Dynamics of pricing and market quality parameters, focusing on two basic factors that impinge on the performance of the market:

- Market microstructure.
- Traders’ horizon.

We will look at

1. Properties of the price process when there is a risk-neutral market making sector.
2. Impact of risk-averse market makers.
8.1 Dynamic Competitive Rational Expectations

Dynamic extension of a version of the static rational expectations model with:

- Informed risk averse agents and noise traders
- Risk neutral competitive market makers
- All traders (except noise traders) conditioning on current prices.
8.1 Dynamic Competitive Rational Expectations

8.1.1 Price Formation with a Competitive Risk-neutral Fringe

Model

- A single risky asset, with random fundamental value $\theta$, and a riskless asset (with unitary return) are traded for $T$ periods in a market with risk averse informed agents and noise traders with the intermediation of risk neutral competitive market makers.
- At period $T + 1$ the fundamental value $\theta \left( p_{T+1} = \theta \right)$ is realized.
- Noise traders’ demands $\{u_t\}_{t=1}^T$ follow a random walk independent of all other random variables in the model with $E[u_t] = 0$ and $\text{Var}[u_t] = \sigma_u^2$.
- At any period there is a continuum of informed traders. Informed trader $i$ in period $t$ receives a normally distributed signal $s_{it} = \theta + \epsilon_{it}$, where $\theta$ and $\epsilon_{it}$ and are uncorrelated, and errors are also uncorrelated across agents and periods (and with noise trade).
- The precision of the signals $\tau_{\epsilon_t}$ is the same across agents in the same period but may be different across periods.
The model captures cases of traders having either a *long* or a *short* horizon: In period $t$ agent $i$ has available the vector of private signals $s^t_i = \{s_{i1}, s_{i2}, \ldots, s_{it}\}$.

- This could be the case if long-lived traders have a long horizon.
- If traders have a short horizon then they may be long-lived but myopic and they do not forget information.
- Alternatively, traders are short-lived with a different generation of informed agents coming to market every period, each member of generation $t$ inheriting the private information of a member of generation $t - 1$. 
8.1 Dynamic Competitive Rational Expectations

8.1.1 Price Formation with a Competitive Risk-neutral Fringe

- Sufficient statistic for $s^t_k$ in the estimation of $\theta$ is
  \[ \tilde{s}_{it} = \frac{\sum_{k=1}^{t} T \epsilon_k s_{ik}}{\sum_{k=1}^{t} T \epsilon_k} . \]

  We restrict attention to linear equilibria.

- In any linear equilibrium the strategy of trader $i$ in period $t$ depends on $\tilde{s}_{it}$ and public information.

- Informed agent $i$ in period $t$ has a demand schedule $X_{it}(\tilde{s}_{it}, p^{t-1}, \cdot)$.

- At period $t$ the competitive market making sector observes the aggregate limit order book $L_t(\cdot)$:

  \[ L_t = \int_0^1 x_{it} \, di - \int_0^1 x_{it-1} \, di + u_t . \]
8.1 Dynamic Competitive Rational Expectations

8.1.1 Price Formation with a Competitive Risk-neutral Fringe

- Trader $i$ submits in period $t$ the (net) demand schedule

$$X_{it}(\tilde{s}_{it}, p^{t-1}, \cdot) - X_{it-1}(\tilde{s}_{it}, p^{t-2}, p_{t-1}).$$

- Consider a candidate linear equilibrium:

$$X_{it}(\tilde{s}_{it}, p^{t-1}, p_t) = a_{it}\tilde{s}_{it} + \varphi_{it}(p^t).$$

- The noisy limit order book at stage $t$ is given by:

$$L_t(p_t) = z_t + \varphi_t(p_t) - \varphi_{t-1}(p^{t-1}),$$

where $z_t = \Delta a_t \theta + u_t$, $a_t = \int_0^1 a_{it} \, di$, $\Delta a_t = a_t - a_{t-1}$ (with $a_0 \equiv 0$).

- This follows using the convention (with $a_{it}$ uniformly bounded for any $t$)

$$\int_0^1 a_{it}\tilde{s}_{it} \, di = a_t \theta + \int_0^1 a_{it} \epsilon_{it} \, di = a_t \theta \text{ (a.s.).}$$
8.1 Dynamic Competitive Rational Expectations

8.1.1 Price Formation with a Competitive Risk-neutral Fringe

- New information in $L_t(\cdot)$: $z_t = \Delta a_t \theta_t + u_t$.
- The competitive market making sector sets $p_t$ conditional on past public information and the new information in $L_t(\cdot)$.
- Past public information is summarized in the sequence $z^{t-1} = \{z_1, z_2, \ldots, z_{t-1}\}$ o.e. to $p^{t-1} = \{p_1, p_2, \ldots, p_{t-1}\}$.
- The price set by the competitive market making sector is then

$$p_t = E[\theta | z^t] = \lambda_t z_t + (1 - \lambda_t \Delta a_t) p_{t-1}.$$
8.1 Dynamic Competitive Rational Expectations

8.1.1 Price Formation with a Competitive Risk-neutral Fringe

- Depth:
  \[ \lambda_t = \frac{\Delta a_t \tau_u}{\tau_t}. \]

- The informativeness or precision of \( p_t \),
  \[ \tau_t = \frac{1}{\text{Var}[\theta|z^t]} \]
  \[ = \tau_\theta + \tau_u \sum_{k=1}^{t} (\Delta a_t)^2. \]

- An explicit expression for the price is
  \[ p_t = \frac{\tau_\theta \bar{\theta} + \tau_u \sum_{k=1}^{t} \Delta a_t z_t}{\tau_t}. \]
8.1 Dynamic Competitive Rational Expectations

8.1.1 Price Formation with a Competitive Risk-neutral Fringe

Proposition

Let \( p_0 \equiv \bar{\theta} \) and \( p_{T+1} \equiv \theta \). At any linear equilibrium for \( t = 1, 2, \ldots, T \):

\[
p_t = E[\theta | z^t]
= \lambda_t z_t + (1 - \lambda_t \Delta a_t) p_{t-1},
\]

with \( \lambda_t = \Delta a_t \tau_u / \tau_t \), \( z_t = \Delta a_t \theta + u_t \) and \( \tau_t = \tau_\theta + \tau_u \sum_{k=1}^t (\Delta a_t)^2 \).
Properties of the Price Process

- Informed trader $i$’s desired position in period $t$:
  \[ X_t(\tilde{s}_{it}, p_t). \]

- Market depth $\lambda^{-1} = \tau_t / \Delta a_t \tau_u$ increases with the amount of public information.

- Historical prices or “technical analysis” are superfluous for decision making, as the current price is a sufficient statistic for all public information.

- Competitive risk neutral market making sector implies that the market is (semi-strong) efficient,
  \[ E[p_t|p^{t-1}] = E[p_t|p_{t-1}], \text{ and } \text{Cov}[\Delta p_t, \Delta p_{t-1}] = 0, \]
  and prices exhibit no drift: $E[\Delta p_t|\Delta p_{t-1}] = E[\Delta p_t] = 0$. ♦
8.1 Dynamic Competitive Rational Expectations

8.1.1 Price Formation with a Competitive Risk-neutral Fringe

- Prices are biased in the sense of regression towards the mean: 
  \[ \text{sign}(E[\theta - p_t|\theta]) = \text{sign}(\theta - \bar{\theta}), \text{ since} \]
  \[ E[\theta - p_t|\theta] = \frac{\tau \theta}{\tau_t} (\theta - \bar{\theta}). \]

- The average of informed investors’ expectations about the fundamental value \( \theta \) at period \( t \) is a convex combination of \( \theta \) and \( p_t \):
  \[ \int_0^1 E[\theta|\tilde{s}_{it}, p_t] \, di = \frac{\sum_{k=1}^t \tau_{t_k \theta} + \tau_t p_t}{\sum_{k=1}^t \tau_{t_k} + \tau_t}. \]
  This implies
  \[ |E[\theta - p_t|\theta]| > \left| E \left[ \theta - \int_0^1 E[\theta|\tilde{s}_{it}, p_t] \, di \right| \theta \right|. \]
In a semi-strong efficient market, trading behavior affects the distribution of volatility over time but not its total magnitude. More informative prices bring forward the resolution of uncertainty.

The martingale property of prices implies:

1. **Unconditional volatility of** $p_t$: $\text{Var}[p_t] = \tau_t^{-1} - \tau_t^{-1}$ is nondecreasing in $t$ ($\tau_t$ is also nondecreasing in $t$).
2. **Conditional volatility of** $p_t$: $\text{Var}[p_t|p_{t-1}] = \tau_{t-1} - \tau_{t-1}$.
3. **Total volatility** is constant:

$$
\sum_{t=1}^{T+1} \text{Var}[p_t|p_{t-1}] = \text{Var}\left[\sum_{t=1}^{T+1} \Delta p_t \right] = \tau_\theta^{-1}.
$$

The dynamics of conditional volatility $\text{Var}[p_t|p_{t-1}]$ depend on the convexity/concavity properties of $\text{Var}[\theta|p_t]$ with respect to $t$. 

