Chapter 4
Understanding Forward Rates for Foreign Exchange
Overview

Introduction to Forward Rates

Links Between Forex & Money Markets
FX & MM Transactions: Ins & Outs
The Matrix: a Diagram of Markets

The Law of 1 Price: Covered Interest Parity
Arbitrage and the LOP
Shopping around under CIP
Infrequently asked Questions on CIP

Market Value of Forward Contract
The formula
Implication 1: Value at Maturity
Implication 2: Value at Inception
Implication 3: F is a risk-adjusted expectation or CEQ
Implication 4: (ir)relevance of hedging?

What have we learned in this chapter?
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What have we learned in this chapter?
How Forward Rates are Quoted

**Quotes:** Two conventions: Outright \( (F) \) vs. swap rate \( (F - S) \)—see e.g. Globe and Mail

<table>
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<tr>
<th></th>
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<tr>
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<td>—</td>
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<td>+0.35</td>
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<td>12 months</td>
<td>1.3266</td>
<td>0.7538</td>
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</tr>
<tr>
<td>...</td>
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<td>...</td>
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</tr>
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<td>10 years</td>
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"at a premium", or "above par" \( + \)  "at a discount", or "below par" \( - \)

\( (a \text{ premium}) \) \( (a \text{ discount}) \)

**Which is used where?** Traders traditionally quoted swap rates. Newspapers have stopped the practice.

Sometimes one uses “p” (= premium), “d” (= discount) instead of “+”, “−”, or one entirely omits the indication.
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"at a premium", or "above par"            "at a discount", or "below par"     (a premium)                  (a discount)

Which is used where? Traders traditionally quoted swap rates. Newspapers have stopped the practice.

Sometimes one uses “p” (= premium), “d” (= discount) instead of “+”, “−”, or one entirely omits the indication.
How we denote risk-free returns

**Effective return** = simple percentage difference between start and end value, as % of start value

<table>
<thead>
<tr>
<th>$T - t$</th>
<th>$V_t$</th>
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<th>$r_{t,T}$</th>
</tr>
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<tbody>
<tr>
<td>3 month</td>
<td>100</td>
<td>102</td>
<td>2%</td>
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**Interest rate** = annualized (“p.a.”) version of $r$. Needs to de-annualized into an effective return.

Examples: 3 months at 6% p.a. means ...

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<td>comp., annual</td>
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<td>comp., monthly</td>
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<td>comp., daily</td>
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<td>cont. comp</td>
<td>$e^{0.06 \times 3/12} - 1$</td>
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What have we learned in this chapter?
The ins & outs of FX & MM Transactions

Assume perfect markets, in this chapter. (See next chapter for imperfections.)

Time-subscripted $HC$, $FC$ refer to amounts of a currency; $t = \text{now}$, $T = \text{future}$.

8 possible transactions in spot/forward/money markets:

<table>
<thead>
<tr>
<th>Output amount</th>
<th>= Input amount $\times$ multiplic. factor</th>
</tr>
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<tbody>
<tr>
<td>sell FX spot</td>
<td></td>
</tr>
<tr>
<td>buy FX spot</td>
<td></td>
</tr>
<tr>
<td>sell FX forward</td>
<td></td>
</tr>
<tr>
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<td></td>
</tr>
<tr>
<td>HC term deposit</td>
<td></td>
</tr>
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Getting our act together into a diagram

1.21

1/1.21

110

1/110

1.10

1/1.10

1210

100

end here

121 000

start here

1331

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Two Key Results—for Perfect Markets

If Covered Interest Parity—CIP—holds, \( \text{ie} \)

\[
F_{t,T} = S_t \frac{1 + r_{t,T}}{1 + r_{t,T}^*} - \quad \text{(IRP or CIP)}
\]

Then

\( \diamond \) **[No-Arb:]** there are no arbitrage opportunities

\( \triangleright \) (With spreads, this will be weakened into an inequality.)

\( \diamond \) **[Shopping Around:]** shopping-around calculations are pointless

\( \triangleright \) (Thus, shopping-around becomes relevant only because of market imperfections)
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Then

\[ \text{[No-Arb:]} \text{ there are no arbitrage opportunities} \]

\[ \text{(With spreads, this will be weakened into an inequality.)} \]

\[ \text{[Shopping Around:]} \text{ shopping-around calculations are pointless} \]

\[ \text{(Thus, shopping-around becomes relevant only because of market imperfections)} \]
Previous data—\( S_t = 100, r_{t,T} = 0.21, r^*_{t,T} = 0.10 \). From CIP, we should have

\[
F_{t,T} = 100 \frac{1.21}{1.10} = 110
\]

(next 3 slides;) if not, there is an arb opp

(2 more slides:) if so, there is no need to check for small differences in outcomes
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- (next 3 slides:) if not, there is an arb opp
- (2 more slides:) if so, there is no need to check for small differences in outcomes
No-arb 1: with CIP, a roundtrip breaks even

Note: the first three steps form a synthetic forward purchase.
No-arb 2a: without CIP, there’s an arb opp

HC\_t

1/100

100

FC\_t

1/1.21

1.21

1/1.10

1.10

what if 1/111?

