

Chapter One

Introduction

1.1 BACKGROUND AND AIMS OF THIS BOOK

The mathematical heritage of the Indian subcontinent has long been recognized as extraordinarily rich. For well over 2500 years, Sanskrit texts have recorded the mathematical interests and achievements of Indian scholars, scientists, priests, and merchants. Hundreds of thousands of manuscripts in India and elsewhere attest to this tradition, and a few of its highlights—decimal place value numerals, the use of negative numbers, solutions to indeterminate equations, power series in the Kerala school—have become standard episodes in the story told by general histories of mathematics. Unfortunately, owing mostly to various difficulties in working with the sources, the broader history of Indian mathematics linking those episodes still remains inaccessible to most readers. This book attempts to address that lack.

The European scholars who encountered Indian mathematical texts in the eighteenth and nineteenth centuries were often completely at sea concerning the ages of the texts, their interrelationships, and even their identities. The sheer number of such works and the uncertainty surrounding even the most basic chronology of Sanskrit literature gave rise to great confusion, much of which survives to this day in discussions of Indian mathematics. This confusion was compounded by the fact that authors of different mathematical texts sometimes had the same name, and different texts themselves sometimes bore the same title. Even when the background and content of the best-known treatises were sorted out in the early nineteenth century, historians still had many vexing problems to contend with. Much mathematical material was embedded in the very unfamiliar context of medieval Indian astronomy and astrology. The style of its presentation, in highly compressed Sanskrit verse, was equally alien in appearance. Yet the material also bore many similarities, from its decimal numerals to its trigonometric formulas, to certain features of Western mathematics.

Into this new historiographic territory came the early authors of general histories of mathematics, foraging for grand narratives. Historians from Montucla to Moritz Cantor and Cajori incorporated into their overviews of world mathematics many of the newly gleaned facts about the Indian tradition. Their accounts established a standard if seriously incomplete picture of Indian mathematics that still serves as the basic framework for its treatment in most modern histories. Meanwhile, in India, researchers such

as Bāpudeva Śāstrī, Sudhākara Dvivedī, and S. B. Dikshit unearthed vast amounts of additional information that, being published mostly in Sanskrit and Hindi, had little impact on the work of non-Indologists.

B. Datta's and A. N. Singh's *History of Hindu Mathematics*, published in the mid-1930s, rapidly became the standard text on the subject in English, with a far broader range of sources and a more careful treatment of original texts than most general histories could boast. Other surveys followed, including C. N. Srinivasiengar's *History of Ancient Indian Mathematics*, in 1967, and T. A. Sarasvati Amma's *Geometry in Ancient and Medieval India* and A. K. Bag's *Mathematics in Ancient and Medieval India*, both in 1979. Indian mathematics has also been featured in several more general studies of Indian science and of non-Western mathematics, such as S. N. Sen's 1966 *Bibliography of Sanskrit Works in Astronomy and Mathematics* and G. G. Joseph's 1991 *Crest of the Peacock*. In addition, a large body of specialist literature on Sanskrit exact sciences—astronomy, mathematics and the disciplines that historically accompanied them, such as astrology—has appeared in English over the last few decades. Examples of this literature include David Pingree's biobibliographical *Census of the Exact Sciences in Sanskrit* and his *Jyotiḥśāstra: Astral and Mathematical Literature*, in Jan Gonda's *History of Indian Literature* series, the articles of R. C. Gupta and others in the journal *Gaṇita Bhāratī*, and editions and translations of Sanskrit texts, such as Takao Hayashi's *Bakhshālī Manuscript* and Pushpa Jain's *Sūryaprakāśa*.

Why, then, is it still so difficult for the nonspecialist to find trustworthy information on many aspects of the Indian mathematical tradition? The inadequacy of the old “grand narratives” in this regard still plagues many modern historians of mathematics who have to rely on them. Early surveys of Indian sources tended to portray them as a record of “discoveries” or “contributions,” classified according to modern mathematical categories and important in proportion to their “originality” or “priority.” The context for understanding Indian mathematics in its own right, as a part of Indian literature, science, and culture, was generally neglected. Up-to-date specialist literature supplying that context is often difficult for nonspecialists to identify or obtain, and sometimes difficult to understand. Finally, much of the desired data is simply absent from India's historical record as presently known, and the resulting informational vacuum has attracted a swirling chaos of myths and controversies to bewilder the uninitiated.