8.1 Dynamic Competitive Rational Expectations

8.1.1 Price Formation with a Competitive Risk-neutral Fringe

In a semi-strong efficient market prices are volatile because they are informative. This implies that when information revelation accelerates the conditional volatility of prices also increases:

We have that $\text{Var}[p_t|p_{t-1}] = \tau_{t-1}^{-1} - \tau_t^{-1}$ and $\text{Var}[\theta|p_t] = \tau_t^{-1}$. Therefore,

$$\text{Var}[p_{t+1}|p_t] \geq \text{Var}[p_t|p_{t-1}] \iff \text{Var}[\theta|p_t] - \text{Var}[\theta|p_{t+1}] \geq \text{Var}[\theta|p_{t-1}] - \text{Var}[\theta|p_t].$$

Rearranging terms the inequality is

$$\text{Var}[\theta|p_t] - \text{Var}[\theta|p_{t-1}] \geq \text{Var}[\theta|p_{t+1}] - \text{Var}[\theta|p_t],$$

and the result follows.
Consider the market set up of the last section and suppose that any informed trader $i$ has a long horizon and maximizes the expectation of the utility of final wealth:

$$ E[U(W_{iT})] = -E[\exp\{-\rho W_{iT}\}] , $$

where

- $\rho > 0$,
- $W_{iT} = \sum_{k=1}^{T} \pi_{ik}$, $\pi_{ik} = (p_{t+1} - p_t)x_{ik}$.
- $W_{i0} = 0$, w.l.o.g.
8.1 Dynamic Competitive Rational Expectations

8.1.2 Dynamic Trading with Long-term Investors

- At period $t$ an informed agent receives a private signal $s_{it}$ about $\theta$.
- A linear equilibrium the information of trader $i$ in period $t$,
  \[ \{s^t_i, p^t\} \]
  is summarized by \[ \{\tilde{s}_{it}, p_t\} \].
- No “infinite-regress” problem (Townsend (1983)).

**Proposition**

*With long-term informed traders there is a unique linear PBE. The strategy for trader $i$ in period $t = 1, 2, \ldots, T$ is given by*

\[
X_t(\tilde{s}_{it}, p_t) = a_t(\tilde{s}_{it} - p_t), \quad \text{with} \quad a_t = \frac{1}{\rho} \sum_{k=1}^{t} \tau_{\epsilon_k}.
\]

“Static” strategies.
8.1 Dynamic Competitive Rational Expectations

8.1.2 Dynamic Trading with Long-term Investors

Sketch of proof:

Consider a candidate linear equilibrium.

- We know that the previous Proposition applies and that prices are normally distributed and a sufficient statistic for public information.

- Starting from the last period:

  \[ X_T(\tilde{s}_iT, p_T) = a_T(\tilde{s}_iT - p_T), \text{ where } a_T = \frac{1}{\rho} \sum_{k=1}^{T} \tau \epsilon_t. \]

- Using the profits obtained in the last period \( x_{iT-1} \) optimizes

  \[ E \left[ -\exp \left\{ -\rho \phi_{iT-1} \right\} \mid s_i^{T-1}, p^{T-1} \right], \]

  where letting \( x_{iT} = a_T(\tilde{s}_iT - p_T), \)

  \[ \phi_{iT-1} = (p_T - p_{T-1}) x_{iT-1} + \frac{\rho}{2} \left( \sum_{k=1}^{T} \tau \epsilon_k + \tau T \right)^{-1} x_{iT}^2. \]
8.1 Dynamic Competitive Rational Expectations

8.1.2 Dynamic Trading with Long-term Investors

- A sufficient statistic for \( \{s_i^{T-1}, p^{T-1}\} \) in the estimation of \( p_T - p_{T-1} \) and \( \tilde{s}_iT - p_T \) is \( \{\tilde{s}_iT-1, p_{T-1}\} \).

- The optimal position consists of a term similar to a static CARA demand

\[
\frac{E[p_T|\tilde{s}_iT-1, p_{T-1}] - p_{T-1}}{\rho \kappa}, \quad \text{for some } \kappa > 0,
\]

plus a term proportional to the expected position in period \( T \), given that expected returns change over time.

- The possibility to trade in the future provides a hedge against adverse price movements. In the present context this hedge exactly compensates for the risk of price changes, and the position of a trader is like in a static market where no price changes occur:

\[
x_{iT-1} = E[x_{iT}|\tilde{s}_iT-1, p_{T-1}] = a_{T-1}(\tilde{s}_iT-1 - p_{T-1}),
\]

where \( a_{T-1} = \rho^{-1} \sum_{k=1}^{T-1} \tau \epsilon_k \).
The net trading intensity at period $t$: $\Delta a_t = \rho^{-1} \tau_{\epsilon_t}$.

As $t$ increases and traders receive more information, $\tau_t$ increases and $\text{Var}[p_t]$ increase.

Market makers take the counterpart of the order flow:

$$-(\Delta x_t + u_t) = -d_t(p_t - p_{t-1}) \text{ with } d_t = \lambda_t^{-1} - a_t.$$  

Two special cases are of interest:

1. Constant flow of information
2. Concentrated information arrival
8.1 Dynamic Competitive Rational Expectations

8.1.2 Dynamic Trading with Long-term Investors

**Constant flow of information**

- If $\tau_{\epsilon_t} = \tau_{\epsilon_1}$, for all $t$ then for any $t$, $\Delta a_t = \rho^{-1}\tau_{\epsilon_1}$.
- Consequence:

  $$\tau_t = \tau_\theta + \tau_u t a^2, \quad \lambda_t^{-1} = \tau_t / \tau_u \Delta a_t,$$

  and the price precision and depth grow linearly with $t$.
- The conditional volatility, $\text{Var}[p_t|p_{t-1}] = \tau_{t-1}^{-1} - \tau_t^{-1}$, decreases with $t$.

**Concentrated information arrival**

- If $\tau_{\epsilon_t} = 0$, for $t \geq 2$ then $\Delta a_t = 0$ for $t \geq 2$.
- “Buy-and-hold” strategy.
- The market price is stationary from period 1 on: $p_t = E[\theta|p_1] = p_1$.
- Foundation for the static rational expectations model because adding more rounds of trade is superfluous.
8.1.3 Summary

1. No room for technical analysis with a competitive risk neutral market making sector.
2. Volatility is driven by information.
3. With concentrated information arrival “buy and hold” is optimal.
8.2 The Impact of Risk-averse Market Makers

- In the market considered above there is no room for “technical analysis.” ♣

- Cespa and Vives (2007) consider a version of the previous model in which there is no competitive risk neutral fringe and where risk averse informed traders perform also a market making activity (the model for $T = 2$ is considered by Brown and Jennings (1989), and Cespa (2002)).
8.2 The Impact of Risk-averse Market Makers
8.2.1 Technical Analysis

**Proposition**

Suppose that there is no risk neutral fringe. Then with long-term informed traders there is a unique linear PBE. The strategy of trader $i$ in period $t = 1, 2, \ldots, T$ is given by:

$$X_t(\tilde{s}_{it}, p^t) = a_t(\tilde{s}_{it} - p_t) + c_t \left(E[\theta|z^t] - p_t\right) \text{ with } a_t = \frac{1}{\rho} \sum_{k=1}^{t} \tau_{\epsilon_t}$$

and $c_t = 1/(\rho \tau_t)$. Prices are given by

$$p_t = \lambda_t z_t + (1 - \lambda_t \Delta a_t)p_{t-1},$$

$p_0 \equiv \bar{\theta}$, $p_T \equiv \theta$ with, $\lambda_t = (\tau_t + \sum_{k=1}^{t} \tau_{\epsilon_t})^{-1}(\rho + \tau_u \Delta a_t)$,

$z_t = \Delta a_t \theta + u_t$, $\Delta a_t = a_t - a_{t-1}$ with $a_0 = 0$, and

$\tau_t = \tau_\theta + \tau_u \sum_{k=1}^{t} (\Delta a_t)^2$. 
8.2 The Impact of Risk-averse Market Makers

8.2.1 Technical Analysis

Now,

- In period $t$, $\{\tilde{s}_i, p_t\}$ is *not* sufficient for $\{s_i^t, p^t\}$ in the estimation of $\theta$.
- At period $t$ demands have the same form as the equilibrium demands in the static model of Chapter 4 with information $\{\tilde{s}_i, p^t\}$.
- Again, dynamic strategies collapse to static ones.
- Two components of a trader’s strategy:

$$a_t(\tilde{s}_i - p_t), \text{ and } c_t \left( E[\theta | z^t] - p_t \right)$$

capturing speculation and market making.
8.2 The Impact of Risk-averse Market Makers

8.2.1 Technical Analysis

- Depth:
  \[
  \lambda_t = \frac{\tau_u \Delta a_t}{\tau_t + \sum_{k=1}^{t} \tau \epsilon_k} + \frac{\rho}{\tau_t + \sum_{k=1}^{t} \tau \epsilon_k}.
  \]

  (1) adverse selection component.
  (2) risk-bearing component.