HC\_T

111?

FC\_T
No-arb 2b: without CIP, there’s an arb opp

HC_t

1/100

100

FC_t

1/1.21

1.21

1/1.10

1.10

what if
1/109?

109?

HC_T

what if 1/109?
In perfect mkts, shopping around is pointless.

Deposits: \( HC \) v swapped \( FC \). (This is why CIP is called CIP.)
In perfect mkts, shopping around is pointless

Try out all $4 \times 3$ trips!
Causality?

CIP in itself has no causality, but you can append stories:

– interest rates: Fisher’s story
– forward rate: (risk-adjusted) expected future spot rate

... and end with a theory on the spot rate:

\[ S_t \]

\[ r^{*},T \]

\[ r_{t,T} \]

\[ F_{t,T} \]

business conditions (incl risk), abroad

expected inflation, abroad

business conditions (incl risk), home

expected inflation, home

expected spot rate

risks of future spot rate

What have we learned?
Causality?

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- **interest rates**: Fisher’s story
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... and end with a theory on the spot rate:

\[
\begin{align*}
S_t &= r^*_{t,T} + S_t \\
F_{t,T} &= r_{t,T} + \text{expected inflation, abroad} \\
&\quad + \text{business conditions (incl risk), abroad} \\
&\quad + \text{expected spot rate} \\
&\quad + \text{risks of future spot rate} \\
\end{align*}
\]
CIP and the swap rate

- **Fact 1:** Sign of swap rate depends just on $r - r^*$:
  
  \[
  F_{t,T} - S_t = S_t \left[ \frac{1 + r_{t,T}}{1 + r^*_{t,T}} - 1 \right],
  \]

  
  \[
  = S_t \left[ \frac{1 + r_{t,T}}{1 + r^*_{t,T}} - \frac{1 + r^*_{t,T}}{1 + r^*_{t,T}} \right],
  \]

  
  \[
  = S_t \left[ \frac{r_{t,T} - r^*_{t,T}}{1 + r^*_{t,T}} \right];
  \]

  
  \[
  \Rightarrow \frac{\partial \cdot}{\partial S_t} = \left[ \frac{r_{t,T} - r^*_{t,T}}{1 + r^*_{t,T}} \right] \approx r_{t,T} - r^*_{t,T}.
  \]

- **Fact 2:** Swap rate has low sensitivity to $S$:

  Traditionally, $r - r^*$ is small: short $T - t$, low p.a. interest.

  This is why traders used to quote swap rates: if you change $S$, the required change in the swap rate is tiny relative to spreads.
CIP and the swap rate

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<tr>
<td>level₁</td>
<td>100.5</td>
<td>100.5 × 1.003333/1.002500 = 100.5835</td>
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<tr>
<td>change</td>
<td>0.5</td>
<td>0.5004</td>
<td>0.0004</td>
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Analytically:

\[
\text{change} \approx (0.003333 - 0.002500) \times 0.5 = 0.000416.
\]
CIP and the swap rate

### Example

\( T - t = 1/12, \) p.a. simple interest 4% (home), 3% (foreign)

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<td>level(_1)</td>
<td>100.5</td>
<td>100.5 ( \times \frac{1.003333}{1.002500} ) = 100.5835</td>
<td>0.0835</td>
</tr>
<tr>
<td>change</td>
<td>0.5</td>
<td>0.5004</td>
<td>0.0004</td>
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Analytically:

\[
\text{change} \approx (0.003333 - 0.002500) \times 0.5 = 0.000416.
\]
CIP and taxes

"Neutral" taxes do not affect decisions

- neutral: there is just one income number, including interest income plus capgains, minus interest paid and caplosses

Example \((S=100, r=0.21, r^*=0.10)\)

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<tr>
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From CIP: 

\[
F \times (1 + r^*) = S \times (1 + r),
\]

\[
capgain + foreign interest = F - S + Fr^* = Sr = domestic interest.
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Introduction to Forward Rates

Links Between Forex & Money Markets
FX & MM Transactions: Ins & Outs
The Matrix: a Diagram of Markets

The Law of 1 Price: Covered Interest Parity
Arbitrage and the LOP
Shopping around under CIP
Infrequently asked Questions on CIP

Market Value of Forward Contract
The formula
Implication 1: Value at Maturity
Implication 2: Value at Inception
Implication 3: $F = CEQ \left( \frac{S_T}{S_0} \right)$
Implication 4: (ir)relevance of hedging?

What have we learned in this chapter?
Who wants to know a contract’s MktVal?

◊ Why do we care?