Additionally, the historiography of science in India has long been co-opted for political purposes. Most notoriously, some nineteenth-century colonial officials disparaged local intellectual traditions, which they termed “native learning,” in order to justify Westernized education for future colonial servants. Many nationalists responded in kind by promoting various separatist or Hindu nationalist historiographies, often including extravagant claims for the autonomy or antiquity of their scientific traditions. The influence of all these attitudes persists today in politicized debates about history, religion, and culture in Indian society.

The present work attempts to trace the overall course of Indian mathematical science from antiquity to the early colonial era. Its chief aim is to do justice to its subject as a coherent and largely continuous intellectual tradition, rather than a collection of achievements to be measured against the mathematics of other cultures. For that reason, the book is divided roughly chronologically, with emphasis on various historical perspectives, rather than according to mathematical topics, as in the classic surveys by Datta and Singh and Sarasvati Amma. Of course, this account remains greatly indebted to the labors of these and other earlier scholars, without whose groundbreaking achievements it would not have been possible.

The rest of this chapter discusses the historical setting and some of the chief historiographic difficulties surrounding Indian mathematics, as well as the role of mathematics in Sanskrit learning. Chapter 2 considers the evidence concerning mathematical concepts in the earliest extant Indian texts, while chapter 3 examines what we know from the (mostly fragmentary) sources in the first several centuries of the Classical Sanskrit period, starting in the late first millennium BCE. These reveal, among other things, the development of written number forms, particularly the now universal decimal place value numerals, and the circulation of mathematical ideas between India and neighboring cultures.

The middle of the first millennium CE saw the appearance of the first surviving complete Sanskrit texts in the medieval Indian tradition of mathematical astronomy. Chapter 4 explores these early texts and the snapshot they provide of mathematical sciences in their day. The establishment of mathematics as an independent textual genre—attested to in works dealing exclusively with the topics and techniques of calculation, rather than their application to astronomical problems—apparently followed soon afterward, as far as we know from the extant texts. The development, subject matter, and structure of this genre and its continuing relation to mathematical astronomy are discussed in chapter 5. Aspects of its social and intellectual context are treated in chapter 6: who were the people who were studying and writing about mathematics in medieval Indian society, what did they perceive its nature and significance to be, and how did this relate to the emergence in the early second millennium CE of important canonical mathematical texts? Chapter 7 continues this theme with a discussion of the best-known (and in many ways the most remarkable) of the pedagogical lineages in Indian mathematics, the famous Kerala school of Mādhava.

Chapter 8 explores the impact of the contacts between Indian and Islamic mathematics, which increased after Central and West Asian incursions into the subcontinent during the second millennium. The story closes in Chapter 9 with a survey of some of the early modern developments that gave place, during the British colonial period, to the cultural and intellectual transition from “Indian mathematics” to Indian participation in modern mathematics. This narrative is supplemented by two appendices at the end of the book. The first supplies some background on the relevant linguistic and literary features of Sanskrit. The second lists the biographical information available

on some of the most historically significant Indian writers on mathematics and attempts to separate out the widespread legends concerning them from the (usually scanty) established facts.

This material includes more discussion of astronomy than is typical for works on Indian mathematics. But it is not really possible to understand the structure and context of mathematics in India without recognizing its close connections to astronomy. Most authors of major Sanskrit mathematical works also wrote on astronomy, often in the same work. Astronomical problems drove the development of many mathematical techniques and practices, from ancient times up through the early modern period.

