premium is pure time value.
A Young person’s Guide to FX Options (2)

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## Exercise Rules

<table>
<thead>
<tr>
<th>(style)</th>
<th>call</th>
<th>put</th>
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</thead>
<tbody>
<tr>
<td>European</td>
<td>$S_T &gt; X$</td>
<td>$X &gt; S_T$</td>
</tr>
<tr>
<td>American</td>
<td>$S_t &gt; X$</td>
<td>$X &gt; S_t$</td>
</tr>
<tr>
<td></td>
<td>$C_{t}^{am} = S_t - X(&gt;0)$</td>
<td>$P_{t}^{am} = X - S_t(&gt;0)$</td>
</tr>
</tbody>
</table>

**European:** – what’s what part of what forward contract?

![European call and put options diagram](image)
Outline

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Are Options too Expensive?
Institutional stuff

**Traded v OTC**

- Traded: Exchanges copied after futures: margin (for writer), clearing
- OTC: professionals

**Option on futures contract**

- Call: if you exercise, you become long side of a contract with historic price $X$, never marked to market.
- Triggers MtM flow of $f_{t,T_f} - X$.
- Exercise rules: Eur:
  - Am:

**Futures-style options**

- initial margin; daily MtM; final payment
- useful for speculators
- price is $[\text{price of regular option}] \times (1 + r_{t,T})$
- if on futures: convenient for put-call arbitrage
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## Traded options: contract info (Liffe)

### US Dollar / Euro Options

**Underlying:**

<table>
<thead>
<tr>
<th>Codes and classification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mnemo</td>
</tr>
<tr>
<td>Exercise type</td>
</tr>
</tbody>
</table>

**US Dollar / Euro Options**

- **Unit of trading:** 100
- **Contract size:** USD 10,000
- **Expiry months:**
  1. Initial lifetime: 1, 2, and 3 months
  2. Initial lifetime: 6, 9 and 12 months
  3. Initial lifetime: 3 years
- **Cycle:**
  - Initial lifetime: 1, 2, and 3 months: all months
  - Initial lifetime: 6, 9 and 12 months: March, June, September and December
  - Initial lifetime: 3 years: September

- **Quotation:** Euros per USD 100
- **Minimum price movement (tick size and value):** EUR 0.01 (= EUR 1 per contract)

- **Last trading day:** Trading in expiring currency derivatives have the EuroFX rate as their settlement basis and ends at 13.00 Amsterdam time on the third Friday of the expiry month, provided this is a business day. If it is not, the previous business day will be the last day of trading.

- **Settlement:** EuroFX rate contracts: Cash settlement, based on the value of the Euro / US Dollar rate set by EuroFX at 13.00 Amsterdam time. For DEX, the inverse value of the EuroFX Euro / US Dollar rate is used and rounded off to four decimal places.

- **Trading hours:** 9.00 – 17.25 Amsterdam time
- **Clearing:** LCH.Clearnet S.A.
- **Option style:** European style.
  Holders of long positions are not entitled to exercise their options before the exercise date.
- **Exercise:** European
- **Last update:** 21/12/04

- **Trading Platform:** Liffe CONNECT®
- **Wholesale Service:** Prof Trade Facility
## Options (1): Concepts and Uses

P. Sercu, *International Finance: Theory into Practice*

### Introduction

### Institutional Aspects

### Using Options (1): Arbitrage

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### What have we learned?

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**Traded options: price info** *(Neue Zürcher Zeitung)*

### DEVISENOPTIONEN

<table>
<thead>
<tr>
<th>Strike/$ Fr.</th>
<th>Sep 1.2235</th>
<th>Dez 1.2250</th>
<th>Mar 1.2500</th>
<th>Jun 1.2750</th>
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<td>1.2000</td>
<td>2.60</td>
<td>3.06</td>
<td>3.41</td>
<td>3.65</td>
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<td>1.07</td>
<td>1.90</td>
<td>2.34</td>
<td>2.66</td>
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<tr>
<td>1.2500</td>
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<td>1.12</td>
<td>1.57</td>
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<tr>
<td>1.2750</td>
<td>0.25</td>
<td>0.67</td>
<td>1.05</td>
<td>1.37</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>Strike/C Fr.</th>
<th>Sep 1.5781</th>
<th>Dez 1.6000</th>
<th>Mar 1.6250</th>
<th>Jun 1.6500</th>
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<tr>
<td>1.5750</td>
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<tr>
<td>1.6500</td>
<td>0.21</td>
<td>0.23</td>
<td>0.26</td>
<td>0.27</td>
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</table>

