MATLAB codes

Number-Crunching: Taming Unruly Computational Problems from Mathematical Physics to Science Fiction by Paul J. Nahin

[In Word format]

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```
%fpu.m
N=32;alpha=0.25;TMAX=10000;DT=20;tspan=[0:DT:TMAX];
options=odeset('Reltol',1e-4,'OutputFcn','odeplot','OutputSel',[1,2,N]);
for I=1:N
   a=1;b(I)=a*sin(pi*I/(N+1));b(I+N)=0;
   omegak2(I)=4*(sin(pi*I/2/N))^2;
end
[T,Y]=ode45('fput1',tspan,b,options,N);
for IT=1:(TMAX/DT)
   TIME(IT)=IT*DT*sqrt(omegak2(1))/2/pi;
   YX(IT,1:N+1)=[0 Y(IT,1:N)];YV(IT,1:N+1)=[0 Y(IT,N+1:2*N)];
   sXF(IT,:)=imag(fft([YX(IT,1:N+1) 0-YX(IT,N+1:-1:2)]))/sqrt(2*(N+1));
   sVF(IT,:)=imag(fft([YV(IT,1:N+1) 0-YV(IT,N+1:-1:2)]))/sqrt(2*(N+1));
   Energ(IT,1:N)=(omegak2(1:N).*(sXF(IT,2:N+1).^2)+sVF(IT,2:N+1).^2)/2;
   for J=2:N-1
      DifY(IT,J)=Y(IT,J+1)-Y(IT,J);
   end
end
plot(TIME,Energ(:,1),'k',TIME,Energ(:,2),'k',TIME,Energ(:,3),'k',TIME,Energ(:,4),'k');
ylabel('energy','FontSize',16)
xlabel('time, in units of lowest frequency mode period','FontSize',16)
title('Figure 3.8.3 - The `classic` FPU experiment','FontSize',16)
text(21,.055,'1','FontSize',16)
text(10,.014,'2','FontSize',16)
text(58,.055,'3','FontSize',16)
```

```
text(35,.035,'4','FontSize',16)
```

```
function dy=fput1(t,y)
N=32;alpha=0.25;
D(N+1)=y(2)-2*y(1)+alpha*((y(2)-y(1))^2-y(1)^2);D(1)=y(N+1);
D(2*N)=y(N-1)-2*y(N)+alpha*(y(N)^2-(y(N)-y(N-1))^2);D(N)=y(2*N);
for I=2:N-1
   D(N+I)=y(I+1)+y(I-1)-2*y(I)+alpha*((y(I+1)-y(I))^2-(y(I)-y(I-1))^2);
   D(I)=y(N+I);
end
dy=D';
```

```
%cp21.m
clear
digits(100)
a=2;
k=input('How many stages (even number)?');
for loop=1:k
   alpha(loop)=a^loop;
end
d=(1/alpha(k-1))+(1/(1+alpha(k)));
power=k-2;
while power>0
   d=1+(1/d);
   d=(1/alpha(power))+(1/d);
   power=power-1;
end
1+(1/d)
```

```
%iso.m
x=linspace(0,pi,1000);
f(1)=0.9;f(2)=0.8;f(3)=0.7;
```

```
num=sin(x);
for i=1:3
   den=tan(pi*f(i)/2);
   y(i,:)=asinh(num/den);
end
for i=1:3
   plot(x,y(i,:),'k-')
   hold on
end
xlabel('x','FontSize',16)
ylabel('y','FontSize',16)
title('Figure S3.3 - Isotherms for infinite strip','FontSize', 16)
```

```
%leapfrog.m
clear
d=1;w=2;r=6;sep=linspace(1,6000,6000);
d=d*5280;w=w*5280/3600;r=r*5280/3600;dt=.01;
for loop=1:6000
   h=sep(loop);x1=0;x2=0;xb=0;bs=1;N=0;s1=r;s2=w;t=0;
   while N==0
      x1=x1+s1*dt;x2=x2+s2*dt;xb=xb+r*dt*bs;t=t+dt;
      if max(x1,x2)>=d
         N=1;
      end
      if N==0
         if bs==0
            if min(x1,x2)>=xb
               if x1<x2
                  xb=x1;s1=r;
               else
                  xb=x2;s2=r;
               end
               bs=1;
```