- With a competitive risk neutral market making sector:
  \[
  \lambda_t = \frac{\tau_u \Delta a_t}{\tau_t}.
  \]

- If \( \tau \epsilon_t = 0 \) then \( \Delta a_t = 0 \), and \( p_t \) is just \( p_{t-1} \) plus noise:
  \( p_t = p_{t-1} + \lambda_t u_t \), with \( \lambda_t = \rho(\tau_t + \sum_{k=1}^{t} \tau \epsilon_k)^{-1} \).

- Even though \( p_t \) reveals \( u_t \) this does not provide any further information on \( \theta \) because \( u_t \) and \( u_{t-1} \) are uncorrelated.
Consider $t = 2$ for illustrative purposes:

- Conditional volatility is given by

$$\text{Var}[p_2|p_1] = \begin{cases} 
\alpha_2^2 \left( \tau_1^{-1} - \tau_2^{-1} \right) & \text{if } \Delta a_2 > 0 \\
\lambda_2^2 \tau_u^{-1} & \text{if } \Delta a_2 = 0,
\end{cases}$$

- The term $\tau_1^{-1} - \tau_2^{-1}$ reflects the arrival of new information.
- The factor

$$\alpha_2 = \frac{\lambda_2 \tau_2}{\tau_u \Delta a_2},$$

arises because of traders’ risk aversion.

- Conditional volatility can be larger or smaller than in a semi-strong efficient market depending on whether $\alpha_2$ is larger or smaller than 1.
8.2 The Impact of Risk-averse Market Makers
8.2.1 Technical Analysis

**Excess volatility**

- Generate examples where the volatility of prices is larger than the volatility of fundamentals:

\[
\text{Var}[p_1] = \alpha_1^2 \left( \tau_{\theta}^{-1} - \tau_1^{-1} \right), \text{ with } \alpha_1 \equiv \frac{\lambda_1 \tau_1}{a_1 \tau_u}.
\]

When \( \tau_{\rho_1} = \rho = 1 \), \( \tau_u = 0.1 \) and \( \tau_{\theta} < 9.9 \) then \( \text{Var}[p_1] > \tau_{\theta}^{-1} \).

- This may be termed “excess volatility” but it is an equilibrium phenomenon derived from the presence of risk averse market making traders who decrease the depth of the market by not fully accommodating shocks.
8.2 The Impact of Risk-averse Market Makers

8.2.1 Technical Analysis

**Technical Analysis**

- Technical analysis is useful because $p_t$ is not a sufficient statistic for $z^t \equiv \{z_1, z_2, \ldots, z_t\}$ or $p^t \equiv \{p_1, p_2, \ldots, p_t\}$.

- Let $T = 2$, then $E[\theta|z^2] = E[\theta|p^2]$ depends on $p_1$ and $p_2$ because $p_1$ has additional information on $\theta$ even conditioning on $p_2$.

- Let $X_t(\theta, p^t) = \int_0^1 X_t(\tilde{s}_{it}, p^t) \, di$, denote the aggregate position demand for the risky asset for informed traders in period $t$.

- Then the market clearing condition at the second period is given by

$$X_2(\theta, p_1, p_2) - X_1(\theta, p_1) + u_2 = X_2(\theta, p_1, p_2) + u_1 + u_2 = 0,$$

since $X_1(\theta, p_1) + u_1 = 0$, from market clearing in the first period. If there is no additional noise trading in period 2 ($u_2 = 0$), $p_1$ and $p_2$ perfectly reveal $\theta$. 
8.2 The Impact of Risk-averse Market Makers

8.2.1 Technical Analysis

**Price Unbiasedness**

- Prices in period 2, as in the static case, are just the average expectations of investors about the fundamental value plus noise:

\[
p_2 = \int_0^1 E[\theta|\tilde{s}_{i2}, z^2] di + \frac{\rho}{\tau_2 + \tau_{\epsilon_1} + \tau_{\epsilon_2}} (u_1 + u_2).
\]

- More interestingly, in the first period:

\[
p_1 = \int_0^1 E[\theta|s_{i1}, z_1] di + \rho(\tau_1 + \tau_{\epsilon_1})^{-1} u_1.
\]

This is not obvious (but follows from the static CARA form of demand and market clearing).

- Cespa and Vives (2007) show that a similar expression can be derived for any \(p_t\) in the \(T\)-period case.
8.2 The Impact of Risk-averse Market Makers
8.2.2 Trade without News and No-trade Theorem

Grundy and McNichols (1989): In a dynamic setting trade can occur without
- New information
- Additional noise trading

In short, trade may be “self-generating.”
Model

- 2 periods, signal in period 1 only: \( s_i = \theta + \eta + \epsilon_i, \eta \sim N(0, \tau_{\eta}^{-1}) \) is a common error term and the average signal reveals now \( \theta + \eta \).

- \( u_2 = 0 \).

Result: in the second round of trade:

1. There is always a “no-trade” outcome.
2. However, a linear fully revealing equilibrium (of \( \theta + \eta \)) also exists at the 2nd round if \( \tau_\eta \) is large enough.

- At the first round and for finite \( \tau_\eta \) there are two (revealing) linear equilibria.
8.2 The Impact of Risk-averse Market Makers

8.2.2 Trade without News and No-trade Theorem

- Prices and allocations may change even with no new information at the second round (and no additional noise trading)!
- A contradiction to the No-Trade-Theorem?
- NO! Beliefs are not **concordant**.
- Role of public signal.
8.2 The Impact of Risk-averse Market Makers

8.2.3 Dynamic Patterns of Volume and Prices

He and Wang (1995): $T$-period generalization, liquidation value $\theta + \eta$, $\eta \sim N(0, \tau_\eta^{-1})$ residual uncertainty.

- Information available to trader $i$ in period $t$ is $I_{it} = \{s^t_i, y^t, p^t\}$, $y_t = \theta + \nu_t$ public signal.

- The stock of noise trade follows AR(1) process:

$$\hat{u}_t = \zeta \hat{u}_{t-1} + e_t, \text{ with } 0 < \zeta < 1$$

with $\{e_t\}_{t=1}^{T}$ an i.i.d. normally distributed noise random process.

- The random noise trade increment in period $t$ is thus $\hat{u}_t - \hat{u}_{t-1}$. If
  - $\zeta = 1$, $\{\hat{u}_t\}$ follows a random walk.
  - $\zeta = 0$, the stock of noise trading is i.i.d. across periods.
8.2 The Impact of Risk-averse Market Makers
8.2.3 Dynamic Patterns of Volume and Prices

- $z_t$ is public information inferred from aggregate orders $\Rightarrow p_t$ is a convex combination of $z_t$ and $E[\theta | z^t, y^t]$.
- The desired position of trader $i$ in period $t$ is a linear function of $E[\theta | I_{it}] - E[\theta | z^t, y^t]$ and $E[\hat{u}_t | I_{it}]$. The first part is linked to speculative trading and the second to accommodating the noise trade shock.
- Multiple equilibria.