Equally crucial for our understanding of this subject is an awareness of some of the historiographic controversies involving ancient Indian texts. The whole framework of the history of Sanskrit mathematical science ultimately hinges on the question of when and how these texts were composed, and it is a question that still has no universally accepted answer. The discussion in this book for the most part hews to the standard or conservative scholarly consensus about the basic chronology of Indian history and science. Many of the generally accepted conclusions in this consensus are nonetheless not definitively proved, and many revisionist or minority views have achieved a wide popular currency.

These issues profoundly affect the inferences that we can draw about mathematics in India, and most readers will probably be much less familiar with them than with the historical background of mathematics in other cultures, such as ancient Greece or seventeenth-century Europe. It therefore seems appropriate to devote some space in the relevant chapters to explaining a few of the most influential debates on these topics. The aim is to steer a middle course between unnecessarily perplexing the reader with far-fetched speculations and ignoring valid criticisms of established hypotheses. Therefore, formerly controversial or surprising claims are not emphasized here if they are now universally accepted or discarded. There should be no need nowadays to point out, for example, that Āryabhaṭa's decimal arithmetic is not associated with Greek sources or that Mādhava's power series for trigonometric functions predate by centuries Newton's and Leibniz's versions of them.

1.2 HISTORY AND SOUTH ASIA

Traditional Indian culture and literature are frequently said to have an ahistorical perspective, supposedly preoccupied with timeless spiritual knowledge rather than the recording of mundane events. This is a rather misleading oversimplification. It is true that chronicles of purely historical events (as opposed to the legends of the ancient Epics and Purāṇas, only distantly inspired by history) are rare in Sanskrit literature. The historian of India, particularly early India, can follow no chronological trail blazed by an ancient predecessor like Thucydides or Sima Qian. Studies of artifacts—archaeology,

epigraphy, numismatics—and some literary references provide most of the known data about what happened and when in premodern South Asia. The current big picture of Indian history has been built up only slowly from these data, and has changed (and continues to change) significantly in the process.

The geographical locus of classical Indian culture is the South Asian subcontinent, encompassing most of the modern nations of India, Pakistan, Nepal, Bangladesh, and Sri Lanka. (Throughout this book the term “India” or “the subcontinent” will generally refer to this larger region rather than the territory bounded by the modern state of India.) Evidence concerning the historical roots of this culture is quite sparse. The earliest known texts in an Indian language are the collections of religious hymns and rituals called the Vedas, composed in an archaic form of Sanskrit known as Vedic Sanskrit, or Old Indo-Aryan. Their language and subject matter clearly reveal their kinship with the various cultures known as Indo-European. For example, the Vedic hymns refer to various Indo-European themes and motifs, such as fire sacrifices to the members of a divine pantheon with many counterparts among, for example, Greek and Norse deities, including a male thunder-god as leader; large herds of cattle; the two-wheeled, two-horse chariots used for battle and sport; and a sacred ritual drink (called *soma* in Vedic and *haoma* in Old Iranian). Moreover, Vedic Sanskrit is unmistakably descended, like the members of the Celtic, Germanic, Hellenic, Italic, Iranian, and other linguistic groups, from a closely related group of ancestral dialects reconstructed by linguists as Proto-Indo-European.

The origin and diffusion of the common ancestral Indo-European cultures are still quite problematic. The similarities and differences among the various reconstructed Proto-Indo-European dialects may provide some clues to their geographical distribution. For example, the Indo-Iranian ancestral dialect appears to have been farthest from the Germanic and Celtic, with ancestors of Greek and Armenian somewhere between them. Many linguists hypothesize that this reflects an Indo-European origin roughly in the middle of the regions over which these languages later spread: somewhere around the Black Sea or Caspian Sea, perhaps. The relative positions of the various dialect groups consequently were more or less maintained as the groups migrated outward into new territories, eventually becoming Celtic and Germanic languages in the northwest, Iranian and Indo-Aryan in the southeast, and so on.