```
          end
        else
          if max(x1,x2)-min(x1,x2)>=h
            bs=0;
            if s1==r
              s1=w;
            else
              s2=w;
            end
          end
        end
    end
end
if x1==max(x1,x2)
   if s1==r
     bf=1;
   else
     bf=0;
   end
 else
   if s2==r
     bf=1;
   else
     bf=0;
   end
end
x=min(x1,x2);
if bf==1
    T=t+(d-x)/w;
else
    if x<xb
      T=t+((xb-x)/w)+((d-xb)/r);
```

```
        else
          T=t+((d-x)/r);
        end
      end
      time(loop)=T;
end
plot(sep,time,'k-')
xlabel('separation (feet)','FontSize',16)
ylabel('time for second boy to finish (seconds)','FontSize',16)
title('FIGURE 9.2.2 - leapfrog.m number-crunches!','FontSize',16)
text(200,1700,'d = 1 mile','FontSize',16)
text(200,1600,'r = 6 mph','FontSize',16)
text(200,1500,'w = 2 mph','FontSize',16)
```

```
%function csitnikov.m
function csitnikov
options=odeset('AbsTol',1e-10,'RelTol',1e-8);
tspan=[0 6*pi];
uzero=[1.3;0];
[t,u]=ode45(@orbit,tspan,uzero,options);
plot(t,u(:,1),'-k')
xlabel('time','FontSize',16)
ylabel('z(t)','FontSize',16)
title('FIGURE 6.8.3 - z-axis oscillations in a three-body problem','FontSize',16)
   function zderiv=orbit(t,u)
   zderiv=[u(2);-u(1)/((u(1)^2+0.25)^1.5)];
   end
end
```

```
%function esitnikov.m
function esitnikov
options=odeset('AbsTol',1e-10,'RelTol',1e-8);
e=0.15;c=sqrt(1-e^2)/2;k=(1-e^2)/2;
```

```
tspan=[0 100*pi];
uzero=[3;0;0];
[t,u]=ode45(@orbit,tspan,uzero,options);
plot(t,u(:,1),'-k')
xlabel('time','FontSize',16)
ylabel('z(t)','FontSize',16)
title('FIGURE 6.8.5 - Elliptical Sitnikov oscillations (\epsilon=0.15)','FontSize',16)
   function zderiv=orbit(t,u)
   r=k/(1+e*cos(u(3)));
   zderiv=[u(2);-u(1)/((u(1)^2+r^2)^1.5);c/r^2];
   end
end
```

```
%function burrau.m
function burrau
tic
clear
global EMAX EMIN
m1=3;m2=4;m3=5;p12=m1*m2;p13=m1*m3;p23=m2*m3;EMAX=-769/60;EMIN=-769/60;
options=odeset('AbsTol',1e-14,'RelTol',1e-12);
tspan=[0 15];
uzero=[1;0;3;0;-2;0;-1;0;1;0;-1;0;];
[t,u]=ode45(@orbit,tspan,uzero,options);
EMAX,EMIN
plot(u(:,1),u(:,3),':k')
hold on
plot(u(:,5),u(:,7),'--k')
hold on
plot(u(:,9),u(:,11),'-k')
title('FIGURE 6.7.4 - Pythagorean orbits for 0 \leq t \leq 63','FontSize',16)
legend('m_1','m_2','m_3')
size(u)
toc
```