Two cases are considered: Homogeneous and Differential information.
8.2 The Impact of Risk-averse Market Makers

8.2.3 Dynamic Patterns of Volume and Prices

Homogeneous (Perfect) Information

- Benchmark: \( \tau_{\varepsilon_1} = \infty \).
- Price equals \( \theta \) plus a risk premium linear in the share position of investors \( \hat{u}_t \).
- Volume is determined by the exogenous random noise trade increments \( E[|\hat{u}_t - \hat{u}_{t-1}|] \) and market depth decreases as \( t \) approaches \( T \).
- Reason: with \( \zeta < 1 \), supply shocks display mean reversion.
8.2 The Impact of Risk-averse Market Makers
8.2.3 Dynamic Patterns of Volume and Prices

Differential Information

- Price at $t$ depends on the investors’ expectations of the future payoff of the stock plus a risk premium, but it is not equal to investors’ average expectations at this date plus a risk premium.
- Reason: correlation in noise trading increments and residual uncertainty in the liquidation value.
- The price process is no longer a martingale.
- With concentrated arrival of information for $t > 1$; investors establish their positions at $t = 1$ but trade persists.
- Effect of public signals: high volume around announcement.
8.2 The Impact of Risk-averse Market Makers
8.2.3 Dynamic Patterns of Volume and Prices

**Price Bias**

- The presence of residual uncertainty and/or correlated patterns of noise trade increments together with market making by risk averse traders can explain departures of prices from fundamentals.
- Prices can be farther away from fundamentals than the average expectations of investors in period $t$:

$$\left| E[p_t - \theta|\theta] \right| > \left| E \left[ \int_0^1 E[\theta|I_{it}] di - \theta|\theta \right] \right|.$$

Then there is over-reliance of traders on public information. Under-reliance when the inequality above is reversed.
8.2 The Impact of Risk-averse Market Makers

8.2.3 Dynamic Patterns of Volume and Prices

Cespa and Vives (2007) characterize in a 2-period model the parameter constellations \((\zeta, \tau_{-1})\) for which over- or under-reliance on public information occurs:

- In period 2 the market behaves as in the static case.
- Same result in period 1 when noise trading follows a random walk and there is no residual uncertainty \((\zeta, \tau_{-1}) = (1, 0)\).
- No longer the case with noise trade persistence/residual uncertainty: For low (high) values of \((\zeta, \tau_{-1})\) there is over-reliance (under-reliance) on public information in period 1.
- The weight of public information on the price depends on the traders reaction to order imbalances.
- This defines a Hayekian and a Keynesian region.
8.2 The Impact of Risk-averse Market Makers

8.2.4 Summary

1. When market makers are risk averse:
   1. Technical analysis is justified.
   2. No price bias (in relation to average expectations) under some conditions.
   3. There may be “excess volatility.”

2. In a dynamic setting trade may be “self-generating.”

3. Asymmetric information may explain peaks in volume around public announcements.

4. Correlation in noise trade demand and the presence of residual uncertainty in the liquidation value influences the bias in prices in relation to the average expectations of investors.
Attention has been drawn recently to the effects of investors’ short horizons in financial markets. This short-termism may come about, for example, because of

1. Liquidity needs of investors.
2. Incentive reasons related to the evaluation of the performance of money managers.
3. Difficulties associated with financing long-term investment in the presence of capital market imperfections.
8.3 Dynamic Trading with Short-term Investors
8.3.1 Short-term Traders and Risk-neutral Market Makers

Consider the context of Section 8.1.1 but now informed traders are assumed to maximize the utility of the short-run return.

- \( \pi_{it} = (p_{t+1} - p_t)x_{it} \) is \( i \)'s short-run profits.
- At stage \( t \) a strategy for agent \( i \): \( X_t(\tilde{s}_{it}, \cdot) \).

**Proposition**

Linear equilibria of the dynamic \( T \)-period trading game with short investment horizons exist. The equilibrium strategy of trader \( i \) is given (implicitly) for \( t = 1, 2, \ldots, T \) by

\[
X_t(\tilde{s}_{it}, p_t) = a_t(\tilde{s}_{it} - p_t), \text{ with } a_t = \frac{\rho}{(\sum_{k=1}^{t} \tau_{\epsilon_k})^{-1} + \tau_t + 1^{-1}},
\]

where \( \tau_t = \tau_\theta + \tau_u \sum_{k=1}^{t} (\Delta a_t)^2 \), and \( a_T = \rho^{-1}(\sum_{k=1}^{T} \tau_{\epsilon_k}) \).
Sketch of proof:

1. From the previous result we know that
   \[ p_{t+1} - p_t = \lambda_{t+1}(\Delta a_{t+1}(\theta - p_t) + u_{t+1}) \]. Then, we can compute
   \[
   E[p_{t+1} - p_t | \tilde{s}_{it}, p_t] = \lambda_{t+1}\Delta a_{t+1} \frac{\sum_{k=1}^{t} \tau_{\epsilon_k}}{\tau_t + \sum_{k=1}^{t} \tau_{\epsilon_k}} (\tilde{s}_{it} - p_t)
   \]
   \[
   \text{Var}[p_{t+1} - p_t | \tilde{s}_{it}, p_t] = \lambda_{t+1}^2 ((\Delta a_{t+1})^2 \text{Var}[\theta | \tilde{s}_{it}, p_t] + \tau_u^{-1}).
   \]
2. Maximization of a CARA utility function by trader \( i \) at time \( t \) yields
   \[
   X_t(\tilde{s}_{it}, p_t) = \frac{E[p_{t+1} - p_t | \tilde{s}_{it}, p_t]}{\rho \text{Var}[p_{t+1} - p_t | \tilde{s}_{it}, p_t]}.
   \]
   Substituting the expressions above an identifying we obtain
   \[
   a_t = \frac{\tau_{t+1}(\sum_{k=1}^{t} \tau_{\epsilon_k})}{\rho(\tau_t + \sum_{k=1}^{t} \tau_{\epsilon_k})}, \text{ for } t = 1, 2, \ldots, T - 1.
   \]
3. The above system of equations \( a_t = g_t(a_1, a_1, \ldots, a_{t+1}) \) always admits a real solution.
8.3 Dynamic Trading with Short-term Investors
8.3.1 Short-term Traders and Risk-neutral Market Makers

Remark

1. With concentrated arrival of information (i.e. \( \tau \epsilon_t = 0 \) for \( t \geq 2 \)),
\[
a_t = \rho^{-1}(\tau^{-1}_{\epsilon_1} + \tau^{-1}_{t+1})^{-1} > 0 \quad \text{and} \quad a_T = \rho^{-1}\tau_{\epsilon_1} = a.
\]

2. With a constant flow of information (i.e. \( \tau \epsilon_t = \tau \epsilon_1 \) for \( t \geq 1 \)),
\[
a_t = \rho^{-1}((t\tau_{\epsilon_1})^{-1} + \tau^{-1}_{t+1})^{-1} \quad \text{and} \quad a_T = Ta.
\]

3. The uniqueness of the linear equilibrium is not asserted. However, for the \( T = 2 \) case uniqueness is easily established (at least for the case of concentrated information arrival) and simulations also support the uniqueness conjecture for larger \( T \). We will see that if market makers are risk averse then there are multiple equilibria.
8.3 Dynamic Trading with Short-term Investors
8.3.1 Short-term Traders and Risk-neutral Market Makers

Properties of the equilibrium.

- $a_t$ is strictly increasing in $t$ and therefore $\Delta a_t > 0$, and $\tau_t$ and $\text{Var}[p_t]$ are strictly increasing in $t$.
- Note: the closer is $p_{t+1}$ to $\theta$, the more intensely informed speculators want to trade.
- $a_t^{LT} > a_t^{ST}$ but $\sum_{k=1}^{T} \Delta a_k = a_T$ independently of trading horizons: in terms of net trading intensities the long-term and the short-term cases differ in the temporal distribution of the same aggregate.
### Numerical Simulations

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8.3 Dynamic Trading with Short-term Investors

8.3.1 Short-term Traders and Risk-neutral Market Makers

Price informativeness and traders’ horizons
Does price informativeness depend on traders’ horizons?

- Price informativeness in period $T$ is given by

$$\tau_T = \tau_\theta + \tau_u \sum_{k=1}^{T} (\Delta a_k)^2,$$

and the total net trading intensity is

$$\sum_{k=1}^{T} \Delta a_k = a_T,$$

in both the long-term and the short-term cases.

- The precision $\tau_t$ is thus akin to an inequality index of the variables $\Delta a_t$ (which add up to $a_T$).

- Thus, $\tau_t$ is smaller the more equally distributed are the increments, the minimum being reached for equal increments every period.
Concentrated arrival of information

- With long-term agents there is informed trading only in the first period.
- Short horizons imply that informed agents will trade in every period.
- Result: $\tau_{LT} > \tau_{ST}$.

Constant flow of information

- With long-term traders net trading intensities are equal across periods to the static trading intensity ($a$).
- With short-term traders net trading intensities differ across periods.
- Result: $\tau_{LT} < \tau_{ST}$. 
In the presence of short-term traders, risk averse market makers are responsible for

- Interesting pricing dynamics (e.g. the appearance of “excess volatility” and substantial divergences of asset prices from fundamental values can be explained without resort to the presence of systematic misperceptions (e.g. De Long et al. (1990 a,b) or Scheinkman and Xiong (2003))).

- Multiple equilibria.
Short-term traders and multiple equilibria

Cespa (2002): two-period version of the model of Section 8.2.1, where

- There is no competitive risk neutral market making sector.
- A proportion of informed traders $\mu$ have a short horizon (one period), while $1 - \mu$ have a long horizon (two periods).