When did this hypothesized diffusion occur? Most reconstructions place it somewhere in the fourth or third millennium BCE. Textual evidence provides some data points concerning later chronology. By the early second millennium BCE, the Anatolian Indo-European language called Hittite was spoken in Asia Minor; a few centuries afterward, an Indo-Aryan language (more archaic than Vedic Sanskrit) was in use in the Mitanni kingdom in what is now Iraq and Syria; an early form of Greek was written in the Linear B script in Crete and the Greek mainland in the thirteenth century BCE; and there are comparatively abundant records by the early first millennium of Indo-European languages and cultures in Iran, Greece, Asia Minor, northwestern

Europe, Central Asia, and elsewhere.

According to this scenario, speakers of Indo-Iranian (the immediate common ancestor of Indian and Iranian languages) were living in eastern Iran and western Afghanistan around the end of the third millennium BCE. Some of them spread westward into Iran, where the Iranian language subfamily then developed. Others moved eastward over the Afghan highlands into the Panjab, where some earlier populations had recently shifted to the east and south, probably due to environmental changes that dried up local rivers. The Indo-Iranian newcomers may have been taking advantage of the resulting increase in elbow room. There, perhaps in the late second millennium, they composed the earliest Vedic hymns in the Old Indo-Aryan tongue that had evolved from Indo-Iranian. (Alternatively, perhaps earlier Indo-Aryan speakers already settled in Iran were split by a wedge of Iranian speakers, which displaced some of them west into what became the Mitanni realm and the rest east into India.) Subsequently they assimilated the cultures, territories, and to a large extent populations of non-Indo-European groups in nearby parts of the subcontinent. By the middle of the first millennium BCE, Indo-Aryan culture was widespread in northern India, and dominant in its political centers. (Languages of the non-Indo-European family called Dravidian, such as Tamil and Telugu, retained their primacy in southern India, although they and their speakers were strongly influenced by Indo-Aryan language and culture.)

This, the standard account of the origin and growth of Vedic India, is sometimes referred to as the Aryan invasion theory (AIT). However, most modern Indologists prefer other terms such as “immigration” or “influx” to “invasion,” which connotes earlier assumptions, now discarded, of large-scale military conquest in the Panjab. The word “Aryan” likewise has unfortunate racist connotations, but it remains the standard linguistic designation for the Indian branch of the Indo-Iranian descendants of Proto-Indo-European. The AIT label itself, however, has become so loaded with ideological overtones that it seems best to avoid it. Here I rely instead on more general terms, such as “standard hypothesis” or “majority view,” to refer to the historical narrative described in the preceding paragraphs.

There are numerous difficulties with most of the features of this hypothesis. In the first place, the archaeological record of Indo-European diffusion is not clearly established. Nor is it clear how relatively small Indo-European population groups might have established so great a cultural, political, and linguistic dominance over such a broad geographical extent between about 3000 and 1000 BCE. But if the Indo-European diffusion was primarily a linguistic and cultural evolution rather than a mass migration of foreign populations, we would expect to find a good deal of continuity in genetics and material culture within the regions of expansion rather than a record of sudden disruption by hordes of new arrivals.

Such continuity is very apparent in northern South Asia, where there is a long record of settled communities with domesticated animals and grain agriculture. The so-called Indus Valley culture, which flourished in and

around Sind and the Panjab in the mid-third millennium, left archaeological traces similar to those found in nearby sites dating from as early as the seventh millennium BCE and as late as the first. The remains of these communities, including major Indus Valley urban centers such as Harappa and Mohenjo-Daro, show extensively developed agriculture, architecture, manufacture, and trade. They also preserve a collection of still undeciphered graphic symbols that may have been part of an ancient script, or perhaps just nonlinguistic signs. More recently discovered sites in Central Asia were probably linked to such centers, which also traded with Sumerian cities in Mesopotamia. Even after the previously mentioned ecological displacement of many inhabitants toward the east and south in the early second millennium and the decline of the major cities, the Indus Valley and related cultures apparently persisted throughout the Vedic period.

These facts have led some historians to suggest that this prehistoric urban-agrarian culture *was* Vedic culture. In this alternative reconstruction, there is no need to link the Vedas and their language to a presumed Indo-European expansion over the Afghan highlands; they can be accounted for as an autonomous development within the Indus Valley culture or one of its relatives (the so-called indigenous Aryan theory). However, this suggestion requires an explanation of the evident cultural and linguistic links between these alleged “autochthonous Aryans” and their counterparts in lands north and west of South Asia.

