

CHAPTER 1

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The Properties of Materials

FORCES: DYNAMICS AND STATICS

We all have some intuitive idea about the mechanics of the world around us, an idea built up largely from our own experience. However, a proper scientific understanding of mechanics has taken centuries to achieve. Isaac Newton was of course the founder of the science of mechanics; he was the first to describe and understand the ways in which moving bodies behave.

Introducing the concepts of inertia and force, he showed that the behavior of moving bodies could be summed up in three laws of motion.

- 1) The law of inertia: An object in motion will remain in motion unless acted upon by a net force. The inertia of an object is its reluctance to change its motion.
- 2) The law of acceleration: The acceleration of a body is equal to the force applied to it divided by its mass, as summarized in the equation

$$F = ma, \quad (1.1)$$

where F is the force; m , the mass; and a , the acceleration.

- 3) The law of reciprocal action: To every action there is an equal and opposite reaction. If one body pushes on another with a given force, the other will push back with the same force in the opposite direction.

To summarize with a simple example: if I give a push to a ball that is initially at rest (fig. 1.1a), it will accelerate in that direction at a rate proportional to the force and inversely proportional to its mass. The great step forward in Newton's scheme was that, together with the inverse square law of gravity, it showed that the force that keeps us down on earth is one and the same with the force that directs the motion of the planets.

All this is a great help in understanding **dynamic** situations, such as billiard balls colliding, guns firing bullets, planets circling the sun, or frogs jumping. Unfortunately it is much less useful when it comes to examining what is happening in a range of no-less-common everyday situations. What is happening when a book is lying on a desk, when a light bulb is hanging from the ceiling, or when I am trying to pull a tree over? (See fig. 1.1b.) In all of these **static** situations, it is clear that there is no acceleration (at least until the tree *does* fall over), so the table or rope must be resisting gravity and the tree must be resisting the forces I am putting on it with equal and opposite

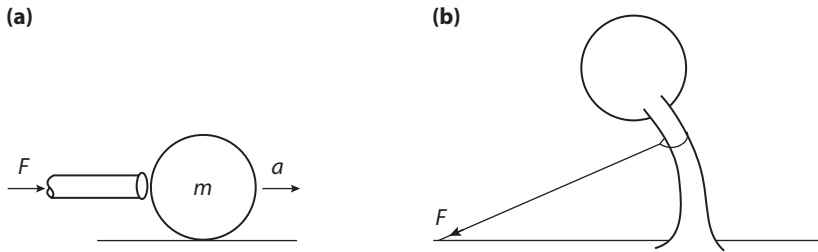


Figure 1.1. Forces on objects in dynamic and static situations. In dynamic situations, such as a pool ball being given a push with a cue (a), the force, F , results in the acceleration, a , of the ball. In static situations, such as a tree being pulled sideways with a rope (b), there is no acceleration.

reactions. But how do objects supply that reaction, seeing as they have no force-producing muscles to do so? The answer lies within the **materials** themselves.

Robert Hooke (1635–1703) was the first to notice that when springs, and indeed many other **structures** and pieces of material, are loaded, they change shape, altering in length by an amount approximately proportional to the force applied, and that they spring back into their original shape after the load is removed (fig. 1.2a). This linear relationship between force and extension is known as **Hooke's law**.

What we now know is that all solids are made up of atoms. In **crystalline materials**, which include not only salt and diamonds but also metals, such as iron, the atoms are arranged in ordered rows and columns, joined by stiff interatomic bonds. If these sorts of materials are stretched or compressed, we are actually stretching or compressing the interatomic bonds (fig. 1.2b). They have an equilibrium length and strongly resist any such movement. In typically static situations, therefore, the applied force is not lost or dissipated or absorbed. Instead, it is opposed by the equal and opposite reaction force that results from the tendency of the material that has been deformed to return to its resting shape. No material is totally rigid; even blocks of the stiffest materials, such as metals and diamonds, deform when they are loaded. The reason that this deformation was such a hard discovery to make is that most structures are so rigid that their deflection is tiny; it is only when we use compliant structures such as springs or bend long thin beams that the deflection common to all structures is obvious.

The greater the load that is applied, the more the structure is deflected until failure occurs; we will then have exceeded the strength of our structure. In the case of the tree (fig. 1.1b), the trunk might break, or its roots pull out of the soil and the tree accelerate sideways and fall over.

INVESTIGATING THE MECHANICAL PROPERTIES OF MATERIALS

The science of elasticity seeks to understand the mechanical behavior of structures when they are loaded. It aims to predict just how much they

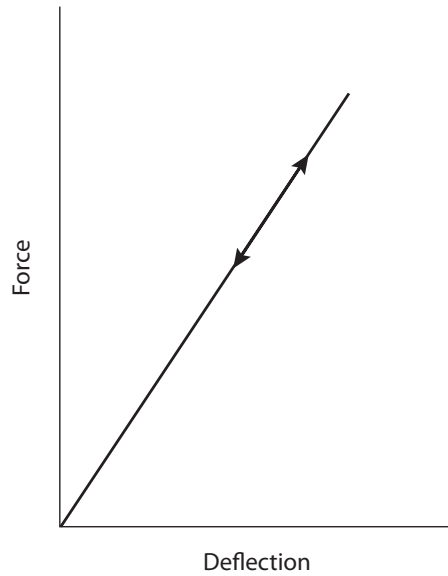
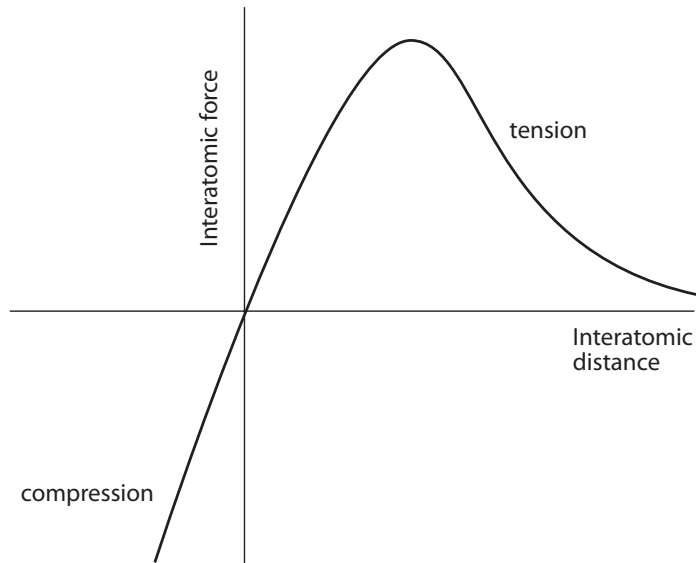
(a)**(b)**

Figure 1.2. When a tensile force is applied to a perfectly Hookean spring or material (a), it will stretch a distance proportional to the force applied. In the material this is usually because the bonds between the individual atoms behave like springs (b), stretching and compressing by a distance that at least at low loads is proportional to the force applied.