<table>
<thead>
<tr>
<th>Strike/C/$</th>
<th>Sep 1.2917</th>
<th>Dez 1.3000</th>
<th>Mar 1.3250</th>
<th>Jun 1.3500</th>
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<tbody>
<tr>
<td>1.2750</td>
<td>2.74</td>
<td>4.01</td>
<td>5.12</td>
<td>6.02</td>
</tr>
<tr>
<td>1.3000</td>
<td>1.02</td>
<td>2.61</td>
<td>3.73</td>
<td>4.66</td>
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<tr>
<td>1.3250</td>
<td>0.42</td>
<td>1.63</td>
<td>2.66</td>
<td>6.54</td>
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<tr>
<td>1.3500</td>
<td>0.26</td>
<td>1.01</td>
<td>1.87</td>
<td>2.66</td>
</tr>
</tbody>
</table>

*Kassamittelkurs: **1.2235**; Rp/$ 100,000 $; Cent/$ 100,000 C*

*Quelle: UBS*
Assume genuine uncertainty: 0 < prob(excse) < 1.

◊ **Calls: lobos**

- \( C_t > \frac{S_t}{1+r_{t,T}^*} - \frac{X}{1+r_{t,T}} \) because ...
  - \( C_t \rightarrow \) [the above] if ...
  - \( C_t > 0 \) because ...
  - \( C_t \rightarrow 0 \) if ...
  - \( C_{t}^{am} \geq C_t \) because ...
  - \( C_{t}^{am} = C_t \) if

**Summary:** \( C_{t}^{am} \geq C_t > \max \left( \frac{S_t}{1+r_{t,T}^*} - \frac{X}{1+r_{t,T}}, 0 \right) \).

- \( C_{t}^{am} > \max(S_t - X, 0) = IV \) because ...

Note: if \( r > r^* = 0 \), the first bound subsumes the second one:

\[
C_{t}^{am} > \max \left( \frac{S_t}{1+r_{t,T}^*}, 0 \right) > \max(S_t - X, 0) = IV
\]

⇒ when \( r > r^* = 0 \), early exercise is ...
Assume genuine uncertainty: $0 < \text{prob(excse)} < 1$.

◦ **Calls: lobos**

$C_t > \frac{S_t}{1+r_{t,T}^*} - \frac{X}{1+r_{t,T}}$ because ...

$C_t \to [\text{the above}]$ if ...

$C_t > 0$ because ...

$C_t \to 0$ if ...

$C_{am}^{t} \geq C_t$ because ...

$C_{am}^{t} = C_t$ if

**Summary:**

$C_{am}^{t} \geq C_t > \text{Max}\left(\frac{S_t}{1+r_{t,T}^*} - \frac{X}{1+r_{t,T}}, 0\right)$.

$C_{am}^{t} > \text{Max}(S_t - X, 0) = IV$ because ...

Note: if $r > r^* = 0$, the first bound subsumes the second one:

$C_{am}^{t} > \text{Max}\left(S_t - \frac{X}{1+r_{t,T}}, 0\right) > \text{Max}(S_t - X, 0) = IV$

⇒ when $r > r^* = 0$, early exercise is ...
Lower Bounds on Prices & Implications

Assume genuine uncertainty: \(0 < \text{prob(excse)} < 1\).