▶ Valuation in financial statements or internal reports
▶ Negotiating an early termination:
  – speculator—wants to lock in gains, or cut losses
  – hedger—underlying hedged position is gone
  – default—file claim for damages
▶ Theory of options:
  – value of unconditional purchase (sale) is lower bound for value option to purchase (sell)
  – needed to explain early exercise issue in American-style option

NOTE: “a contract” means a purchase of FC 1.
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NOTE: “a contract” means a purchase of FC 1.
The valuation formula

- A forward contract has two legs, each of which can be thought of as a promissory note:
  - (asset:) you receive a PN from the bank \( ad \) FC1
  - (liability:) you write a PN to the bank \( ad \) HC \( F_{t_0,T} \)

- So the contract’s value is equal to the net value of this small portfolio.

Example \((S=100, r=0.21, r^*=0.10; F_{t_0,T}=115)\)

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\text{Market value of forward purchase at } F_{t_0,T} = \left( \frac{1}{1 + r^*_t} \times S_t \right) - \frac{F_{t_0,T}}{1 + r_t}.
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\text{Market value of forward purchase at } F_{t_0,T} = \frac{1}{1 + r_{t,T}} \left[ 1 + \frac{r_{t,T}}{1 + r_{t,T}^*} S_t - \frac{F_{t_0,T}}{1 + r_{t,T}} \right] = F_{t,T} - F_{t_0,T} \frac{1}{1 + r_{t,T}}
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Implication 1: Value at Maturity

If \( t = T \), then \( r_{T,T} = \ldots = r^*_{T,T} \); so

\[
\frac{S_t}{1 + r^*_{t,T}} - \frac{F_{t_0,T}}{1 + r_{t,T}} = \text{t=T}
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Buy forward

Sell forward
Understanding Forward Rates

P. Sercu, *International Finance: Theory into Practice*

### Implication 2: Value at Inception

If $t_0 = t$, then

$$\frac{F_{t,T} - F_{t_0,T}}{1 + r_{t,T}} = t$$

### NOTES

- Holds only at the moment the contract is signed—otherwise the contract would be pointless.

- Major implication: at the moment of hedging, the value of an asset *per se* is the same as the value of the hedged asset.

$$\text{PV}(A + B \times \tilde{S}_T) = \frac{A + B \times F_{t,T}}{1 + r_{t,T}} :$$

- We replace $\tilde{S}_T$ by $F_{t,T}$, and
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\[
\begin{align*}
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&= A + B \times F_{t,T} - r_{t,T} \tilde{S}_T
\end{align*}
\]

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Introduction

Links Between Markets

The LOP and CIP

MktVal of Forward Contract

The formula

Implic1: Value at Maturity

Implic2: Value at Inception

Implic3: \( F = CEQ_t(\tilde{S}_T) \)

Implic4: (ir)relevance of hedging?

What have we learned?

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**Implication 2: Value at Inception**

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Implic3: F is a risk-adjusted expectation

- Two ways to value a unit FC TBill:
  - Way #1: General asset pricing approach:
    \[
    PV_t(\tilde{S}_T) = \frac{E_t(\tilde{S}_T)}{1 + E_t(\tilde{r}_{\tilde{S},t,T})}
    \]
    where \(E_t(\tilde{r}_{\tilde{S},t,T}) = \) the expected return, given risk of \(\tilde{S}_T\).
  - Way #2: value the hedged asset
    \[
    PV_t(\tilde{S}_T) = \frac{F_{t,T}}{1 + r_{t,T}} \left( = \frac{CEQ_t(\tilde{S}_T)}{1 + r_{t,T}} \right)
    \]
  - For completeness: Way #3: translated FC value: \(PV_t(\tilde{S}_T) = \frac{S_t}{1 + r_{t,T}}\).
  - Re-interpretation of CEQ as risk-adjusted expectation:
    \[
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Implic3: F is a risk-adjusted expectation

◊ Two ways to value a unit FC TBill:

▷ Way #1: General asset pricing approach:

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Implication 4: (ir)relevance of hedging?

Does the zero initial value mean that hedging adds no value?

◊ Criterion of firm’s MktVal as the yardstick of relevance:
  ▶ takes into account effects of hedging on expected cash flow and risk
  ▶ MM’s criterion

◊ At $t$, adding a hedge does not add/destroy any value provided the firm’s other cash flows are unaffected.

◊ In many cases, the firm’s other cash flows are likely to be affected:
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  ▶ taxes
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Outline

Introduction to Forward Rates

Links Between Forex & Money Markets

The Law of 1 Price: Covered Interest Parity

Market Value of Forward Contract

What have we learned in this chapter?
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- Forward quotes can be *outright* or in *swap-rate* format.

- The Matrix:
  - Spot, forward, and money markets are so closely related that we have to study them together.
  - In a perfect market one could eliminate one of them and not lose anything.

- The perfect replicability implies a no-arb result, Covered Interest Parity—no causality suggested, here:
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... which simplifies to \(S_T - F_{t0,T}\) when \(t = T\), and to zero when \(t = t_0\).

- Zero initial value does not necessarily mean that hedging adds no value: other cash flows may be affected.

- The forward rate is also a risk-adjusted expectation, or certainty equivalent:

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