One proposed explanation is that the Indus Valley region was actually the original homeland of Indo-European culture: instead of a few Indo-Europeans trickling into the subcontinent through the mountain passes, most Indo-Europeans trickled out of it (whence the alternative name, Out of India, for this hypothesis). But this proposal creates at least as many problems as it solves. It is difficult to compare the evidence of Vedic Sanskrit culture with that of the Indus Valley and related cultures: the former is mostly textual while the latter is exclusively archaeological. But there do seem to be some significant differences between the two. For example, early Vedic hymns do not refer to cities or wheat, well known in the Indus culture. At the same time, Indus culture sites do not contain remains of characteristic Indo-European goods such as horses or chariots.

Linguistically, the Out of India hypothesis is seriously inadequate. Vedic Sanskrit exhibits some linguistic influences from non-Indo-European Indian languages that are not found in other Indo-European language families. How could this have happened if all Indo-European languages originated together in India? In addition, a number of plants and animals whose names occur in different Indo-European language families, allowing reconstruction of corresponding words in Proto-Indo-European, are found only in temperate climates north of the subcontinent, suggesting that Proto-Indo-European dialects were spoken outside India. Finally, as noted above, the reconstructed relationships among these dialects appear to correspond roughly to the relative spatial locations of the language families they ultimately evolved into. This correspondence is hard to explain if we assume that all the dialects

diffused in the same direction, via the same narrow channel, from a place of origin near the southeastern edge of the Eurasian continent. And of course, the Out of India hypothesis still leaves us with all the abovementioned difficulties in accounting for Indo-European expansion in other regions.

Consequently, the standard historical narrative, in which Vedic culture is largely based on Indo-European influence from northwest of the subcontinent in the second millennium BCE, still appears the simplest and most consistent explanation. However, it must be stressed that there is little definite evidence concerning the ways in which this influence operated, the genetic makeup or geographic origin of the people involved, and the relationships between Vedic and other early Indian cultures.

Events in Indo-Aryan India began to connect to recorded history elsewhere only around the middle of the first millennium BCE. This period saw what is known as the “second urbanization” of the subcontinent, with new major urban centers, the first to emerge since the decline of the Indus civilization, arising mostly in the eastern valley of the Ganges. By the late sixth century, the Persian empire had expanded as far as the northwestern Gandhara region on the Indus River. Alexander seized control of Gandhara from the Persians in the 320s. Almost simultaneously, a large kingdom was consolidated in northern India under Candragupta Maurya, who may have participated in the battles to check Alexander’s advance across the Panjab.

The birth of the religious-philosophical traditions of Jainism and Buddhism also occurred in the middle of the first millennium. Mahāvīra, the founder of Jainism, was born probably in the late sixth century, and the Buddha perhaps somewhat later. Their teachings, frequently linked to reformist movements within late Vedic thought, are possibly derived from non-Vedic religious beliefs in northeastern India, based on the concepts of karmic retribution and cycles of rebirth. Their influence in the late first millennium was considerable, even among the elite. Alexander’s contemporary Candragupta Maurya is said to have embraced Jainism; his grandson, the emperor Aśoka, in the mid-third century BCE adopted Buddhist beliefs.

The inscribed stone monuments of Aśoka’s reign contain the oldest securely dated writing in an Indian language (in this case, a Middle Indo-Aryan language related to Sanskrit). It may be that writing systems had been in use in India before Aśoka’s monuments were carved, but we have no positive evidence for this. The Vedas are the only extant Indian texts known to be much older than Aśoka’s time, and they were preserved by a sacred oral tradition rather than in written form. On the other hand, it may be that writing was a fairly recent innovation in Aśoka’s India, possibly stimulated by contact with the Persian empire.