◊ **Calls: lobos**

\[ C_t > \frac{S_t}{1+r_{t,T}^*} - \frac{X}{1+r_{t,T}} \]

because ...

\[ C_t \rightarrow \text{[the above]} \text{ if ...} \]

\[ C_t > 0 \text{ because ...} \]

\[ C_t \rightarrow 0 \text{ if ...} \]

\[ C_t^{am} \geq C_t \text{ because ...} \]

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**Summary:** \( C_t^{am} \geq C_t > \text{Max} \left( \frac{S_t}{1+r_{t,T}^*} - \frac{X}{1+r_{t,T}}, 0 \right) \).

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Note: if \( r > r^* = 0 \), the first bound subsumes the second one:

\[ C_t^{am} > \text{Max} \left( S_t - \frac{X}{1 + r_{t,T}}, 0 \right) > \text{Max}(S_t - X, 0) = IV \]

\[ \Rightarrow \text{ when } r > r^* = 0, \text{ early exercise is ...} \]
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Assume genuine uncertainty: $0 < \text{prob(excse)} < 1$.

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\[ C_t \to \text{[the above]} \text{ if ...} \]

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Assume genuine uncertainty: $0 < \text{prob(excse)} < 1$.

◊ **Calls: lobos**

▷ $C_t > \frac{S_t}{1+r_{t,T}^*} - \frac{X}{1+r_{t,T}}$ because ...

$C_t \rightarrow [\text{the above}]$ if ...

▷ $C_t > 0$ because ...

$C_t \rightarrow 0$ if ...

▷ $C_{am}^t \geq C_t$ because ...

$C_{am}^t = C_t$ if

**Summary:** $C_{am}^t \geq C_t > \text{Max} \left( \frac{S_t}{1+r_{t,T}^*} - \frac{X}{1+r_{t,T}}, 0 \right)$.

▷ $C_{am}^t > \text{Max}(S_t - X, 0) = IV$ because ...

Note: if $r > r^* = 0$, the first bound subsumes the second one:

$$C_{am}^t > \text{Max} \left( S_t - \frac{X}{1 + r_{t,T}}, 0 \right) > \text{Max}(S_t - X, 0) = IV$$

$\Rightarrow$ when $r > r^* = 0$, early exercise is ...
Lower Bounds on Prices & Implications

Assume genuine uncertainty: 0 < prob(excse) < 1.

◊ **Calls: lobos**

▷ $C_t > \frac{S_t}{1+r_{t,T}^*} - \frac{X}{1+r_{t,T}}$ because ...

$C_t \to \text{[the above]}$ if ...

▷ $C_t > 0$ because ...

$C_t \to 0$ if ...

▷ $C_{tam} \geq C_t$ because ...

$C_{tam} = C_t$ if

**Summary:** $C_{tam} \geq C_t > \text{Max} \left( \frac{S_t}{1+r_{t,T}^*} - \frac{X}{1+r_{t,T}}, 0 \right)$.

▷ $C_{tam} > \text{Max}(S_t - X, 0) = IV$ because ...

Note: if $r > r^* = 0$, the first bound subsumes the second one:

$C_{tam} > \text{Max} \left( S_t - \frac{X}{1 + r_{t,T}}, 0 \right) > \text{Max}(S_t - X, 0) = IV$

⇒ when $r > r^* = 0$, early exercise is ...
Assume genuine uncertainty: $0 < \text{prob(excse)} < 1$.

- **Calls: lobos**
  - $C_t > \frac{S_t}{1+r^{*}_{i,T}} - \frac{X}{1+r_{i,T}}$ because ...
    - $C_t \to \text{[the above]}$ if ...
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    - $C_t \to 0$ if ...
  - $C_t^{am} \geq C_t$ because ...
  - $C_t^{am} = C_t$ if
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  - $C_t^{am} > \text{Max}(S_t - X, 0) = IV$ because ...