```
   function xyderiv=orbit(t,u)
   v21=[u(5)-u(1),u(7)-u(3)];r21=norm(v21);denom21=r21^3;
   v31=[u(9)-u(1),u(11)-u(3)];r31=norm(v31);denom31=r31^3;
   v32=[u(5)-u(9),u(7)-u(11)];r32=norm(v32);denom32=r32^3;
   a=(u(5)-u(1))/denom21;b=(u(9)-u(1))/denom31;c=(u(7)-u(3))/denom21;
   d=(u(11)-u(3))/denom31;e=(u(9)-u(5))/denom32;f=(u(11)-u(7))/denom32;
   KE=(m1*(u(2)^2+u(4)^2)+m2*(u(6)^2+u(8)^2)+m3*(u(10)^2+u(12)^2))/2;
   PE=-(p12/r21)-(p13/r31)-(p23/r32);
   E=KE+PE;
   EMAX=max(EMAX,E);EMIN=min(EMIN,E);
   xyderiv=[u(2);m2*a+m3*b;u(4);m2*c+m3*d;u(6);...
     -m1*a+m3*e;u(8);-m1*c+m3*f;u(10);-m1*b-m2*e;...
      u(12);-m1*d-m2*f;];
   end
end
```

```
%function rbinary.m
function rbinary
alpha=1;l=2.1;fps=4*pi*pi;
mu=1/(1+alpha);a=mu*l;b=(1-mu)*l;w=2*pi/(sqrt(mu)*(l^(3/2)));
c1=2*w;c2=w*w;
options=odeset('AbsTol',1e-8,'RelTol',1e-5);
tspan=[0 2.2];
uzero=[-0.05;0;0;2*pi];
[t,u]=ode45(@orbit,tspan,uzero,options);
plot(u(:,1),u(:,3),'-k')
axis([-1.5 2.5 -1.3 1.3])
title('FIGURE 6.5.2 - Aliens destroy the Earth!','FontSize',16)
xlabel('unit of distance = 1 AU','FontSize',16)
ylabel('unit of distance = 1 AU','FontSize',16)
text(-0.05,0,'X','FontSize',14)
text(-1.05,0,'*','FontSize',30)
legend('orbit duration = 2.2 years')
```

```
   function xyderiv=orbit(t,u)
   r1=[u(1)-b,u(3)];denom1=norm(r1)^3;
   r2=[u(1)+a,u(3)];denom2=norm(r2)^3;
   c3=1/denom1;c4=alpha/denom2;c5=c2-fps*(c3+c4);c6=b*c3-a*c4;
   xyderiv=[u(2);c1*u(4)+c5*u(1)+fps*c6;u(4);-c1*u(2)+c5*u(3)];
   end
end
```

```
%function fbinary.m
function fbinary
alpha=1;l=2.1;fps=4*pi*pi;
mu=1/(1+alpha);a=mu*l;b=(1-mu)*l;w=2*pi/(sqrt(mu)*(l^(3/2)));
options=odeset('AbsTol',1e-8,'RelTol',1e-5);
tspan=[0 2.2];
uzero=[-0.05;0;0;2*pi];
[t,u]=ode45(@orbit,tspan,uzero,options);
plot(u(:,1),u(:,3),'-k')
title('FIGURE 6.4.3 - Aliens destroy the Earth?','FontSize',16)
xlabel('unit of distance = 1 AU','FontSize',16)
ylabel('unit of distance = 1 AU','FontSize',16)
text(-0.05,0,'X','FontSize',14)
text(-1.1,-.06,'*','FontSize',30)
legend('orbit duration = 2.2 years')
   function xyderiv=orbit(t,u)
   theta=w*t;c4=cos(theta);c5=sin(theta);
   r1=[u(1)-b*c4,u(3)-b*c5];denom1=norm(r1)^3;
   r2=[u(1)+a*c4,u(3)+a*c5];denom2=norm(r2)^3;
   c1=1/denom1;c2=alpha/denom2;c3=c1+c2;
   xyderiv=[u(2);-fps*(c3*u(1)-b*c4*c1+a*c4*c2);u(4);...
      -fps*(c3*u(3)-b*c5*c1+a*c5*c2)];
   end
end
```

