Chapter 6

The Market for Currency Futures

P. Sercu,
*International Finance: Theory into Practice*

Overview
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Handling Default Risk in Forward Markets: Old & New Tricks

How Futures Contracts Differ from Forwards

Effect of Marking to Market on Futures Prices

Hedging with Futures Contracts
   The general MinVar problem
   The delta hedge
   The cross hedge

Conclusion: pros and cons
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Handling Default Risk in Forward Markets

◊ **Issues**
  ▶ Default risk—limited by right of offset to $\tilde{S}_T - F_{t,T}$
  ▶ Illiquidity: early settlement is a favor, not a right

◊ **Forwards: Standard Ways of Reducing Default Risk**
  ▶ towards firms: credit agreements, security
  ▶ towards firms: restricted use
  ▶ towards banks: credit lines
  ▶ towards all: short lives; rolling over

◊ **New gimmicks**
  ▶ start with small collateral, covering 1-day risk instead of N
  ▶ [variable collateral:] every day: ask new collateral (or release old) depending on change mkt value—OR:
  ▶ [mk2mkt:] every day, settle yesterday’s contract, “buy” a new one
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### Variable collateral / Mk2Mkt: Example

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<thead>
<tr>
<th>data</th>
<th>Variable Collateral</th>
<th>Periodic Recontracting</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>time 0:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$F_{0,3} = 40$</td>
<td>Smitha buys forward USD 1m at $F_{0,3} = 40$</td>
<td>Smitha buys forward USD 1m at $F_{0,3} = 40$</td>
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<tr>
<td>$r_{0,3} = 3%$</td>
<td>Market value of old contract is $\frac{38m - 40m}{1.02} = -1.961m$</td>
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<td>Smitha puts up T-bills worth at least 1.961m</td>
<td></td>
<td>Smitha buys back the old contract for 1.961m and signs a new contract at $F_{1,3} = 38$.</td>
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<td><strong>time 1:</strong></td>
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<td></td>
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<tr>
<td>$F_{1,3} = 38$</td>
<td>Market value of old contract is $\frac{36m - 38m}{1.01} = -3.960m$</td>
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<td>$r_{1,3} = 2%$</td>
<td>Smitha increases the T-bills put up to at least 3.960m</td>
<td>Smitha buys back the old contract for 1.980m and signs a new contract at $F_{2,3} = 36$.</td>
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<tr>
<td><strong>time 2:</strong></td>
<td></td>
<td></td>
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<tr>
<td>$F_{2,3} = 36$</td>
<td>Smitha pays the promised INR 40m for the USD 1m, and gets back her T-bills</td>
<td>Smitha pays the promised INR 36m for the USD 1m</td>
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<td>$r_{2,3} = 1%$</td>
<td>(adjusted for time value:)</td>
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<tr>
<td><strong>time 3:</strong></td>
<td></td>
<td></td>
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<tr>
<td>$F_{3,3} = S_3 = 34$</td>
<td>$\frac{36m - 38m}{1.01} = -1.980m$</td>
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<td>$r_{3,3} = 0%$</td>
<td>Smitha pays the promised INR 40m for the USD 1m</td>
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<td><strong>total paid:</strong></td>
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- time 3: 36m
- time 2: 1.980 \times 1.01 = 2m
- time 1: 1.961 \times 1.02 = 2m
- total: 40m
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♦ Feature#1: Marking to Market

▷ Mk2Mkt using undiscounted change of price

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▷ loser pays winner via margin accounts held with broker / clearing members

▷ payment based on settlement price – or trade price in case of exit/entry during the day

▷ reduces loser’s incentive to run away, and counterpart’s loss if loser still runs away

♦ Feature#2: Clearing Corporation

▷ central counterpart between buyer and seller ⇒ guarantor

▷ also nets a player’s purchases against sales
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How Futures Differ from Forwards

◊ **Feature#3: initial margin; maintenance**
  ▶ *initial margin*—interest bearing
  ▶ small losses can accumulate until *maintenance margin* is reached; then a *margin call* is issued
  ▶ failure to pay up = order to close out

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**Example**

Nick Leeson had accumulated losses *ad* GBP 800m—Barings’ entire equity—but the Singapore Exchange lost “only” 50m:
  – 500m was paid as m-to-m with Barings’ money
  – 250m was paid as m-to-m with other customers’ money
The balance was lost by Simex while liquidating (immense price pressure).