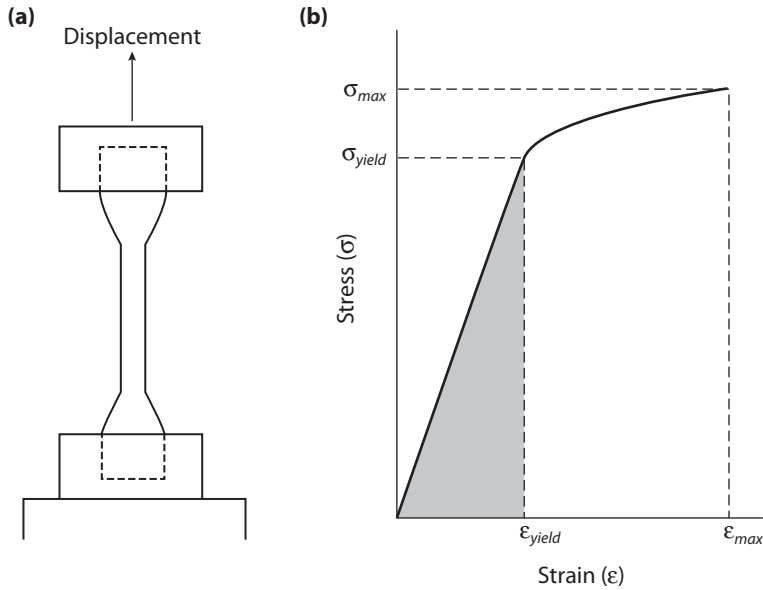


Figure 1.3. In a tensile test, an elongated piece of a material is gripped at both ends (a) and stretched. The sample is usually cut into a dumbbell shape so that failure does not occur around the clamps, where stresses can be concentrated. The result of such a test is a graph of stress against strain (b), which shows several important mechanical properties of the material. The shaded area under the graph is the amount of elastic energy the material can store.

should deflect under given loads and exactly when they should break. This will depend upon two things. The properties of the material are clearly important—a rod made of rubber will stretch much more easily than one made of steel. However, geometry will also affect the behavior: a long, thin length of rubber will stretch much more easily than a short fat one.

To understand the behavior of materials, therefore, we need to be able separate the effects of geometry from those of the material properties. To see how this can be done, let us examine the simplest possible case: a **tensile test** (fig 1.3a), in which a uniform rod of material, say a rubber band, is stretched.

The Concept of Stress

If it takes a unit force to stretch a rubber band of a given cross-sectional area a given distance, it can readily be seen that it will take twice the force to give the same stretch to two rubber bands set side by side or to a single band of twice the thickness. Resistance to stretching is therefore directly proportional to the cross-sectional area of a sample. To determine the mechanical state of the rubber, the force applied to the sample must

consequently be normalized by dividing it by its cross-sectional area. Doing so gives a measurement of the force per unit area, or the intensity of the force, which is known as **stress** and which is usually represented by the symbol σ , so that

$$\sigma = P/A, \quad (1.2)$$

where P is the applied load and A the cross-sectional area of the sample. Stress is expressed in SI units of newtons per square meter (N m^{-2}) or pascals (Pa). Unfortunately, this unit is inconveniently small, so most stresses are given in kPa ($\text{N m}^{-2} \times 10^3$), MPa ($\text{N m}^{-2} \times 10^6$), or even GPa ($\text{N m}^{-2} \times 10^9$).

The Concept of Strain

If it takes a unit force to stretch a rubber band of a given length by a given distance, the same force applied to two rubber bands joined end to end or to a single band of twice the length will result in twice the stretch. Resistance to stretching is therefore inversely proportional to the length of a sample. To determine the change in shape of the rubber as a material in general, and not just of this sample, the deflection of the sample must consequently be normalized by dividing by its original length. This gives a measure of how much the material has stretched relative to its original length, which is known as **strain** and which is usually represented by the symbol, ε , so that

$$\varepsilon = dL/L, \quad (1.3)$$

where dL is the change in length and L the original length of the sample. Strain has no units because it is calculated by dividing one length by another.

It is perhaps unfortunate that engineers have chosen to give the everyday words *stress* and *strain* such precise definition in mechanics, since doing so can confuse communications between engineers and lay people who are used to the vaguer uses of these words. As we shall see, similar confusion can also be a problem with the terms used to describe the mechanical properties of materials.

DETERMINING MATERIAL PROPERTIES

Many material properties can be determined from the results of a tensile test once the graph of force against displacement has been converted with equations 1.2 and 1.3 into one of stress versus strain. Figure 1.3b shows the stress-strain curve for a typical tough material, such as a metal. Like many, but by no means all, materials, this one obeys Hooke's law, showing linear elastic behavior: the stress initially increases rapidly in direct proportion to the strain. Then the material reaches a **yield** point, after which the stress increases far more slowly, until finally failure occurs and the material breaks.

The first important property that can be derived from the graphs is the **stiffness** of the material, also known as its **Young's modulus**, which is represented by the symbol E . Stiffness is equal to the initial slope of the stress-strain curve and so is given mathematically by the expression

$$E = d\sigma/d\varepsilon \quad (1.4)$$

or by the original force-displacement curve

$$E = \frac{LdP}{AdL}. \quad (1.5)$$

Stiff materials therefore have a high Young's modulus. **Compliance** is the inverse of stiffness, so compliant materials have a low Young's modulus. In many materials, the slope of the curve changes as the material is stretched. For such materials one can distinguish between the **initial stiffness** and the **tangent stiffness**, which is the slope at higher strains.

The second important property that can be derived is the **strength**, or **breaking stress**, of the material; this is simply the maximum value of stress, σ_{\max} , along the y-axis. Breaking stress can alternatively be calculated from the original force-displacement curve using the formula

$$\sigma_{\max} = P_{\max}/A. \quad (1.6)$$

Strong materials have a high breaking stress, whereas weak ones have a low breaking stress. The **yield stress**, σ_{yield} , can also be read off the graph, being the stress at which it stops obeying Hooke's law and becomes more compliant; this is the point at which the slope of the graph falls.

A third useful property of a material is its **extensibility**, or **breaking strain**, ε_{\max} , which is simply the maximum value of strain along the x-axis. Breaking strain can alternatively be calculated from the original force-displacement curve using the formula

$$\varepsilon_{\max} = (L_{\max} - L)/L. \quad (1.7)$$

The **yield strain** can also be determined from this curve, being the strain at which the slope of the graph falls.