Aśoka’s inscriptions also testify to a remarkable geographical range for the political influence, or at least the scattered political penetration, of the Mauryan empire: they occur as far north as Gandhara and as far south as modern Karnataka, and on both the western and the eastern coasts. Moreover, they record the launch of Buddhist missionary expeditions to Greek kingdoms in the west and to Sri Lanka. The teachings of Buddha

and Mahāvīra diffused rapidly throughout the subcontinent, although in the early Common Era these movements lost ground to an emerging complex of beliefs and practices that we now call Hinduism, namely, the worship of a modified pantheon combining Vedic and pre-Vedic deities and dominated by the gods Viṣṇu and Śiva.

For several centuries during and after Aśoka's reign, Indian contacts with neighboring cultures were frequent and often turbulent. In the northwest, successors to Alexander (the so-called Indo-Greeks) blended Greek and Indian cultures in their dominions. They in turn were followed by the Śakas or "Indo-Scythians" and "Indo-Parthians" arriving from central Asia, starting around the first century BCE. Some Śaka groups subsequently expanded southward into western India, under pressure from incursions by the Yuezhi of Mongolia, founders of the Kuṣāṇa empire. The Kuṣāṇas were strongly established in northern and western India by the second century CE, and traded extensively with the Roman empire, as did kingdoms in South India. Southern Indian ports also maintained a thriving trade with Southeast Asia.

The spread of Buddhist traditions in China inspired some Chinese Buddhists to make pilgrimages to India, where the empire or federation of the Gupta rulers held sway north and east of the Deccan plateau in the fourth and fifth centuries CE. In the sixth century, Gupta power was undermined by yet another invasion spurred by tumult in Central Asia, that of the Hūnas or Huns. Direct trade between India and Europe decreased with the decline of the Kuṣāṇa and Roman empires, but communication by sea between Southeast Asia and India's east and southwest coasts continued to flourish. In fact, much of Southeast Asia became heavily Indianized, with vigorous Buddhist and Hindu traditions.

After the rise of Islam, southern India's sea trade came to be largely dominated by Muslim Arab traders with commercial ties to West Asia. Arabs also established realms in northern and western India during the Islamic expansion of the early eighth century. At the start of the second millennium, strife in Central Asia once again impelled invaders across the Afghan passes: in this case, Turkic and Persian Muslims who turned from struggles with other Central Asian peoples to raids and conquests in northern India. The resulting Indo-Muslim empires of the mid-second millennium were later supplanted by European colonies, leading to the almost complete political control of the subcontinent by Great Britain in the nineteenth century.

It is plain even from the foregoing brief sketch that India has never been historically isolated from or irrelevant to the rest of Eurasia but rather has constantly exchanged goods and ideas with its neighbors. At the same time, from classical antiquity until the modern period, its multiple strands of influence and innovation were woven into a web of Sanskritized culture and learning that linked the entire subcontinent.

1.3 SANSKRIT LITERATURE AND THE EXACT SCIENCES

Sanskrit texts frequently refer to the “ocean of knowledge,” an appropriate metaphor for the vast abundance of subjects covered by the varieties of Sanskrit literature. The sacred Vedas, whose name literally means “knowledge,” are often considered the foundation of learning. The genre of “Vedic texts” embraces the four *saṃhitās* or collections of hymns and rituals—namely, the *Ṛg-veda*, *Yajur-veda*, *Sāma-veda*, and *Atharva-veda*—as well as exegetical and philosophical works like the Brāhmaṇas and Upaniṣads. In the first millennium BCE, the divisions of learning included not only the Vedic texts themselves but also the six “limbs,” or supporting disciplines, of the Vedas. These were phonetics, grammar, etymology, and poetic metrics, which ensured the proper preservation and comprehension of the archaic verses of the hymns; ritual practice, which specified the details of the various rites; and *jyotiṣa* or astronomy and calendrics, which determined the proper times for performance of the rites. The Vedic texts are generally known as *śruti*, “heard” via divine revelation; the limbs of the Veda, on the other hand, are called *smṛti*, “remembered” from human tradition.