Linear equilibria exist and display a combination of elements from the short-term and long-term traders’ cases:

- In the first period traders have an additional market making motive on top of absorbing liquidity shocks.
- Short-term traders also modify their trading intensity in period 1 with respect to the case with competitive risk neutral market makers by a factor of $\alpha_2$, which is a measure of the effect of the trader’s risk aversion on second period market depth.
With concentrated arrival of information there are two equilibria: one with low (conditional) volatility and another with high (conditional) volatility.

- In the first there is high trading intensity and price informativeness in the first period, and positive slope of aggregate excess demand in the second period. ♣

- In the second there is low trading intensity and price informativeness in the first period, and negative slope of aggregate excess demand in the second period. ♠
8.3 Dynamic Trading with Short-term Investors
8.3.2 Short Horizons with Risk-averse Market Makers

**Short term trading and price departures from fundamentals**
Consider the same model as in Section 8.3.1 with short term traders:

- **Assume** \( T = 2 \).
- **A general pattern for noise trade correlation** \( \hat{u}_t = \zeta \hat{u}_{t-1} + e_t \) with \( 0 < \zeta < 1 \).
- **No competitive risk neutral market making sector.**

In the second period a trader’s optimal position is

\[
X_2(\tilde{s}_{i2}, p^2) = \frac{E[\theta|\tilde{s}_{i2}, p^2] - p_2}{\rho \text{Var}[\theta|\tilde{s}_{i2}, p^2]}.
\]

Imposing market clearing and solving for \( p_2 \) yields

\[
p_2 = \bar{E}_2[\theta] + \rho \text{Var}_2[\theta] \hat{u}_2,
\]

where \( \bar{E}_t[Y] \equiv \int_0^1 E[Y|\tilde{s}_{it}, p^t] \, di \) and \( \text{Var}_t[Y] \equiv \text{Var}[Y|\tilde{s}_{it}, p^t] \).
In the first period we have

\[ X_1(s_{i1}, p_1) = \frac{E[p_2|s_{i1}, p_1] - p_1}{\rho \text{Var}[p_2|s_{i1}, p_1]}, \]

and imposing market clearing

\[ p_1 = \bar{E}_1[p_2] + \rho \text{Var}_1[p_2]\hat{u}_1 \]

\[ = \bar{E}_1[\bar{E}_2[\theta] + \rho \text{Var}_2[\theta]\zeta\hat{u}_1] + \rho \text{Var}_1[p_2]\hat{u}_1, \]

since \( \hat{u}_2 = \zeta\hat{u}_1 + e_2 \) and \( e_2 \) is independent of the other random variables. Thus in period 1 the price of an asset depends on:

1. The market average expectation of the market average expected liquidation value plus
2. a risk term associated with holding a position in the asset (due to the presence of noise traders).
This result generalizes to a $T$-period horizon where one can show (Cespa and Vives (2007)) that for $1 \leq t \leq T$:

$$p_t = \bar{E}_t \left[ \bar{E}_{t+1} \left[ \cdots \bar{E}_{T-1} \left[ \bar{E}_T[\theta] + \rho \text{Var}_T[\theta] \zeta^{T-t} \hat{u}_t \right] + \rho \text{Var}_{T-1}[\theta] \zeta^{T-(t-1)} \hat{u}_t \right] \right.$$

$$+ \rho \text{Var}_{t+1}[p_{t+2}] \zeta \hat{u}_t \right] + \rho \text{Var}_t[p_{t+1}] \hat{u}_t.$$

When the stock of noise trading is independent, $\zeta = 0$, it is immediate that

$$p_t = \bar{E}_t \left[ \bar{E}_{t+1} \left[ \cdots \bar{E}_{T-1} \left[ \bar{E}_{T+1}[\theta] \right] \cdots \right] \right] + \rho \text{Var}_t[p_{t+1}] \hat{u}_t,$$

and

- $p_t$ is the average expectation at $t$ of the average expectation at $t + 1$ of the average expectation at $t + 2$ of... the liquidation value in period $T + 1$ plus the corresponding period, risk-adjusted noise shock.

- This, as pointed out by Allen et al. (2006), is reminiscent of Keynes’ vision of the stock market as a beauty contest.
8.3 Dynamic Trading with Short-term Investors

8.3.2 Short Horizons with Risk-averse Market Makers

- Allen et al. (2006): when averaging over the realizations of noise trading, the price at date $t$ will not equal in general the period $t$ average expectation of the fundamental value.

- The consensus value of the fundamentals $\bar{E}_t[\theta]$ does not coincide with the mean price $E[p_t|\theta]$, with the exception of the last period which is like in a static market.

- In particular, the price gives a higher weight to history: Bias towards public information.

- Consequence: current price will be always farther away from fundamentals than the average of investors’ expectations and it will be more “sluggish” to adjust.
8.3 Dynamic Trading with Short-term Investors

8.3.2 Short Horizons with Risk-averse Market Makers

- While with $\zeta = 0$ there is a unique linear equilibrium, when $\zeta \in (0, 1]$ there are two linear equilibria (Cespa and Vives (2007)).

- In one of them prices are closer to fundamentals than the average expectations of investors (corresponding to the first equilibrium in the previous section) and in the other prices are farther away from fundamentals than the average expectations of investors (corresponding to the second equilibrium in the previous section). However, only the second equilibrium is stable (in the sense of having a negative slope for the aggregate excess demand function).

- Higher order expectations introduce a wedge in the pricing equation.
8.3 Dynamic Trading with Short-term Investors
8.3.3 Endogenous Information Acquisition and Short Horizons

In the previous section:

- We have seen how short-term investors care only about the next period price and not directly about the fundamental value of the asset.

- Consequence: short-term investors prefer short-term to long-term information and want their information to be reflected in the price and this happens only if other traders also have access to short-term information.

- This means that there may be strategic complementarities in information acquisition.
8.3 Dynamic Trading with Short-term Investors
8.3.3 Endogenous Information Acquisition and Short Horizons

Froot, Scharfstein and Stein (1992)

- 4-period model in which agents unwind their position in period 3 and decide whether to acquire information on one of the two additive components of the fundamental value.
- For a low (high) probability of revelation of the fundamental value in period 3 information acquisition by short-term traders displays strategic complementarity (substitutability).

Hirshleifer, Subrahmanyam and Titman (1994) and Holden and Subrahmanyam (1996) consider models where

- Short horizons of traders can also arise endogenously because of risk aversion when traders acquire information.
- Investors care about prices in the short term making herding on short-term information acquisition possible.
8.3 Dynamic Trading with Short-term Investors

8.3.4 Summary

Trading with a short horizon has important implications for market quality parameters.

1. In the presence of a risk neutral market making sector:
   - Short horizons make traders less responsive to their information on the fundamental value of the asset.
   - The pattern of evolution of market depth and volatility is very rich and contrasts with the case with long-term traders.

2. When market makers are risk averse:
   - Multiple (high and low volatility) equilibria may arise.
   - There is always an equilibrium, which is stable, in which prices are “farther away” from fundamentals than average investors’ expectations.
   - Short horizons may induce strategic complementarities in information acquisition.
8.4 Explaining Crises and Market Crashes

8.4.1 Crashes in Rational Expectations Models

In October 1987 the Dow Jones index fell by 23% in a single day and there was no obvious connection with news of changed fundamentals. Within the rational expectations equilibrium setup, three types of explanation of crashes have been advanced:

1. Small events or price changes may lead to a chain reaction that ends up in a crash (e.g. abrupt information revelation with small price movements as in Romer (1993)).

2. Multiple equilibria and a discontinuous equilibrium price function (e.g. Gennote and Leland (1990)).

3. Liquidity shortages (e.g. Grossman (1988)).

Further explanations are provided by herding models and models of bubbles.
8.4 Explaining Crises and Market Crashes
8.4.1 Crashes in Rational Expectations Models

Small events and chain reactions

- Romer (1993): some traders are uncertain about the fundamental value of the asset and the **precision** of information of other traders.
- Traders can receive signals of high, intermediate, or low precision.
- Two-dimensional uncertainty: 1st period price cannot reveal everything. Then, 2nd period price movement caused by a (known) supply shock may reveal a lot about the precision of traders’ information, and imply a large change in prices.
  - Suppose traders do not know the precision of information of other traders and see an **extreme** price in period 1.
  - They infer that it is more likely that other investors received a very noisy signal but they do not know for sure.
  - Now, a small – observable – supply shock in period 2, may reveal the type of other investors and correct the misalignment of prices with respect to the joint information of traders.