```
%function twobody.m
function twobody
alpha=1;M=1;d=2.1;fps=4*pi*pi;
options=odeset('AbsTol',1e-8,'RelTol',1e-5);
tspan=[0 1.8];
uzero=[1;0;0;2*pi];
[t,u]=ode45(@orbit,tspan,uzero,options);
plot(u(:,1),u(:,3),'-k')
title('FIGURE 6.3.2 - Aliens destroy the Earth!','FontSize',16)
xlabel('unit of distance = 1 AU','FontSize',16)
ylabel('unit of distance = 1 AU','FontSize',16)
axis([-0.4 1.2 -1 1])
text(1,0,'X','FontSize',14)
text(-.03,-.04,'*','FontSize',30)
grid on
   function xyderiv=orbit(t,u)
   r1=[u(1),u(3)];r2=[d-u(1),u(3)];
   denom1=norm(r1)^3;denom2=norm(r2)^3;
   xyderiv=[u(2);fps*M*((-u(1)/denom1)+(alpha*(d-u(1))/denom2));...
      u(4);-fps*M*u(3)*((1/denom1)+(alpha/denom2))];
   end
end
```

```
%mass2.m
function mass2
options=odeset('AbsTol',1e-8,'RelTol',1e-5);
tspan=[0 50];
uzero=[1;0;1;0];
[t,u]=ode45(@hanging,tspan,uzero,options);
figure(1)
plot(t,u(:,1),'-k')
hold on
plot(t,u(:,3),'--k')
```

```
title('FIGURE 4.7.1 - Oscillations of the upper (solid) and lower (dashed) masses','FontSize',16)
xlabel('time (t)','FontSize',16)
ylabel('normalized amplitude','FontSize',16)
figure(2)
plot(u(:,1),u(:,2),'-k')
axis([-1.1 1.1 -1 1])
title('FIGURE 4.7.2 - Phase plane portrait for upper mass','FontSize',16)
xlabel('x_{1}','FontSize',16)
ylabel('dx_{1}/dt','FontSize',16)
text(1,0,'X','FontSize',16)
figure(3)
plot(u(:,3),u(:,4),'-k')
axis([-1.5 1.5 -1.1 1.1])
title('FIGURE 4.7.3 - Phase plane portrait for lower mass','FontSize',16)
xlabel('x_{2}','FontSize',16)
ylabel('dx_{2}/dt','FontSize',16)
text(1,0,'X','FontSize',16)
   function xderivative=hanging(t,u)
      xderivative=[u(2);-2*u(1)+u(3);u(4);u(1)-u(3)];
   end
[r,c]=size(u)
u(r,1)
u(r,2)
u(r,3)
u(r,4)
end
```

```
%mass5.m
function mass5
options=odeset('AbsTol',1e-8,'RelTol',1e-5);
tspan=[0 50];
uzero=[1;0;1;0;1;0;1;0;1;0];
[t,u]=ode45(@hanging,tspan,uzero,options);
```

```
figure(1)
plot(t,u(:,1),'-k')
ylabel('normalized amplitude','FontSize',16)
xlabel('time (t)','FontSize',16)
title('FIGURE 4.7.4 - Oscillation of upper-most mass for N=5','FontSize',16)
figure(2)
plot(u(:,1),u(:,2),'-k')
axis([-1 1.1 -1 1])
xlabel('x_{1}(t)','FontSize',16)
ylabel('dx_{1}(t)/dt','FontSize',16)
text(1,0.1,'X','FontSize',16)
title('FIGURE 4.7.5 - Phase plane portrait of upper-most mass for N=5','FontSize',16)
   function xderivative=hanging(t,u)
      xderivative=[u(2);-2*u(1)+u(3);u(4);u(5)+u(1)-2*u(3);...
         u(6);u(7)+u(3)-2*u(5);u(8);u(9)+u(5)-2*u(7);...
         u(10);u(7)-u(9)];
   end
end
```

```
function osc
options=odeset('AbsTol',1e-7,'RelTol',1e-4);
tspan=[0 2];
c=100;
x0=sqrt(0.25-19.62/c);
uzero=[x0;0];
[t,u]=ode45(@wire,tspan,uzero,options);
subplot(311)
plot(t,u(:,1),'-k')
text(0.6,0.2,'c=100')
xlabel('t (seconds)','FontSize',16)
ylabel('x(t)','FontSize',16)
title('FIGURE 4.2.2 - Sinusoidal solutions to (4.2.4)','FontSize',16)
c=400;
```