A further material property that can be derived by examining the shape of the stress-strain curve is its susceptibility or resistance to breakage. A **brittle** material, such as glass, will not have a yield region but will break at the end of the straight portion (fig 1.4), whereas a **tough** material, such as a metal, will continue taking on load at strains well above yield before finally breaking.

LOADING, UNLOADING, AND ENERGY STORAGE

A final useful aspect of stress-strain graphs is that the area under the curve equals the energy, W_v , that is needed to stretch a unit volume of the material to a given strain. This factor is given in units of joules per cubic meter (J m^{-3} , which is dimensionally the same as N m^{-2}). Under the linear part of the

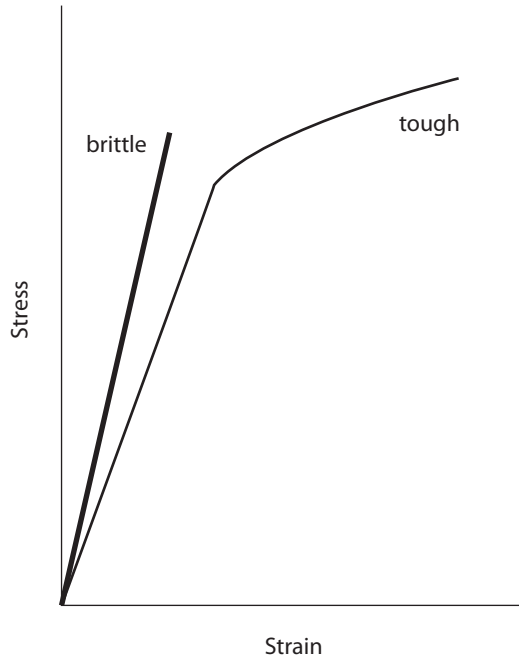


Figure 1.4. Contrasting stress-strain graphs of brittle and tough materials. The tough material shows appreciable stretching after yield.

stress-strain curve, this energy equals half the stress times the strain, so

$$W_e = \sigma \varepsilon / 2.$$

But strain equals stress divided by stiffness, so

$$W_e = \sigma(\sigma/E)/2 = \sigma^2/2E. \quad (1.8)$$

The **elastic storage capability**, W_c , of a material is the amount of energy under the curve up to the point at which yield occurs and is given by the equation

$$W_c = \sigma_{\text{yield}}^2/2E. \quad (1.9)$$

The amount of energy an elastic material can store, therefore, increases with its yield stress but decreases with its stiffness, because stiffer materials do not stretch as far for a given stress. So the materials that store most energy are ones that are strong but compliant.

In a perfectly elastic material, all of this energy would be stored in the material and could be recovered if it were allowed to return to its original length. However, no materials are perfectly elastic; the percentage of energy released by a material, known as its **resilience**, is never 100% and falls dramatically in tough materials after yield, since yield usually involves irreversible damage to the sample. The resilience of a material can be readily

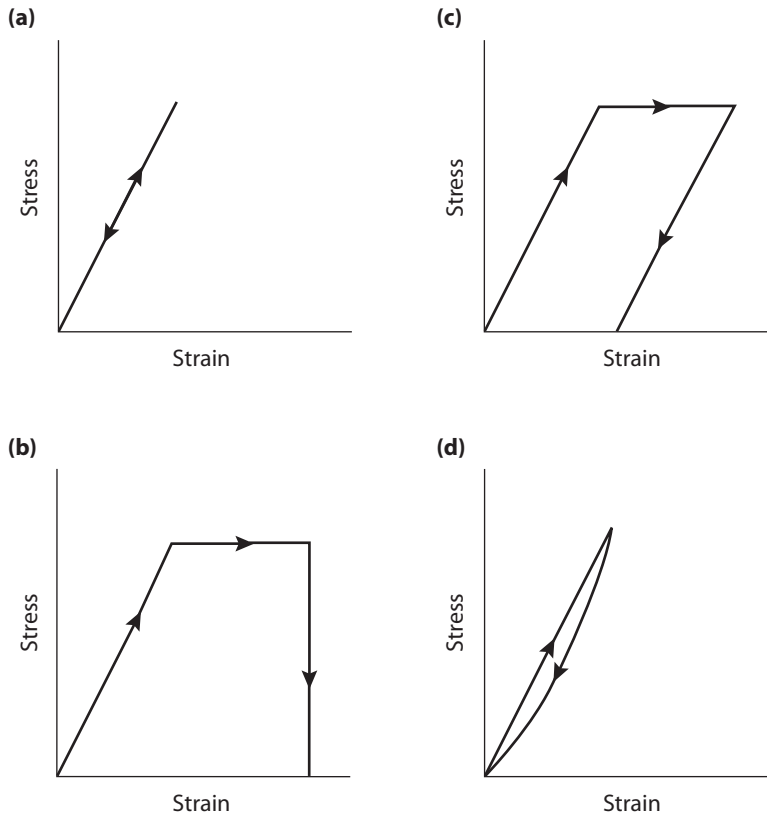


Figure 1.5. The results of loading/unloading tests for (a) a perfectly elastic material, (b) a perfectly plastic material, (c) an elastic-plastic material, and (d) a viscoelastic material.

measured using a modified tensile test in which the sample is stretched to a point before yield occurs and then allowed to return to its rest length. The unloading curve will always be below the loading curve. The resilience is the percentage of the area under the unloading curve divided by the area under the loading curve; the percentage of energy that is lost is known as the **hysteresis** and is the remainder of 100% minus the resilience.

Loading/unloading tests can be used to differentiate between different sorts of materials. In a perfectly **elastic** material (fig 1.5a), the unloading curve follows the loading curve exactly, there is no hysteresis, and the material returns to its original shape after the test. In a perfectly **plastic** material, on the other hand (fig 1.5b), the material will be permanently deformed by the load, and all the energy put into it will be dissipated. Tough materials often show **elastic-plastic** behavior (fig 1.5c), acting elastically before and plastically after yield, in which case the sample will return only part of the way to its original shape and some energy will be dissipated in deforming it permanently. Finally, even before yield, materials often show

viscoelastic behavior (fig 1.5d), in which energy is lost as they deform, just as it does in liquids, due to internal friction. The amount of energy lost and hence the shape of the loading/unloading curve will vary with the speed at which the test is carried out, as we shall see in Chapter 3, but unlike with elastic-plastic behavior, the material will eventually return to its original shape.

THE EFFECT OF DIRECTION

Many engineering materials, such as metals, plastics, and concrete, are essentially homogenous and have the same material properties in all directions. These are said to be **isotropic**. Many other materials, on the other hand, particularly those with a complex internal structure (including many if not most biological materials), have very different mechanical properties in different directions. These materials are said to be **anisotropic**, and to fully characterize them, materials tests must be carried out in all three planes.