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$$C_t^{am} > \text{Max} \left( S_t - \frac{X}{1 + r_{i,T}}, 0 \right) > \text{Max}(S_t - X, 0) = IV$$

⇒ when $r > r^{*} = 0$, early exercise is ...
Assume genuine uncertainty: \(0 < \text{prob(excse)} < 1\).

\[ C_t > \frac{S_t}{1+r^{*}_{t,T}} - \frac{X}{1+r_{t,T}} \]

because ...

\[ C_t \rightarrow \text{[the above]} \text{ if ...} \]

\[ C_t > 0 \text{ because ...} \]

\[ C_t \rightarrow 0 \text{ if ...} \]

\[ C_{t}^{am} \geq C_t \text{ because ...} \]

\[ C_{t}^{am} = C_t \text{ if} \]

**Summary:** \( C_{t}^{am} \geq C_t > \text{Max} \left( \frac{S_t}{1+r^{*}_{t,T}} - \frac{X}{1+r_{t,T}}, 0 \right) \).

\[ C_{t}^{am} > \text{Max}(S_t - X, 0) = \text{IV} \text{ because ...} \]

Note: if \( r > r^{*} = 0 \), the first bound subsumes the second one:

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\( \Rightarrow \text{when } r > r^{*} = 0 \), early exercise is ...

\[ \Rightarrow \text{when } r > r^{*} = 0 \], early exercise is ...
Lower Bounds on Prices & Implications

Assume genuine uncertainty: $0 < \text{prob(exercise)} < 1$.

◊ Puts: lobos

$P_t > \frac{X}{1+r_{t,T}} - \frac{S_t}{1+r_{t,T}^*}$ because ...

$P_t \rightarrow \text{[the above]}$ if ...

$P_t > 0$ because ...

$P_t \rightarrow 0$ if ...

$P_t^{am} \geq P_t$ because ...

$P_t^{am} = P_t$ if

**Summary:** $P_t^{am} \geq P_t \geq \text{Max} \left( \frac{X}{1+r_{t,T}} - \frac{S_t}{1+r_{t,T}^*}, 0 \right)$.

$P_t^{am} > \text{Max}(X - S_t, 0) = IV$ because ...

Note: if $r^* > r = 0$, the first bound subsumes the second one:

$P_t^{am} > \text{Max} \left( X - \frac{S_t}{1+r_{t,T}^*}, 0 \right) > \text{Max}(X - S_t, 0) = IV$

⇒ when $r^* > r = 0$, early exercise is ...
Lower Bounds on Prices & Implications

Assume genuine uncertainty: 0 < prob(exercise) < 1.

◊ Puts: lobos

▷ $P_t > \frac{X}{1+r_{t,T}} - \frac{S_t}{1+r^{*}_{t,T}}$ because ...

$P_t \to \text{[the above]}$ if ...

▷ $P_t > 0$ because ...

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Lower Bounds on Prices & Implications

Assume genuine uncertainty: $0 < \text{prob(ercise)} < 1$.

**Puts: lobos**

- $P_t > \frac{X}{1+r,t} - \frac{S_t}{1+r^*,t}$ because ...
  
  $P_t \to \text{[the above]}$ if ...
  
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- $P^\text{am}_t \geq P_t$ because ...
  
  $P^\text{am}_t = P_t$ if

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Lower Bounds on Prices & Implications

Assume genuine uncertainty: $0 < \text{prob(exrcise)} < 1$.

◊ **Puts: lobos**

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Lower Bounds on Prices & Implications

Assume genuine uncertainty: $0 < \text{prob(exercise)} < 1$.

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$P_t > \frac{X}{1+r_{t,T}} - \frac{S_t}{1+r_{t,T}^*}$ because ...

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Lower Bounds on Prices & Implications

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Lower Bounds on Prices & Implications

Assume genuine uncertainty: $0 < \text{prob(exercise)} < 1$.

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- $P_t > \frac{X}{1+r_{t,T}} - \frac{S_t}{1+r_{t,T}}^*$  
  because ...

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Assume genuine uncertainty: $0 < \text{prob(exercise)} < 1$.

\[ P_t > \frac{X}{1+r_{t,T}} - \frac{S_t}{1+r_{t,T}^*} \]

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**Summary:**

\[ P_{t}^{am} \geq P_t > \text{Max} \left( \frac{X}{1+r_{t,T}} - \frac{S_t}{1+r_{t,T}^*}, 0 \right). \]

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- **Puts: lobos**
  
  $P_t > \frac{X}{1+r_{t,T}} - \frac{S_t}{1+r^*_{t,T}}$ because ...
  