```
x0=sqrt(0.25-19.62/c);
uzero=[x0;0];
[t,u]=ode45(@wire,tspan,uzero,options);
subplot(312)
plot(t,u(:,1),'-k')
text(0.3,0.25,'c=400')
xlabel('t (seconds)','FontSize',16)
ylabel('x(t)','FontSize',16)
c=1600;
x0=sqrt(0.25-19.62/c);
uzero=[x0;0];
[t,u]=ode45(@wire,tspan,uzero,options);
subplot(313)
plot(t,u(:,1),'-k')
text(0.37,0.35,'c=1600')
xlabel('t (seconds)','FontSize',16)
ylabel('x(t)','FontSize',16)
   function xderivative=wire(t,u)
   xderivative=[u(2);-(4*u(1)*u(2)^2+2*c*u(1)^3+19.62*u(1))/(1+4*u(1)^2)];
   end
end
```

```
%walk.m
N=input('How many walks?');
X=input('X=?');Y=input('Y=?');
X1=X;Y1=Y;
fraction=zeros(1,4);
for loop=1:N
   X=X1;Y=Y1;
   while X>0&&X<100&&Y>0&&Y<100
      xdir=rand;ydir=rand;
      if xdir<0.5
         X=X+1;
```

```
        else
            X=X-1;
        end
        if ydir<0.5
            Y=Y+1;
        else
            Y=Y-1;
        end
    end
    if Y==0
        i=1;
    elseif Y==100
        i=3;
    elseif X==0
        i=2;
    else
        i=4;
    end
    fraction(i)=fraction(i)+1;
end
fraction=fraction/N;
fraction,100*fraction(1)
```

```
%jacobi.m
T=input('What is T?');
U=zeros(101,101);
for c=1:101
    U(1,c)=T;
end
for r=2:100
    for c=2:100
        U(r,c)=T/4;
    end
```

```
end
A=U;
for loop=1:5000
   for r=2:100
      for c=2:100
         A(r,c)=(U(r-1,c)+U(r+1,c)+U(r,c-1)+U(r,c+1))/4;
      end
   end
   U=A;
end
```

```
%space.m
sum=0;f=1/3;
for loop=1:10000000
   X1=rand/2;X2=0.5+rand/2;
   if X2-X1<f
      sum=sum+1;
   end
end
sum/10000000
```

```
%all3.m
h=sqrt(3);total=0;
for loop=1:10000000
   x=rand;y=h*rand;
   if (y/x)>h
      x=1+x;y=h-y;
   end
   d1=x^2+y^2;
   d2=(x-2)^2+y^2;
   d3=(x-1)^2+(y-h)^2;
   if d1<2&&d2<2&&d3<2
      total=total+1;
```

```
   end
end
total/10000000
```

```
%cf.m
clear
digits(100)
a=10;
k=input('How many stages (at least 2)?');
for j=1:k-1
   alpha(j)=a^j;
end
d=alpha(k-1);
d=d+1/d;
d=1/d;
for loop=k-2:-1:1
   d=alpha(loop)+d;
   d=1/d;
   d=alpha(loop)+d;
   d=1/d;
end
1+1/(1+d)
```

```
%eyeb.m
function eyeb
options=odeset('AbsTol',1e-112,'RelTol',1e-9);
t=[0 15.9];
uzero=[0;0];
[t,u]=ode45(@frontis,t,uzero,options);
plot(u(:,1),u(:,2),'-k')
title('FIGURE 1.5.1 - The Eyes Of Murphy Are Watching You!','FontSize',16,'FontAngle','ital')
xlabel('x','FontSize',16)
ylabel('dx/dt','FontSize',16)
   function xderivative=frontis(t,u)
```

```
    xderivative=[u(2);3*sin(3*0.9284513988*t)-u(1)+u(1)^3/6];
  end
end
```