CHANGES IN SHAPE DURING AXIAL LOADING

When a typical material sample is put into **axial loading**, that is, being stretched or compressed, it does not only get longer or shorter; it also gets narrower or thicker, necking or bulging under the load (fig 1.6). As a consequence, in a tensile test the load will be spread over a smaller area, and so the actual stress in the sample will be greater than the stress given by dividing the load by the original area. The shape of the sample will also be elongated by more than the value given by dividing the change in length by the original area. In other words, both the stress and the strain will be underestimated. In most engineering materials, which deform by no more than 0.1–1% of their original length before they break, this is not a great problem. Engineers usually do not bother to try and calculate the **true stress** and **true strain** in their samples. Instead they use the convention of ignoring the change in shape and instead calculating what are known as **engineering stress** and **engineering strain** from the original dimensions of the sample. With such small changes in shape, the error would in any case be small.

For many biological materials, on the other hand, strains can be far greater, reaching values up to 10, meaning stretches of 1000%! In these cases the differences between true stress and strain and engineering stress and strain can be very great indeed. However, because it is difficult to measure changes in shape during the course of materials tests, even biologists usually use engineering stress and strain, although, as we shall see, measuring the actual changes in shape can also provide other information about the material.

The degree to which a material necks or bulges when stretched or compressed is given by its **Poisson's ratio**, which is denoted by the symbol

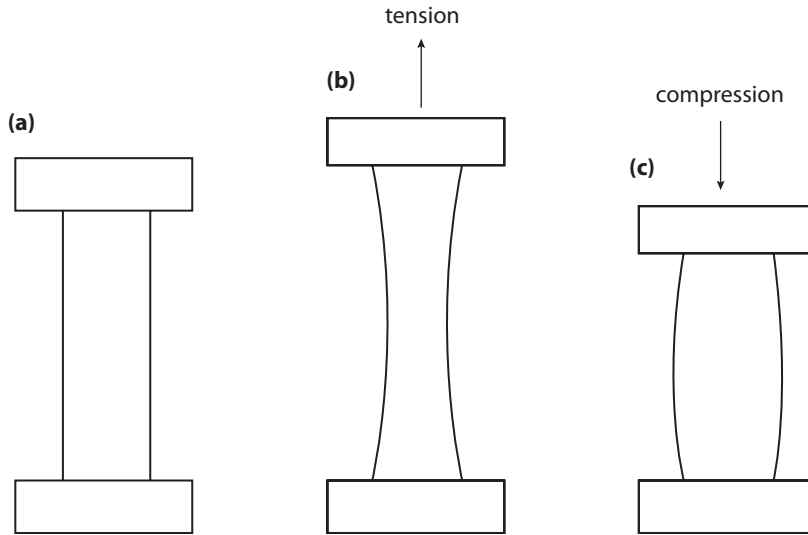


Figure 1.6. Shape changes during loading. If a specimen (a) is stretched (b), it will also tend to get narrower, whereas if it is compressed (c), it will tend to bulge outwards.

ν and calculated using the equation

$$\nu = - \frac{\text{lateral strain}}{\text{axial strain}} \quad (1.10)$$

For engineering materials that are isotropic, ν is usually between 0.25 and 0.33. The upper theoretical limit for ν is supposed to be 0.5, since at this value the volume of material will be unchanged as it is stretched; if the length increases by 1%, both the thickness and width will decrease by 0.5%, and the total volume will remain the same. If the lateral strain in both directions were greater than half the longitudinal strain, it would result in the volume decreasing when a material was stretched and increasing when compressed, which would seem to be physically improbable.

Many biological materials behave in rather odd ways, however; being anisotropic they may have different Poisson's ratios in different directions. As we shall see later in the book, some biological materials also have very high Poisson's ratios, whereas others, such as **cork**, have values near zero; it is even possible to design materials with *negative* Poisson's ratios, materials that expand laterally when stretched.

SHEAR

We have seen how the axial stresses of **tension** and **compression** deform materials, but materials can also be deformed by a different kind of stress, **shear stress**. Shear stress acts parallel to a material's surface (fig 1.7),

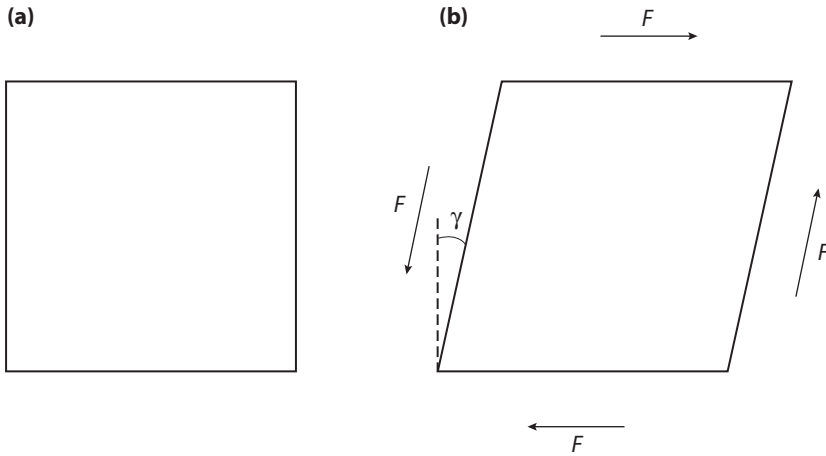


Figure 1.7. Shear stress deforms a square piece of material (a) into a rhombus shape (b) with shear strain γ .

tending to deform a rectangle into a parallelogram, a deformation known as **shear strain**.

Shear Stress

Just as for axial stresses, the shear stress, which is denoted by τ , is a measurement of the intensity of the shear force and is therefore given by the expression

$$\tau = F/A, \quad (1.11)$$

where F is the shearing force that has been applied, and A is the area *parallel* to that force over which the force is applied. The units of shear stress are the same as those of axial stress: newtons per square meter (N m^{-2}), or pascals (Pa). Note that if a unit of material is put into shear (fig 1.7b), the right-hand element being pushed upward, an equal and opposite shear force must act downward on the left-hand face for the element to be in equilibrium. However, if those were the only forces on the element, they would form a couple, spinning the material counterclockwise. Therefore two other shear stresses are set up, a stress on the top surface acting toward the right, and one on the lower surface acting toward the left.

Shear Strain

Just as axial stresses cause axial strains, so shear stresses set up shear strains, which are the change in the angles within the elements, denoted by γ (fig 1.7). Shear strains are expressed in radians, which, being ratios of the

angular displacement relative to a portion of the full circumference, are dimensionless, just like axial strains.