Other papers along this line are Caplin and Leahy (1994), Hong and Stein (2003), Lee (1998), and Zeira (1999).
Multiple equilibria

- Gennote and Leland (1990): comparative statics analysis in a static model with multiple equilibria showing how a small supply shift can entail a large price drop.
- This happens if the market is sitting at a high equilibrium price level and this equilibrium disappears with the exogenous shock (think of an inverted-S shaped demand curve and a vertical supply that moves to the right with the shock).
- The base model is a noisy rational expectations model. There are three types of traders:
  - Informed traders.
  - Traders that are better informed about the noisy supply of the asset.
  - Portfolio traders that buy when demand increases and sell when demand falls (crucial for the result).
- Demange (2002) shows that multiple equilibria and crashes can occur under standard assumptions when market participation (or the risk tolerance of traders) is uncertain.
- Barlevy and Veronesi (2003) obtain a result similar to Gennote and Leland (1990) doing away with the presence of portfolio insurance.
8.4 Explaining Crises and Market Crashes

8.4.1 Crashes in Rational Expectations Models

Liquidity shortages

- Grossman (1988): large price movements come from a liquidity shortage due to market timers’ underestimation of the degree of dynamic hedging activity.

- Reason: non informational equivalence between traded and synthetical options even when the payoff of the former can be replicated with the latter (with dynamic trading strategies).

- Market timers can stabilize the market by setting aside capital to smooth price movements caused by the dynamic trading strategies of a fraction of agents with increasing risk aversion when their wealth declines.
8.4 Explaining Crises and Market Crashes
8.4.1 Crashes in Rational Expectations Models

**Herding**

Variants of herding models can also explain crashes.

- Avery and Zemsky (1998) generate crashes in their dynamic model where traders are uncertain about the precision of information of other traders (like in Romer (1993)).
- Auction literature offers related insights (see Bulow and Klemperer (1994, 1999)).
- Veldkamp (2006) explains sporadic surges in asset prices (frenzies) and herding in a dynamic version of the Grossman-Stiglitz model where payoff volatility is increasing in the payoff level and there are increasing returns to information production.
In this section we present a basic coordination game that models crises such as a currency attack, or a bank or creditors run.

- Coordination games tend to have multiple equilibria since they are games of strategic complementarities. In a game of strategic complementarities the marginal return of the action of a player is increasing in the level of the actions of rivals.

- This leads to best replies being monotone increasing. We first provide conditions under which the equilibrium is unique and then some applications.

- We reexamine the model introducing a financial market and making public information endogenous.
8.4 Explaining Crises and Market Crashes
8.4.2 Crises and Coordination Problems

Model
The basic model is a symmetric binary action game of strategic complementarities.

- Consider a game with a continuum of players of mass one where the action set of player is $A_i \equiv \{0, 1\}$, with $y_i = 1$ interpreted as “acting” and $y_i = 0$ “not acting.”
- To act may be to attack a currency (Morris and Shin (1998)), not renewing debt (Morris and Shin (2004), Corsetti et al. (2006), Rochet and Vives (2004)), run on a bank or not renew a certificate of deposit in the interbank market (Goldstein and Pauzner (2005), Rochet and Vives (2004)), but also invest, adopt a technology, or revolt against the statu quo.
8.4 Explaining Crises and Market Crashes

8.4.2 Crises and Coordination Problems

- Let \( \pi^1 = \pi(y_i = 1, \tilde{y}; \theta) \) and \( \pi^0 = \pi(y_i = 0, \tilde{y}; \theta) \) where \( \tilde{y} \) is the fraction of investors acting and \( \theta \) is the state of the world.

- The differential payoff to acting is

\[
\begin{align*}
\pi^1 - \pi^0 &= \begin{cases} 
B > 0 & \text{if } \tilde{y} \geq h(\theta) \\
-C < 0 & \text{if } \tilde{y} < h(\theta)
\end{cases}
\end{align*}
\]

where \( h(\theta) \) is the critical fraction of investors above which it pays to act.
The game is of strategic complementarities since $\pi^1 - \pi^0$ is increasing in $\tilde{y}$.

It is assumed that $h(\cdot)$ is strictly increasing, crossing 0 at $\theta = \theta_L$ and 1 at $\theta = \theta_H$.

It follows from these payoffs, if the state of the world is known, that

\[
\text{Dominant strategy} = \begin{cases} 
\text{Act} & \text{if } \theta \leq \theta_L \\
\text{Not to Act} & \text{if } \theta \geq \theta_H 
\end{cases}
\]

For $\theta \in [\theta_L, \theta_H]$ there are multiple equilibria. Both everyone acting and no one acting are equilibria.

Since the game is a game of strategic complementarities there is a largest and a smallest equilibrium.

The largest equilibrium is $y_i = 1$, for all $i$ if $\theta \leq \theta_H$, and $y_i = 0$ for all $i$ if $\theta \geq \theta_L$, and it is (weakly) decreasing in $\theta$. 
Currency attacks
The model above is a streamlined version of Morris and Shin (1998) where

- $\theta$ represents the reserves of the central bank: with $\theta < 0$ meaning that reserves are depleted.
- Each speculator has one unit of resources to attack the currency ($y_i = 1$) at a cost $C$. Letting $h(\theta) = \theta$, the attack succeeds if $\tilde{y} \geq \theta$.
- The capital gain if there is a depreciation is fixed and equal to $\hat{B} = B + C$. 
Coordinate failures in the interbank market
Rochet and Vives (2004)

- Consider a market with three dates: \( t = 0, 1, 2 \).
- At date \( t = 0 \), the bank has equity \( E \) and collects uninsured certificates of deposit (CDs) in amount \( D_0 \equiv 1 \). These funds are used to finance risky investment \( I \) and cash reserves \( M \).
- The returns \( \theta I \) on these assets are collected at date \( t = 2 \). If the bank can meet its obligations the CDs are repaid at their face value \( D \), and the equityholders of the bank obtain the residual (if any).
- A continuum of fund managers make investment decisions in the interbank market. At \( \tau = 1 \) each fund manager decides whether to cancel \( (y_i = 1) \) or renew his CD \( (y_i = 0) \).
- If \( \tilde{y} \geq M \) then the bank has to sell some of its assets to meet payments. A fund manager is rewarded for taking the right decision (i.e., withdrawing if and only if the bank fails).
8.4 Explaining Crises and Market Crashes
8.4.2 Crises and Coordination Problems

- Let $m \equiv M/D$ be the liquidity ratio; $\theta_L \equiv (D - M)/I$, the solvency threshold of the bank; $\lambda > 0$ the fire sales premium of early sales of bank assets; and $\theta_H \equiv (1 + \lambda)\theta_L$ the “supersolvency” threshold such that a bank does not fail even if no fund manager renews his CDs.
- Under these conditions the bank fails if

$$\tilde{y} \geq h(\theta) \equiv m + \frac{1 - m}{\lambda} \left( \frac{\theta}{\theta_L} - 1 \right),$$

for $\theta \geq \theta_L$ and $h(\theta) < 0$ otherwise.
- This example can be reinterpreted replacing bank by country and CD for foreign-denominated short-term debt.
A game of incomplete information

- Consider now an incomplete information version of the game where players have a normal prior on the state of the world

\[ \theta \sim N(\bar{\theta}, \tau^{-1}_\theta). \]

- Player \( i \) observes a private signal \( s_i = \theta + \epsilon_i \) with normally i.i.d. distributed noise \( \epsilon_i \sim N(0, \tau^{-1}_\epsilon) \).

- Morris and Shin (2003) show that in the incomplete information game there is a unique Bayesian equilibrium, and it is in threshold strategies – of the type “act if and only if the signal received is below a certain threshold,” provided that \( \tau_\theta / \sqrt{\tau_\epsilon} \) is small.
The equilibrium is the outcome of iterated elimination of strictly dominated strategies. The result can be shown with the help of the tools of supermodular games:

1. The game is one of strategic complementarities with a monotone information structure. The game is “monotone supermodular” since $\pi(y_i, \tilde{y}; \theta)$ has increasing differences in $(y_i, (\tilde{y}, -\theta))$ and signals are affiliated.

2. This means that extremal equilibria exist, are symmetric (because the game is symmetric), and are in monotone (decreasing) strategies in type (Van Zandt and Vives (2007) and Vives (2005)).
Since there are only two possible actions, the strategies must then be of the threshold form:

\[ y_i = \begin{cases} 
1 & \text{if } s_i < \hat{s} \\ 
0 & \text{otherwise} 
\end{cases} \]

where \( \hat{s} \) is the threshold.

It follows also that the extremal equilibrium thresholds \( \bar{s}, \underline{s} \) bound the set of strategies which are the outcome of iterated elimination of strictly dominated strategies.

If \( \bar{s} = \underline{s} \) the game is dominance solvable and the equilibrium is unique.
8.4 Explaining Crises and Market Crashes

8.4.2 Crises and Coordination Problems

- An equilibrium will be characterized by two thresholds \((s^*, \theta^*)\) where
  - \(s^*\) is the signal threshold to act and
  - \(\theta^*\) is the state-of-the-world critical threshold, below which the acting mass is successful and an acting player obtains the payoff.