◊ **Feature#4: organized markets**
  ▶ Fwd: OTC—so no info on prices, volumes; just an informal snapshot around noon
  ▶ Futures: formal exchanges. CME/CBOT, Eurex, LIFFE, etc
  ▶ More and more via computerized PLOB or mixed system
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1.1. Organized markets

1. What are Currency Futures

- **Forward contracts:**
  - decentralized pricing. OTC, market makers.
  - No information on when a transaction took place, and at what price.
  - No secondary market.

- **Futures contracts:**
  - organized exchanges. Price results from centralized meeting of demand & supply.
  - "open outcry" system (US, LIFFE, MATIF)
  - computerized Public Limit Order Book (many continental European exchanges).
  - price and transaction information.
  - secondary market.

### Feature#5: Standardized contracts

- **contract size** (far smaller than OTC currency)
- **expiry dates:** e.g. monthly (≤ 3mo), mar/jun/sept/dec (< 12mo), annual (12 to 4 mo)

| Rate:GBP | IMM | 62,500 | Other exchanges |
| USD:EUR | IMM | 125,000 | LIFFE, PBOT, SIMEX, MACE, FINEX |
| EUR:USD | OM-S | 50,000 | EUREX |
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- **Feature#5: Standardized contracts** to stop fragmentation and facilitate secondary dealing
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**FUTURES PRICES**

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<th>CURRENCY</th>
<th>Lifetime</th>
<th>Open</th>
<th>High</th>
<th>Low</th>
<th>Settle</th>
<th>Change</th>
<th>High</th>
<th>Low</th>
<th>Interest</th>
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<tr>
<td>JAPAN YEN (CME)</td>
<td>— 12.5 million yen ; $ per yen (.00)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sept</td>
<td>.9458</td>
<td>.9466</td>
<td>.9386</td>
<td>.9389</td>
<td>—.0046</td>
<td>9540</td>
<td>.7945</td>
<td>73,221</td>
<td></td>
</tr>
<tr>
<td>Dec</td>
<td>.9425</td>
<td>.9470</td>
<td>.9393</td>
<td>.9396</td>
<td>—.0049</td>
<td>9529</td>
<td>.7970</td>
<td>3,455</td>
<td></td>
</tr>
<tr>
<td>Mr94</td>
<td>.9417</td>
<td>.0051</td>
<td>.9490</td>
<td>.8700</td>
<td></td>
<td></td>
<td></td>
<td>318</td>
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Est vol 28,844; vol Wed 36,595; open int 77,028, + 1.820

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Effect of Mk2Mkt on Futures Prices

Q: Does Mk2Mkt drive a wedge between futures and forward prices?
A: Yes, downward—but it’s absolutely tiny.

Example (data):

3 dates (0, 1, T=2). \( F_{0,2} = 100, F_{1,2} = \begin{cases} 105, & p = \frac{1}{2} \\ 95, & p = \frac{1}{2} \end{cases}, \tilde{F}_{2,2} = \tilde{f}_{2,2} = \tilde{S}_3. \)

– \( f_{1,2} \) must be equal to \( T_{1,2} \) because ...

– Q: is \( f_{1,2} = F_{1,2} \)? We verify/falsify this conjecture.

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<td>105</td>
<td>105 - 100 = +5</td>
<td>-105</td>
<td>0</td>
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<td>+5</td>
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<td>$-95$</td>
<td>0</td>
<td>$-100$</td>
<td>$-5$</td>
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– Q rephrased: do investors mind/love/loathe the no-interest loan/deposit in the up/down state, resp.?
– assume risk neutrality
Effect of Mk2Mkt on Futures Prices

Q: Does Mk2Mkt drive a wedge between futures and forward prices?
A: Yes, downward—but it’s absolutely tiny.