Determining Material Properties in Shear

The shear properties of a material can be determined by carrying out direct shear tests or **torsion** tests (see chapters 3 and 10), the results of which can be worked up just like the results of axial tests to give a graph of shear stress against shear strain. The most important shear property, the **shear modulus**, G , is determined similarly to Young's modulus, using the equation

$$G = d\tau/d\gamma. \quad (1.12)$$

The Relationship between Axial Forces and Shear

At first glance it seems as if axial and shear forces are quite different, unconnected forces. However, if we look at what happens during axial and shear loading, it becomes apparent that they are inextricably linked. In tensile and compressive tests, a square element at 45° to the loading will be sheared (fig 1.8a–c), whereas in a shear test, a square element at 45° to the loading will be stretched in one direction and compressed in the other (fig 1.8d,e). The amount of shear produced by a tensile test depends on the Poisson's ratio of the material: materials with a larger ν will contract more laterally than those with a smaller ν , so the shear strain caused by a given tensile strain will be greater. For this reason materials with a high Poisson's ratio will have a relatively lower shear modulus, G , compared with their Young's modulus, E . It can be readily shown by a geometrical argument (Gere, 2004) that E and G are related by the expression

$$G = \frac{E}{2(1 + \nu)} \quad (1.13)$$

so the shear modulus G is typically between 1/3 and 1/2 of the Young's modulus of a material, depending on its Poisson's ratio. Note that this expression is valid only for isotropic materials and so should *not* be used for biological materials, where it can prove highly misleading!

PERFORMING MATERIAL TESTS

Many of the mechanical properties of a material can therefore be readily determined by carrying out one of two sorts of mechanical tests in which materials are put into axial loading: tensile tests and **compressive tests**. Both of these are most conveniently carried out in universal testing machines on specially prepared samples.

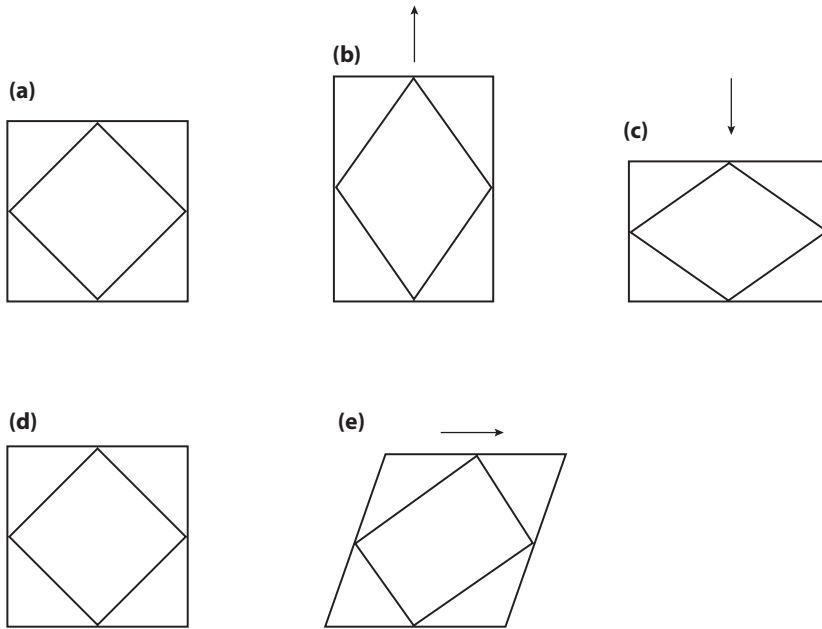


Figure 1.8. The relationship between axial and shear strains. If a square element at 45° (a) is stretched (b) or compressed (c), it will be sheared into a rhombus. Similarly if a square element at 45° (d) is sheared (e), it will be stretched and compressed into a rectangle.

Tensile Tests

For a tensile test, the test piece typically has a “dumbbell” form (fig 1.3a) with a relatively long, thin central portion and broad shoulders at each end. The sample is gripped firmly at its shoulders by two clamps: the lower one is mounted in the base of the machine; the upper one is attached via a **load cell** to a movable **crosshead**. To carry out the test, the crosshead is driven upward at a constant speed, while the force required to stretch the sample is measured by the load cell. This data is transferred to a computer that produces a readout of force versus deflection and, given the original dimensions of the sample, has the ability to calculate its material properties. It is assumed (fairly accurately) that all the stretching has occurred in the narrow central section of the sample. The widening at the ends ensures that the sample breaks in this central section and not at the clamps, where stresses can be concentrated.

Tensile tests have three main pitfalls. The first is that with relatively thick samples of stiff materials, the rigidity of the sample may approach that of the testing machine. When a test is carried out in this situation, the machine itself will deform significantly meaning that the readout overestimates the deflection of the sample and stiffness is underestimated. There are three

solutions to this problem: you can use a longer, narrower sample to reduce its rigidity; you can attach an **extensometer**, which directly measures strain, to the sample; or you can attach an electronic **strain gauge** to the sample (Biewener, 1994). The second problem with tensile tests is the difficulty in producing the complex shape of the sample. Biological materials can be prepared by cutting around a machined template, but doing so can prove difficult particularly for samples cut from small pieces of tissue. The third problem is that many biological materials, particularly the soft, wet, and slippery ones, can prove extremely difficult to clamp. In such cases biomechanists may resort to a range of techniques: using sandpaper to roughen the clamps; freezing the clamps to harden the material within the jaws; gluing the sample to the clamps using a cyanoacrylate glue that binds to water; or simply wrapping the sample around purpose-built attachments.

Compressive Tests

Some of the problems of tensile tests can be overcome by carrying out compressive tests, in which a relatively thicker rod of material is squashed between two plates. The sample is much easier to machine because no expanded ends are needed, but the sample and plates must both be machined flat. Because the sample is relatively thicker, it will also be more rigid than a tensile sample, so it is much more likely that strain will have to be measured with an extensometer. Compressive tests usually give values of stiffness very similar to those of tensile tests, but as we shall see, materials often have very different tensile and compressive strengths. Therefore to fully characterize a material both tests may be needed.

Torsion Tests

The shear properties of materials can be determined using the sorts of torsion tests we will examine more thoroughly in chapter 10.

Mechanical Testing with Homemade Equipment

Not everyone has access to a materials testing machine or can afford to buy one. It may also be impossible to transport samples to the laboratory (for instance, if you want to investigate the properties of wood in a tropical rainforest). Finally, most commercially available testing machines are just not sensitive enough to measure the material properties of structures such as lengths of spider **silk**, which are very thin and compliant. For these situations, it is often necessary to construct purpose-built apparatus, which can work perfectly well. Nowadays, electronic force and displacement transducers are fairly inexpensive and data logging into laptop computers is fairly straightforward. However, in certain situations electronic equipment

may not be practicable or affordable, and good results may instead be achieved by purely mechanical means, either by measuring force with a spring gauge or by hanging weights on the end of a sample. Whichever way the forces are measured, though, it must be remembered that tests can be divided into two main types. For larger samples, **displacement-controlled tests**, in which the length of the sample is progressively altered while the force required to do this is measured, are recommended. Not only is one controlling the independent variable of the stress-strain curve, but these tests are also fairly safe, since when the material breaks the only energy released is that which is stored in the sample. In contrast, **load-controlled tests**, such as those in which weights are hung on the end of a sample, are very easy to perform, but failure of the sample can result in potentially damaging deflection of the clamps and of the mechanism that is applying the load.