- Let \(\Phi\) denote the cumulative distribution of the standard normal random variable \(N(0, 1)\).

- In equilibrium:
  - the fraction of acting players must equal the critical fraction above which it pays to act:
    \[
    \tilde{y}(\theta^*, s^*) = \Pr(s \leq s^*|\theta^*) \equiv \Phi(\sqrt{\tau}(s^* - \theta^*)) = h(\theta^*). \tag{1}
    \]
  - at the critical signal threshold the expected payoff of acting and not acting should be the same:
    \[
    E[\pi(1, \tilde{y}(\theta, s); \theta) - \pi(0, \tilde{y}(\theta, s); \theta)|s = s^*] = \\
    Pr(\theta \leq \theta^*|s^*)B + Pr(\theta > \theta^*|s^*)(-C) = 0.
    \]
8.4 Explaining Crises and Market Crashes
8.4.2 Crises and Coordination Problems

- The last equation can be expressed as

\[ Pr(\theta \leq \theta^* | s^*) \equiv \Phi \left( \sqrt{\tau_\theta + \tau_\epsilon} \left( \theta^* - \frac{\tau_\theta \bar{\theta} + \tau_\epsilon s^*}{\tau_\theta + \tau_\epsilon} \right) \right) = \gamma, \]

for \( \gamma \equiv (B + C)^{-1} C < 1. \)

- Combining it with (1) yields the following equation in \( \theta^* \)

\[ \varphi(\theta^*; \gamma, \bar{\theta}) \equiv \tau_\theta (\theta^* - \bar{\theta}) - \sqrt{\tau_\epsilon} \Phi^{-1}(h(\theta^*)) - \sqrt{\tau_\theta + \tau_\epsilon} \Phi^{-1}(\gamma) = 0. \]

- Assume \( h(\cdot) \) is linear and \( h' > 0 \), then it can be checked that the above equation admits a unique solution in \( \theta^* \) if and only if

\[ \frac{\tau_\theta}{\sqrt{\tau_\epsilon}} < \sqrt{2\pi} h'. \]

- The equilibrium is unique, and \( \theta^*, s^* \) move together.
8.4 Explaining Crises and Market Crashes

8.4.2 Crises and Coordination Problems

- Intuition: think in terms of the best reply of a player to the (common) signal threshold used by the other players.
- Let \( P(s, \hat{s}) \) be the conditional probability that the acting players succeed if they use a (common) threshold \( \hat{s} \) when the player considered receives a signal \( s \). That is,

\[
P(s, \hat{s}) \equiv Pr \left( \theta < \hat{\theta}(\hat{s}) | s \right) = \Phi \left( \sqrt{\tau_\theta + \tau_\epsilon} \left( \hat{\theta}(\hat{s}) - \frac{\tau_\theta \bar{\theta} + \tau_\epsilon \bar{\epsilon}}{\tau_\theta + \tau_\epsilon} \right) \right).
\]

- \( \hat{\theta}(\hat{s}) \) is the critical \( \theta \) below which there is success when players use a strategy with threshold \( \hat{s} \).
- It is immediate then that \( \partial P/\partial s < 0 \) and \( \partial P/\partial \hat{s} \geq 0 \).
- Given that other players use a strategy with threshold \( \hat{s} \), the best response of a player is to use a strategy with threshold \( s^* \) where \( P(s^*, \hat{s}) = \gamma \): act if and only if \( P(s, \hat{s}) > \gamma \). This defines a best-response function in terms of thresholds

\[
r(\hat{s}) = \frac{\tau_\theta + \tau_\epsilon}{\tau_\epsilon} \hat{\theta}(\hat{s}) - \frac{\tau_\theta \bar{\theta}}{\tau_\epsilon} - \sqrt{\tau_\theta + \tau_\epsilon} \Phi^{-1}(\gamma).
\]
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- We have \( r' = - (\partial P / \partial \hat{s}) / (\partial P / \partial s) \geq 0 \), and the game is of strategic complementarities: a higher threshold by others induces a player to use also a higher threshold.

- If at a candidate equilibrium \( r'(\hat{s}) < 1 \), \( r(\cdot) \) crosses the 45\(^o\) line only once the equilibrium is unique:

\[
\begin{align*}
\text{Figure 8.1. } & \text{Best response of a player to the threshold strategy used by rivals (the flatter best response corresponds to the case } \\
& \tau_0 / \sqrt{\pi} \leq \sqrt{2\pi} h' \text{ while the steeper one to the case } \\
& \tau_0 / \sqrt{\pi} > \sqrt{2\pi} h').
\end{align*}
\]
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- The equilibrium is unique with small noise in the signals in relation to the prior \((\tau_\theta / \sqrt{\tau_\epsilon})\) because decreasing the amount of noise decreases the strength of the strategic complementarity among the actions of the players.

- Multiple equilibria come about when the strategic complementarity is strong enough (the steeper best response in the Figure). With small noise a player faces greater of uncertainty about the behavior of others and the strategic complementarity is lessened, the best response is “flattened” and \(r' \leq 1\).

- Consider the limit cases \(\tau_\epsilon \to \infty\) (or, equivalently, a diffuse prior \(\tau_\theta \to 0\)). Then it is not hard to see that the distribution of the proportion of acting players \(\tilde{y}(\theta, s^*)\) is uniformly distributed over \([0, 1]\) conditional on \(s_i = s^*\).

- This means that players face maximal strategic uncertainty. In contrast, at any of the multiple equilibria with complete information when \(\theta \in (\theta_L, \theta_H)\), players face no strategic uncertainty.
In the region where the equilibrium is unique we can obtain several useful results: **Coordination failure**

- When $\theta < \theta^*$, the acting mass of players succeeds.
- In the range $[\theta^*, \theta_H)$ there is coordination failure from the point of view of players, because if all them were to act then they would succeed.
  - For example, in this range if currency speculators were to coordinate their attack then they would succeed, but in fact the currency holds or, in the interbank example where the equilibrium failure threshold of the bank is $\theta \in [\theta_L, \theta_H]$, in the range $[\theta_L, \theta^*)$ the bank is solvent but illiquid. This provides a rationale for a Lender of Last Resort intervention with the discount window.
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Comparative statics

- Both $\theta^*$ and $s^*$ (and the probability that the acting mass succeeds) are decreasing in the relative cost of failure $\gamma \equiv C/(B + C')$ and in the expected value of the state of the world.

- Thus, the probability of a currency crisis is decreasing in the relative cost the attack $C/B$ and in the expected value of the reserves of the central bank $\bar{\theta}$.

- In the interbank example the critical $\theta^*$ (and probability of failure) is:
  - a decreasing function of the liquidity ratio $m$ and the solvency $(E/I)$ of the bank, of the critical withdrawal probability $\gamma$, and of the expected return on the bank’s assets $\bar{\theta}$.
  - It is an increasing function of the fire-sale premium $\lambda$ and of the face value of debt $D$. 
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Multiplier effect of public information

- The prior mean $\bar{\theta}$ of $\theta$ can be understood as a public signal of precision $\tau_\theta$.
- The equilibrium threshold is determined by $r(s^*; \bar{\theta}) - s^* = 0$. From which it follows that

$$\left| \frac{ds^*}{d\bar{\theta}} \right| = \left| \frac{\partial r}{\partial \bar{\theta}} \right| \frac{1 - r'}{1 - r'} > \left| \frac{\partial r}{\partial \bar{\theta}} \right|$$

whenever the uniqueness condition $r' < 1$ is met and $r' > 0$.

- Then, an increase in $\bar{\theta}$ has a larger effect on the equilibrium threshold than the direct impact on the best response of a player.

- This multiplier effect is largest when $r'$ is close to 1, i.e. when strategic complementarities are strong and we approach the region of multiplicity of equilibria.

- The multiplier effect of public information is emphasized by Morris and Shin (2003).
This happens because public information becomes common knowledge and affects the equilibrium outcome. This phenomenon may be behind the apparent overreaction of financial markets to Fed announcements or may explain why the per-viewer price of advertising in TV is higher for big sports events (Chew (1998)).

In the region where there is a unique equilibrium the probability of occurrence of a “crisis” (successful mass action) depends on the state of the world. In contrast, in the complete information model there are multiple self-fulfilling equilibria in the range \((\theta_L, \theta_H)\).