Example (data):

3 dates (0, 1, T=2). \( F_{0,2} = 100, F_{1,2} = \begin{cases} 105, & p = 1/2 \\ 95, & p = 1/2 \end{cases} \), \( \tilde{F}_{2,2} = \tilde{f}_{2,2} = \tilde{S}_3 \).

- \( f_{1,2} \) must be equal to \( T_{1,2} \) because ...
- Q: is \( f_{0,2} = F_{1,2} \) ?? We ¿verify/falsify? this conjecture.

| \( F_{1,2} \) | HC flows: futures | \( F_{1,2} \) | HC flows: forward | \( F_{1,2} \) | difference |
|---|---|---|---|---|
| 105 | 105 − 100 = +5, −105 | 0, −100 | +5, −5 |
| 95 | 95 − 100 = −5, −95 | 0, −100 | −5, +5 |

- Q rephrased: do investors mind/love/loathe the no-interest loan/deposit in the up/down state, resp.?
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Undo Mk2Mkt by borrowing 5 (down) or lending 5 (down). Costly? Profitable?

Example

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<thead>
<tr>
<th>state</th>
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</tr>
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<tr>
<td></td>
<td>r</td>
<td>net time value</td>
<td>r</td>
</tr>
<tr>
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<td>10</td>
</tr>
<tr>
<td>down</td>
<td>0</td>
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◇ Case 1: market interest rate is zero. You don’t even notice interest-free deposits/loans. Conjecture acceptable.

◇ Case 2: market interest rate is a positive “constant”. You still don’t mind. Conjecture acceptable.


▷ f must be below $F$
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<td>up</td>
<td>$r$ (5 - 5 = 0.00)</td>
<td>$r$ (5 \times 1.10 - 5 = 0.50)</td>
<td>$r$ (5 \times 1.08 - 5 = 0.40)</td>
</tr>
<tr>
<td>down</td>
<td>$0 - 5 + 5 = 0.00$</td>
<td>$10 - 5 \times 1.10 + 5 = -0.50$</td>
<td>$8 - 5 \times 1.12 + 5 = -0.60$</td>
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To induce investors to hold futures contracts, futures prices must be lower than forward prices.

But ...

- correlation between $\Delta r$ and $\Delta f$ is lowish
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The Market for Currency Futures

P. Sercu, 
*International Finance: Theory into Practice*

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- The general MinVar problem
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Conclusion: pros and cons
What’s special?: Approximate hedging

✧ **Limited choice:**
  - only a few currencies (or grades / delivery points, for commodities)
  - only a few expiry dates

Almost surely there is no perfect hedge

✧ **How to deal with it?**
  - Chose a hedge ratio that reduces the remaining risk to a minimum
  - Let $T_1$ be the firm’s hedging horizon
  - Let $\tilde{y} := \tilde{C}_{T_1}$ be the firm’s cash flow at $T_1$, in $HC$
  - Let $T_2 (\geq T_1)$ be the expiry date of the hedge
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A firm once commissioned a hedging program for its commodity purchases, ignoring the fact that price increases were passed on to the consumers with a delay of about 2 weeks.

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A standardized problem

- a unit inflow of FC $e$ at $T_1$ — e.g. SEK (in USD/SEK)
- hedge contract is available for FC $h$, contract size one unit — e.g. EUR (in USD/EUR)

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- Rule-of-thumb solution for regression:
  - If time-\( T_1 \) interest rates were known in advance, then so would be \( B \):

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S_{T_1}^{(e)} = \frac{1 + r_{T_1}^{(e)}}{1 + r_{T_1}} \frac{1 + r_{T_1}^{(e)}}{1 + r_{T_1}^{(e)}} S_{T_1}^{(e)}
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