FAILURE AND BREAKING

As we have seen, it is relatively easy to explain how and why materials resist being deformed; one just has to consider the forces set up between their atoms. The fracture behavior of materials is more difficult to understand. It might be expected that the strength of a piece of material will be directly proportional both to the strength of its interatomic bonds and to its cross-sectional area. Hence its breaking stress should be high and independent of the sample size. However, most materials have much lower breaking stresses than would be predicted from the strength of their chemical bonds, and larger pieces of material often have far lower breaking stresses than small ones. Brittle materials also tend to be much easier to break than tough ones, even if they have the same breaking stress.

Throughout the last century, with the pioneering work of C. E. Inglis and A. A. Griffith (recounted very clearly by Gordon [1968]), it has been shown that to explain fracture, it is necessary to consider not only the overall stress in materials but also the distribution of stress within the sample and the changes in energy involved.

STRESS CONCENTRATIONS AND NOTCH SENSITIVITY

Let us first examine the distribution of stress within a material that is being stretched in a tensile test. If the test piece used is perfectly smooth and free of internal flaws, the stresses will be evenly distributed throughout the material and the strength of the sample will equal the breaking stress of the material times its cross-sectional area. However, if there is a small scratch or ridge in the surface, or a flaw within the material, the stresses will have to divert around these obstructions, and **stress concentrations** will be set up at their sides (fig. 1.9a). The stress concentration factor will depend on the shape of these imperfections. For a circular hole or semicircular notch,

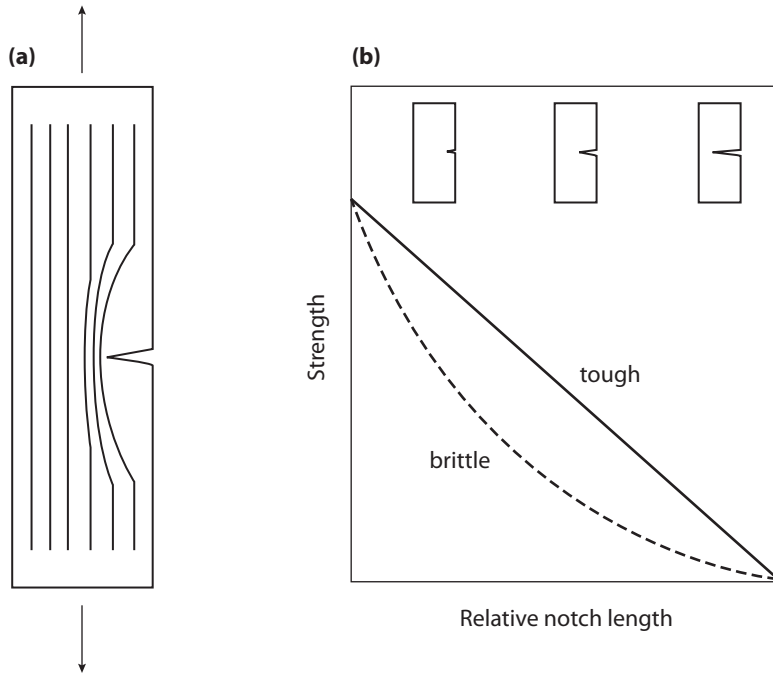


Figure 1.9. The effect of stress concentrations on the strength of materials. In a brittle material, stress concentrations will form at the tip of cracks (a). Therefore the strength of a piece of brittle material will fall rapidly (b, dashed line) if a notch is introduced. In contrast, in tough materials the strength will fall only linearly (b, solid line) in proportion to the length of the notch.

it has been calculated that the stress at the sides will be three times the mean stress, whereas for an elliptical hole or semielliptical notch, the stress concentration, C , is given by the formula

$$C = 1 + (2r_{pe}/r_{pa}), \quad (1.14)$$

where r_{pe} is the radius perpendicular to the force and r_{pa} is the radius parallel to it. The longer the crack and the smaller the crack tip, therefore, the higher the stress concentration. Long, narrow cracks or holes oriented at right angles to the applied force will therefore increase stress far more than ones oriented parallel to it.

If a brittle material with a notch cut in its side is stretched, the stress at its tip will increase more rapidly than in the material as a whole until the breaking stress of the material is reached and a crack opens up; this opening makes an even sharper notch, which quickly runs through the material. The strength of a piece of brittle material will therefore fall rapidly with the size of any flaws or notches at its surface, which is illustrated by a concave graph of strength against notch size, such as that shown in figure 1.9b. Such a material can be said to be notch sensitive. One reason why large pieces of

glass have a lower breaking stress than small ones is that they are more likely to have larger, sharper notches.

In contrast, if a tough material with a notch cut into it is stretched, the material will yield rather than break at the tip of the notch, deforming markedly and so blunting the crack tip. As a consequence the strength of a piece of tough material will fall slowly and linearly with notch length (fig. 1.9b) so that the strength is proportional to the area of intact tissue at the end of the notch. Materials showing this sort of behavior are said to be notch insensitive. Tough materials therefore show low **notch sensitivity**, and brittle materials show high notch sensitivity.

ENERGY CHANGES AND THE WORK OF FRACTURE

The argument above based on stress concentrations works well qualitatively, but it is less successful in quantitatively predicting and understanding the behavior of tough materials. Another, even more useful, way of thinking about what happens during failure is to look at the energy changes involved.

When an object breaks, interatomic bonds are broken, creating two new surfaces; this process requires energy, and at first glance it appears difficult to understand where that energy might come from. Let us examine the situation shown in figure 1.10, in which a plate of material of thickness t and with a crack in it of length a is being stretched, producing an overall tensile stress in the plate, σ . Elastic energy is stored in the plate, but because the stresses are being diverted around the tip of the crack, small areas above and below the crack will be unstressed and will store no energy. Here it is assumed that these areas have the shape of a right triangle, but this is just a rough approximation. If the crack extends by a distance da , a greater volume of material will become unstressed. The amount of elastic energy, W_e , that this will release is equal to the energy stored per unit volume of material (which we have seen from equation 1.8 is $\sigma^2/2E$) times the extra volume, which from geometry can readily be seen to be $2t a da$.