Thus, the model builds a bridge between the self-fulfilling theory of crisis (e.g., Diamond and Dybvig (1983)) and the theory that links crisis to the fundamentals (e.g, Gorton (1985, 1988)).
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**Transparency**

- Releasing more public information (e.g., by the Central Bank in the banking example), is not necessarily good.
- At one extremal equilibrium the probability of crisis will increase substantially from the the situation without the public signal.
- The analysis may therefore rationalize oblique statements by central bankers and other regulatory authorities which seem to add noise to a basic message.
We can still perform comparative statics analysis even if there are multiple equilibria. This is so because the game is a monotone supermodular game.

Suppose we are in the multiple equilibrium region and that \( \bar{\theta} \) increases. The comparative statics result that the critical thresholds \( \theta^* \), and \( s^* \) decrease still holds for extremal equilibria.

Indeed, extremal equilibria of monotone supermodular games are increasing in the posteriors of the players (see Van Zandt and Vives (2007)).
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- A sufficient statistic for the posterior of a player under normality is the conditional expectation $E[\theta|s]$, which is increasing in $\bar{\theta}$. It follows then that extremal equilibrium thresholds $(-\theta^*, -s^*)$ increase with $\bar{\theta}$.

- The same result holds for reasonable out-of-equilibrium dynamics that eliminate the middle “unstable” equilibrium. For example, best-reply dynamics where, at any stage after the perturbation from equilibrium, a new $\theta$ is drawn independently and a player responds to the strategy threshold used by other players at the previous stage.
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Extensions:

- Large players (see Corsetti, Dasgupta, Morris and Shin (2004) on currency attacks).
- Dynamic settings (see Morris and Shin (2003), Dasgupta (2007) for a noisy social learning application)
- Endogenous public information.
Endogenous public information

- The public signal comes from a price in a financial market with liquidation value $\theta$ (Angeletos and Werning (2006), see also Tarashev (2007)).

- The financial market meets previously to the coordination game and is as in Section 4.2.1 but without uniformed traders. There is a unit mass of CARA informed traders ($\mu = 1$), each informed receives a signal about $\theta$ of precision $\tau_\epsilon$, and the variance of noise trading is given by $\tau_u^{-1}$.

- We know that in this market the precision of prices is given by $\tau = \tau_\theta + a^2 \tau_u$ where $a = \rho^{-1} \tau_\epsilon$.

- Suppose that in the coordination game $h(\theta) = \theta$ and note, as before, that the prior can be interpreted as a public signal.
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- It follows that there will be multiple equilibria if and only if
  \[
  \frac{\tau}{\sqrt{\tau} \epsilon} > \sqrt{\frac{2\pi}{\epsilon}}
  \]
  
  this is the result we had before replacing \( \tau \theta \) by \( \tau \).

- Contrary to the case where public information is exogenous the inequality holds if \( \tau \epsilon \) is large (and also if \( \tau u \) is large).

- Both everyone acting and no one acting are equilibria in the range \( \theta \in (\theta_L, \theta_H) \) as in the complete information case.

- In many circumstances crises will affect the return in the asset market.

- Asset market prices also affect the outcome of coordination game.
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- The reason for the backward bending demand curve for the asset is that a high price realization decreases the incentives to attack (by signaling a low $\theta$) but a reduced attack in turn raises the return of the asset.
- If the second effect is strong enough demand may increase with the price. Multiple equilibria obtain again in the low noise scenarios when either or is large.
- In the speculative attacks model of Tarashev (2007) the information role of the interest rate allows to explain abrupt currency attacks without resorting to sunspots and multiple equilibria. Furthermore, the analysis finds that an intervention in the foreign exchange market may reinforce a currency peg because of its influence on the precision of public information.
- Hellwig, Mukherji and Tsyvinski (2006) analyze in depth the role of interest rates in self-fulfilling currency crises. In their model multiplicity of equilibrium arises in a primary market, in which interest rates have direct effects on payoffs as well as aggregate information.
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- This in contrast with the model above by Angeletos and Werning (2006) where information aggregation occurs through the price of a derivative asset, which has no direct effect on the payoffs of players. The root of the multiplicity is similar to the models in Gennotte and Leland (1990) and Barlevy and Veronesi (2003): the non-monotonicity of demand and supply schedules for assets. The authors conclude that the global games results on the uniqueness of equilibrium do not apply models of currency markets where markets provide endogenous signals.

- In Angeletos, Hellwig, and Pavan (2006) public information is endogenously generated by policy interventions. To a coordination game of the type studied at the start of the section the authors add a previous stage at which a policy maker can influence the payoff from acting (i.e. attacking the currency or running on the bank).

- Whenever there are different types of policy maker, and since his actions are observed by the agents before deciding whether to act, signaling is introduced in the coordination game.
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- The consequence is that multiple equilibria are introduced as a combination of signaling and coordination. For example, by intervening the policy maker may reveal that he is in a range of intermediate types and this can be used by agents to coordinate on multiple reactions.

- Angeletos, Hellwig, Pavan (2007) extend the basic static global coordination game of regime change (the binary action game of strategic complementarities with incomplete information) to a dynamic context where each can act in many periods and can learn about the fundamentals.

- Here endogenous public information at any point in time comes in the form of knowledge that the regime has survived past attacks. The authors find equilibrium multiplicity under the same conditions that guarantee uniqueness in the static benchmark.

- Multiplicity is generated by the combination of the endogenous public information and the arrival of new private signals. It is found also that the fundamentals may determine the eventual outcome (i.e. whether the statu quo is overturned) but not the timing or the number of attacks.
Furthermore, the dynamics of the economy alternate between phases of calm, where no attack is possible and agents only accumulate information, and phases of distress where a large attack can occur triggered by a small change in information (or in the fundamentals).


Angeletos, Lorenzoni and Pavan (2007) look at a two-way feedback between investment and prices in financial markets when there is dispersed information on investment opportunities. Asset prices tend to increase with aggregate investment (the public signal) since higher investment is linked with high profitability. In turn, high asset prices induce high investment.

The authors show this endogenous complementarity makes investment react too much to noise (or correlated errors in private signals of profitability) and too little to fundamentals. This is a form of beauty-contest inefficiency in the interaction between financial and real activity.
8.4 Explaining Crises and Market Crashes

8.4.3 Summary

1. Crashes can be explained in a range of competitive rational expectations models with some of the following ingredients:
   - Uncertain precision of the traders’ information.
   - Multiple equilibria.
   - Liquidity shortages.

2. Crises with an underlying coordination problem of investors, such as currency attacks or runs in banking markets, can be modeled as a game of strategic complementarities.

3. Endogenous public signals and multiple equilibria.
This chapter reviews financial dynamics with heterogeneous information in the rational expectations tradition where informed traders condition on current prices.

- With a competitive risk neutral market making sector:
  - The market is semi-strong efficient.
  - Closed form solution is provided for a standard dynamic noisy rational expectations market with long-term investors and heterogeneous information.
  - Compared to the market with short-term traders, predictions on price informativeness that depend on patterns of information arrival.
  - With no risk-neutral market makers: explanations for technical analysis, “excess” volatility, trade with no news, general dynamic patterns of volume in the presence of private and public information, and departures of prices from fundamentals.
Summary

• In particular

  1 With concentrated arrival of information, short horizons reduce final price informativeness; with diffuse arrival of information, short horizons enhance it.

  2 Short horizons may imply that traders have incentives to herd in information acquisition, either in terms of the assets researched or the type of information, or concentrate only on short-term information.

• The chapter also examines markets with no risk neutral market makers, where market making involves a risk premia, and provides explanations for technical analysis, “excess” volatility, trade with no news, general dynamic patterns of volume in the presence of private and public information, and departures of prices from fundamentals.
Summary

- We find, in particular that:
  - Similar to the static model and as long as there is no residual uncertainty in the liquidation value and noise trade increments are uncorrelated, price at any period equals the average expectations of investors plus a risk-bearing component; otherwise, traders speculate both on the terminal value and on price changes and prices may be farther away from or closer to fundamental values than the average expectations of investors. The predictability of noise trading and lessened residual uncertainty push prices away from fundamental values in relation to average expectations.
  - Market depth has a risk-bearing component on top of the adverse selection component.
  - The combined presence of risk averse market makers and traders with short horizons induces multiple equilibria (a high and a low volatility equilibrium) except if the stock of noise trading is independent across periods; there is always an equilibrium, which is stable, in which prices are farther away from fundamental values than the average expectations of investors.
Summary

- Crashes and crises, as well as other market anomalies, can be explained without recourse to the irrationality of market participants.
- Games of strategic complementarities with incomplete information prove useful to study crises with an underlying coordination problem and the role of policy interventions. In particular it is possible to obtain comparative static results even in the presence of multiple equilibria.
- In some circumstances the degree of strategic complementarity is lessened and a unique equilibrium is obtained. The conditions under which uniqueness holds however are stringent, in particular when public information is endogenously generated.