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$\Rightarrow$ when $r^* > r = 0$, early exercise is ...
Put-Call Parity — European Options!

Note the replication possibilities:

\[ \tilde{S}_T - X + \tilde{P}_T = \tilde{C}_T \Rightarrow \text{synth call} \]
\[ \tilde{S}_T + \tilde{P}_T - \tilde{C}_T = X \Rightarrow \text{synth HC PN} \]
\[ X - \tilde{S}_T + \tilde{C}_T = \tilde{P}_T \Rightarrow \text{synth put} \]
\[ X - \tilde{P}_T + \tilde{C}_T = \tilde{S}_T \Rightarrow \text{synth FC PN} \]
Put-Call Parity

- **A no-arb relation**: if at $T$: $C_T - P_T = S_T - X$, by arb there must be parity also at $t$:
  
  $$C_t - P_t = \frac{F_{t,T} - X}{1 + r_{t,T}} = \frac{S_t}{1 + r_{t,T}^*} - \frac{X}{1 + r_{t,T}}$$

  (Put-Call Parity—Eur. options only!)

- **Three implications**
  - **At-the-forward (ATF)**: if $X = F_{t,T}$ then
    $$C_t = P_t,$$
    i.e. ATF puts and calls have equal prices.
  - **At-the-money (ATM)**: if $X = S_t$ then
    $$C_t - P_t = S_t \frac{r_{t,T} - r^* t, T}{(1 + r_{t,T})(1 + r_{t,T}^*)} > 0 \text{ if } r_{t,T} = r^* t, T.$$
    i.e. ATM call (=upward potential) is more valuable than put (downward potential) if $F_{t,T} > S_t$ (i.e. FC “strong”) & vv.
  - As soon as we have a Call option price model, PCParity implies the Put option pricing model.
Put-Call Parity

◇ **A no-arb relation:** if at $T$: $C_T - P_T = S_T - X$, by arb there must be parity also at $t$:

$$C_t - P_t = \frac{F_{t,T} - X}{1 + r_{t,T}} = \frac{S_t}{1 + r_{t,T}^*} - \frac{X}{1 + r_{t,T}}$$

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Put-Call Parity

♦ **A no-arb relation:** if at $T$: $C_T - P_T = S_T - X$, by arb there must be parity also at $t$:

\[
C_t - P_t = \frac{F_{t,T} - X}{1 + r_{t,T}} = \frac{S_t}{1 + r^*_{t,T}} - \frac{X}{1 + r_{t,T}}
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(Put-Call Parity—Eur. options only!)

♦ **Three implications**

▷ **At-the-forward (ATF):** if $X = F_{t,T}$ then

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Outline

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**One-edged hedging of contractual exposure**

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- **S**T

- **P**hedged A/R
- **S**T

- **Hedging exposures with big quantity risks**
  - Examples: Int tender, risky A/R etc, risky stock investments, reinsurance,
  - “Advantage”: no risk of two bad tidings—losing on the exposed position *and* on the hedge: OTM option not exercised
  - Dubious argument: option’s added flexibility is still 100% tied to Xrisk, not to quantity risk
Using Options 2: hedging

◇ One-edged hedging of contractual exposure

◇ “Hedging exposures with big quantity risks”

▷ Examples: Int tender, risky A/R etc, risky stock investments, reinsurance,

▷ “Advantage”: no risk of two bad tidings—losing on the exposed position and on the hedge: OTM option not exercised

▷ Dubious argument: option’s added flexibility is still 100% tied to $X_{risk}$, not to quantity risk
More on Options as hedges

- **Hedging nonlinear exposure**

  **Example**: exports as an option

  - your perifraxes can be sold either at home at EUR 1, or exported at USD 1 net (price takership).
  - Thus—see graph—\( V_T = 1 + \text{Max}(\tilde{S}_T - 1, 0) \), quite option-like

  ![Diagram](image)