Extending the crack increases its surface area by the amount $2t da$, since the crack has both an upper and lower surface, and if the surface energy (in J m^{-2} , the energy required to produce a unit area of new surface) of the material is g , the surface energy required to extend the crack, W_s , is $2tg da$. For the crack to spontaneously extend, the energy released by unstressing the material around the crack must at least equal the surface energy required to extend it, so that

$$W_e \geq W_s$$

so

$$\sigma^2/2E \times 2ta da \geq 2tg da.$$

Therefore

$$\sigma^2 \geq 2Eg/a.$$

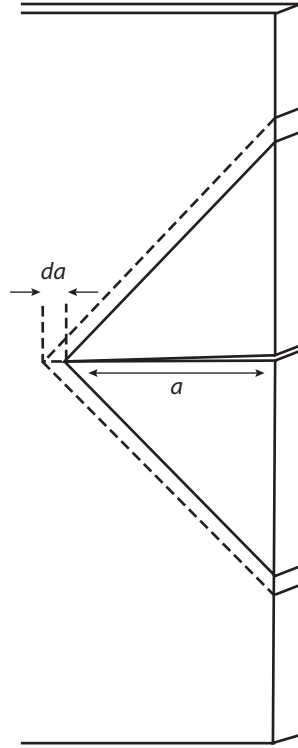


Figure 1.10. The effect of increasing crack length on the volume of unstressed material. If the crack extends a distance da , the material between the dashed and solid lines becomes unstressed, releasing energy.

More sophisticated and precise analysis gives a slightly different figure for σ of

$$\sigma \geq (2Eg/\pi a)^{0.5} \quad (1.15)$$

At low stresses a crack cannot extend because not enough energy is released from the relaxation around the crack tip to open the crack. Once σ exceeds the critical value, however, the crack can and will extend, and as it lengthens it becomes more and more energetically favorable for it to do so. The crack will run very rapidly across the material and break it. Note that the longer the crack is initially, the lower the stress required to break the material. The critical crack length a_{crit} can readily be obtained by rearranging 1.15 to give

$$a_{\text{crit}} = 2Eg/\pi\sigma^2. \quad (1.16)$$

It is greater for stiffer materials and ones with higher surface energy and decreases rapidly with the stress applied.

In fact, the surface energy of most materials, the energy that is required to break the top layer of interatomic bonds, is very low, approximately 1 J m^{-2} , so for a brittle material even tiny scratches can make it much weaker.

In tough materials, the amount of energy needed to make new surfaces, or the **work of fracture**, is far higher, because yielding may involve a wide range of mechanisms that absorb energy; it may involve deforming the material near the crack tip plastically, as in metals, or creating a rough fracture surface with a much greater surface area, as in fiberglass and green wood. Many different fracture tests may be used to calculate the work of fracture, W_f , which is defined as the energy to produce the crack, e , divided by the crack's area, A (not the total new area produced, which would be twice the area of the crack) giving the expression

$$W_f = e/A. \quad (1.17)$$

Work of fracture, like surface energy, therefore has the units J m^{-2} . The critical crack length for a tough material is derived by substituting work of fracture into equation 1.16 to give the expression

$$a_{\text{crit}} = E W_f / \pi \sigma^2. \quad (1.18)$$

As we shall see, many biological materials have particularly sophisticated toughening mechanisms.

MEASURING WORK OF FRACTURE

You might think it should be very easy to measure the work of fracture of a biological material. All you would need to do would be to perform a simple tensile test, and the work of fracture could be estimated from the area under the stress-strain curve up to the point of failure. Unfortunately things are not that simple. When a tensile piece is broken, some of the energy that was stored elastically may not be used to break the material but may instead be released explosively, making a snapping noise and flinging material about. The area under the stress-strain curve will therefore overestimate the work of fracture, and the error will be most severe when using long test specimens that store more energy. The problem could be minimized by using a very short test specimen, but machine compliance would then become a major difficulty. Instead, materials scientists have developed a range of tests to measure the work of fracture, although each of these has its own limitations.

Controlled Cracking

One method of overcoming the loss of stored elastic energy is to carry out a more controlled test in which the crack grows in a stable fashion. One way of doing this is to sequentially load and unload a test piece that is clamped asymmetrically in the testing machine. Examples of such methods include the compact tension test (Vincent, 1992) and the double cantilever beam (fig 1.11a). The specimen may be sequentially loaded and unloaded several times, driving the crack across the specimen, between which actions it should return to its original shape. The work of fracture can be calculated

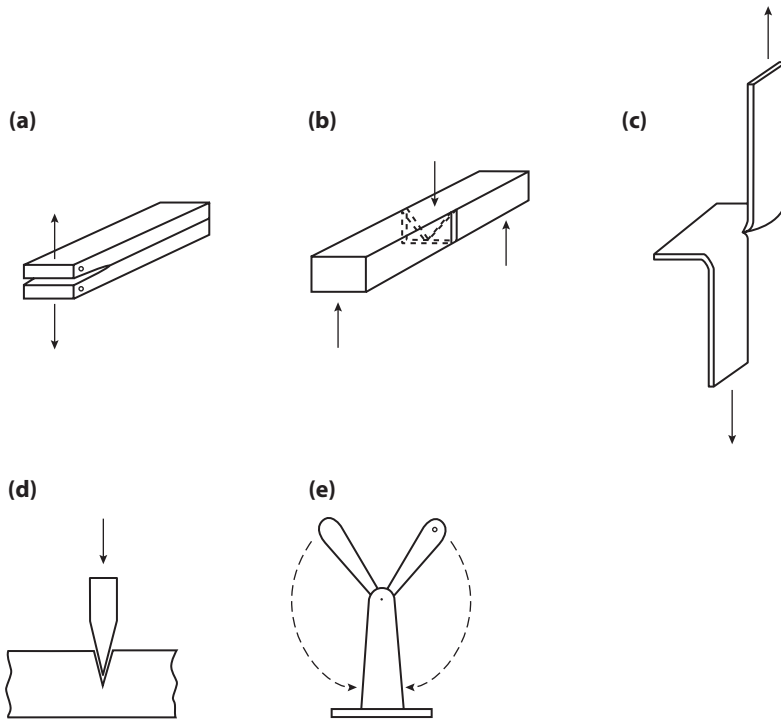


Figure 1.11. Mechanical tests used to determine the work of fracture of materials: (a) a double cantilever test, (b) a Tattersall and Tappin notch test, (c) a trouser tear test, (d) a cutting test, and (e) an impact test.

by dividing the area between the loading and unloading curves by the area of new crack formed. One problem with this test is that it can prove difficult to drive the crack in the right direction, though this can be overcome by cutting notches and guide slots to weaken the material in the required direction. Another, more intractable problem is the difficulty in machining and clamping suitable samples. A similar test method is to carry out a Tattersall and Tappin notch test (fig 1.11b; Tattersall and Tappin, 1966), very slowly bending a sample that is cut in such a way as to drive a crack gently through a beam.