  - Selling an option replaces this *potential* extra income by its (PV’d) CEQ, the premium income.
  - Naive forward hedging (USD 1 per perifrax) cannot remove the exposure at all.
More on Options as hedges

**Hedging nonlinear exposure**

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- Thus—see graph—\( V_T = 1 + \max(\tilde{S}_T - 1, 0) \), quite option-like

\[
\begin{align*}
\text{export} & \quad \text{optimal use} \\
\text{sell at home} & \quad \tilde{S}_T \\
1 & \quad 1
\end{align*}
\]

- Selling an option replaces this *potential* extra income by its (PV’d) **C\text{EQ}**, the premium income.
- Naive forward hedging (USD 1 per perifrax) cannot remove the exposure at all.
Hedging nonlinear exposure

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Piecewise Linear Approximations & Options

Options (1): Concepts and Uses
P. Sercu, *International Finance: Theory into Practice*

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What have we learned?

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<td>194.0</td>
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Non-constant exposure

![Graph showing piecewise linear approximations and options](image)
Options (1): Concepts and Uses

- P. Sercu, *International Finance: Theory into Practice*

**Outline**

**Introduction**
- Puts and Calls
- Some Jargon: IV, I-A-OTM, TV
- Rational Exercising

**Institutional Aspects**

**Using Options (1): Arbitrage**
- Lower Bounds
  - (European) Put-Call Parity

**Using Options (2): Hedging**
- Advantages

**Using Options (3): Speculation**

**What have we learned?**
- Summary
- Are Options too Expensive?
Speculating on $S$ or on $\sigma_S$

- **Speculation on $S$**
  - Bulls buy calls or sell puts, Bears buy puts or sell calls
  - Buying options limits your risk to the premium
    - ... but the chance of losing all is usually big ($\approx 50\%$, ATM)
  - Selling options is risky

- **Speculating on volatility**
  - Wait till $T$ and cash in big-time—or so you hope

<table>
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<th>prob($T$)</th>
<th>expectation for $C$, $P$ ($X=1$)</th>
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</thead>
<tbody>
<tr>
<td>prob($0.9$)</td>
<td>your opinion 0.25 0.50 0.25 $E_{\text{you}}(C$ or $P) = 0.025$</td>
</tr>
<tr>
<td>prob($1.0$)</td>
<td>mkt opinion 0.15 0.70 0.15 $E_{\text{mkt}}(C$ or $P) = 0.015$</td>
</tr>
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<td>prob($1.1$)</td>
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- or cash in as soon as the market has seen the error of its ways and revalued the options—or so you hope
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<table>
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<th>Straddle</th>
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  ![Diagram showing straddle and strangle options](image)

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◊ **Chopped-up Forward Contracts**

- European options provide the **holder** with the positive part of the payoff of the comparable forward contract—below $X$ for the **put**, above $X$ for the **call**. The **writer** gets the negative parts.

- Options being **zero-sum games**, the parties can agree only if the holder pays the writer a **premium**, which should be the risk-adjusted and discounted expected value.

◊ **Lower Bounds on prices.**

- As a European option provides the nice part of the comparable forwards, the value of the latter is a lower bound on the $E$ option’s price.

- Zero is another lower bound.

- American options are worth at least the $E$ option, and also at least the intrinsic value.

For some interest-rate combinations the latter bound can never be reached, or is unlikely to ever be reached.
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Are Options too Expensive?

◊ The most expensive option is cheap
  ▶ The most expensive option is a VeryDeep ITM one,
  ▶ and it is priced as a forward,
  ▶ which cannot be controversially expensive.

◊ Outrageous Bid-ask Spreads?
  ▶ Bid-Ask for options is easily 5% or more. but ...
  ▶ cannot be compared to spread on forwards, since the premium
    is a levered net value while the forward is the price of one leg
    Example: If $F_t = 100$ and $F_{t0} = 98$ and $r \approx 0$ then the market value is
    2; and a 0.10% spread on $F$ would already be a 5% spread on 2.
  ▶ In addition, hedging the option is much more costly and risky, to the bank, than hedging a forward

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