Tearing and Peeling

Another method, which is useful for thin flexible material samples, is to carry out a trouser tear test (Vincent, 1992), in which two legs of a thin specimen are pulled apart (fig 1.11c). The work of fracture is the area under the force-deflection curve divided by the area of the new crack formed. A similar test can also be used to peel a narrow sliver of a material from the rest. Unfortunately, although they are easy to perform, these tests are only

really useful for materials that have preferred lines of failure, such as plant tissues, in which longitudinal fiber constrain the cracks to run between them; otherwise the tears are all too readily diverted in the wrong direction.

Cutting Tests

To overcome the problem of cracks moving in the wrong direction, several different kinds of cutting test have recently been devised. Materials may be cut through with a sharp blade (fig 1.11d), whether it be that of an instrumented microtome (Atkins and Vincent, 1984), a guillotine (Atkins and Mai, 1979), an inclined razor blade (Ang et al., 2008), or nail clippers (Bonser et al., 2004). Alternatively, one can use the double blades of a sharp pair of scissors (Darvell et al., 1996). In all of these cases the fracture is constrained to run in the desired direction. All of these tests are straightforward to perform, and the work of fracture is found by dividing the energy needed to cut through the specimen by the area of the fracture surface produced, although in scissor and guillotine tests, the friction of the devices also must be taken into account. The tests can also detect particularly tough regions of the material, such as fiber or veins, since the force needed to cut through them is greater and allows their toughness to be calculated separately. However, because the cuts constrain the direction of the crack very precisely, these tests measure the minimum work of fracture of the material and cannot detect how much toughening is given to the material by the sorts of mechanisms that involve diverting the crack.

Impact Tests

A final series of tests to measure work of fracture involves specimens being struck by the impact of a moving pendulum and measuring the energy required to snap them (Vincent, 1992). Typically a notched bar or rod of material is mounted in one arm of the apparatus and is hit by the other swinging pendulum (fig 1.11e). The specimen is broken transversely, the energy required being supplied by the kinetic energy of the pendulum, which consequently does not rise to so great a height after the impact as before. The work of fracture is the change in potential energy of the pendulum before and after the test divided by the cross-sectional area of the bar. This test works well for many stiffer materials, but since the precise conditions of loading are usually unknown, it is often hard to relate the results from this test to those of the other tests. In particular, the impacts tend to be very rapid, so less energy is used to break viscoelastic materials than in the other slower methods, and the work of fracture is consequently underestimated.

Other Measurements of Work of Fracture

The work of fracture of a material is usually regarded as being the same thing as its **toughness**. However, other, quite different definition of toughness

Table 1.1

Properties of Some Man-Made and Biological Materials.

Material	Stiffness (MPa)	Tensile Strength (MPa)	Extensibility	Strain energy (MJ m ⁻³)	Resilience (%)	Work of fracture (J m ⁻²)
Steel	200,000	400	0.008	1		100,000 –1,000,000
Glass	70,000	170	0.002	0.2		1–10
Concrete	100,000	5	0.00005	0.0001		3–40
Rubber	3	7	3	10	90	
Resilin	2	4	2	4	93	
Abductin	4				75–90	
Elastin	1.2	2	1	0.8	75	
Mucus	0.0002	0.0005	5		55	
Mesoglea	0.001					1,200
Moth Silk	4000	2000	0.3	200	35	
Tendon	2000	100	0.1	3	93	
Keratin	3000	300	0.2			15,000
Cuticle	5000	60	0.01	0.3		2,000
Unlignified plant cell wall	3,000	100	0.05			3,000
Wood	4,000	40	0.01	0.5		30,000
Bone	17,000	200	0.006	0.6		2,000
Dentine	15,000	50	0.003	0.1		500
Enamel	50,000	35	0.0005	0.02		200
Shell (nacre)	30,000	50	0.006	0.15		1600

are sometimes presented in papers. Some authors (see, for instance, Gosline et al., 2002) present values for toughness with the units J m⁻³. These values were actually derived from the area under the stress-strain graphs and so are more correctly measurements of the energy-storage capabilities of the materials. It is better, therefore, to use the term *work of fracture*.

COMPARING THE PROPERTIES OF MATERIALS

All the complexity of the properties of materials and of materials testing means that it is surprisingly complicated to compare the properties of different materials. They can differ in their stiffness, their strength, their ability to store and release energy, and in their toughness. Some materials are also better at resisting tension, whereas others resist compression better; and some materials have the same properties in all directions, whereas with others, their performance depends on the direction in which they are stressed.

A good way to get an instinctive idea of how to compare materials is to think of some everyday objects familiar even to children. Jell-O has very

low stiffness and strength and is also very brittle, making it easy for even very young children to cut up and eat. Cookies, in contrast, are stiff but not very strong, so a child can carry one around without breaking it yet still readily bite into it. Toffees are both stiff and strong, although they can be smashed to smithereens when hit by a hammer, showing that they are brittle. To eat them a child has to put them whole into their mouth, before softening them and dissolving them with their warm saliva. Pencils are stiff, strong, and extremely tough, so they can take a lot of punishment without breaking, although the brittle leads are easy to snap off. Rubber bands have low stiffness, but they are very stretchy, so they are reasonably strong and can store a lot of energy, which makes them ideal for use as slingshots.

What about “grown-up” materials? Well, steel is one of the most popular materials in engineering because it is stiff, strong, and tough and so is ideal for taking loads and resisting impacts. This is why it is used to make the shells of cars. Glass is far less useful, because although it is almost as stiff and strong as steel, it is extremely brittle and so shatters on impact. Concrete is widely used to build walls and floors because it is stiff and extremely strong in compression, but because it cannot take tension, it cannot be used to make the roofs of buildings without being reinforced with steel.

The properties of some important man-made materials and those of the natural materials we will encounter in the next few chapters are shown for ease of comparison in table 1.1. Perhaps the material that acts as the best single benchmark is rubber, since we can deform it fairly easily by hand and so get an intuitive feel for its properties. Most of its properties also, have small integer values for stiffness and strength (in MPa), strain energy (in MJ m^{-3}), and maximum strain. Some materials, such as **mucus** and mesoglea, are more compliant than rubber, but most “rigid materials” are thousands of times stiffer and tens of times stronger but many times less extensible.

As we shall see in the next few chapters, it is practically impossible to produce materials in which all the properties are maximized; in general, the stiffer and stronger a material is, the less it can be stretched. There is therefore no one “super” material that is ideal for all purposes, and the mechanical design of organisms consists of making materials that are suitable for a particular role and arranging them in the right way within the body. That topic is what most of the rest of this book is about.