The post-Vedic era of what is known as Classical Sanskrit, beginning in the late first millennium BCE, saw an expansion of the recognized categories within which knowledge was produced and organized. The plethora of Classical literary genres included works treating *dharma*, or religiously mandated law and right conduct; narrative and legend, such as the great epics *Mahābhārata* and *Rāmāyana*, and the Purāṇas; various philosophical, theological, liturgical, and devotional subjects; different types of literary composition, such as stories and poetry, and their aesthetic characteristics; performing arts; building arts; and several sciences, including an enhanced form of *jyotiṣa* that incorporated not only astrology but also computational methods in general, known as *gaṇita*. The exact sciences and most other branches of *smṛti* learning were called *śāstras*, “treatises” or “teachings.”

Vernacular languages—Indo-Aryan vernaculars like Pali and Prakrit, as well as classical Dravidian languages such as Tamil—played a large role in the development of Indian literature. Many religious and philosophical works, stories, poems, plays, and grammatical treatises were composed in languages other than Sanskrit. This was especially true among Jains and Buddhists, for whom the ancient Sanskrit Vedas were not as significant as their own sacred canons in Prakrit and Pali, respectively. (A number of Buddhist and Jain scholars in the Classical period, however, wrote in Sanskrit chiefly or exclusively.) The number and variety of surviving texts in vernacular languages increased with the passage of time and included, in the second millennium, many works on astronomy and mathematics. Sanskrit, like Latin in medieval Europe, nevertheless remained central as a widely shared language of scholarship: as the Indologist Sheldon Pollock writes, “There was nothing unusual about finding a Chinese traveler studying Sanskrit grammar in Sumatra in the seventh century, an intellectual from Sri Lanka writing Sanskrit literary theory in the northern Deccan in the tenth,

or Khmer princes composing Sanskrit political poetry for the magnificent pillars of Mebon and Pre Rup in Angkor in the twelfth” ([Pol2000], p. 599). But the place of the vernaculars in the culture of learning was never negligible. A view of Indian mathematics drawn almost exclusively from Sanskrit texts, as in the present work, is necessarily partial and incomplete; its only excuse—apology, rather—lies in the limitations on the size of the book and the abilities of the author.

The Vedic veneration of Sanskrit as a sacred speech, whose divinely revealed texts were meant to be recited, heard, and memorized rather than transmitted in writing, helped shape Sanskrit literature in general. The privileged position of orality may have inspired the fascination with, and advanced development of, phonetics and grammar among Indian scholars. Its influence is also visible in the conventional forms of Sanskrit works. Even treatises on secular and technical subjects were ideally considered as knowledge to be learned by heart, not merely kept in a book for reference. (In practice, of course, written manuscripts were crucial to the preservation and transmission of learning, and were produced probably in the hundreds of millions over the last two millennia.) Thus, texts were composed in formats that could be easily memorized: either condensed prose aphorisms (*sūtras*, a word later applied to mean a rule or algorithm in general) or verse, particularly in the Classical period. Naturally, ease of memorization sometimes interfered with ease of comprehension. As a result, most treatises were supplemented by one or more prose commentaries, composed sometimes by the author of the treatise, sometimes by later scholars, either in Sanskrit or in a local vernacular.

In addition to emphasizing the significance of the spoken word, Sanskrit intellectual traditions generally considered knowledge to be founded upon divine teachings. True knowledge of whatever sort was necessarily part of the fundamental truth of the Veda (or, for Buddhists and Jains, of their own sacred principles). Again, it would be misleading to characterize Indian thought simply as “static” or “timeless.” It changed over time to accommodate new ideas and new lines of argument, but innovations were generally worked into existing traditions rather than flaunted as revolutionary novelties.

Furthermore, the distinction between *śruti* and *smṛti* did not imply a sharp division of the sacred from the secular; many texts, even on technical subjects like *jyotiṣa*, were ascribed to the revelations of gods or legendary sages. These attributions expunged the historical context of the works to stress the divine importance of their content. Similarly, even historical human authors frequently omitted biographical information and other contextual details as irrelevant or unnecessary to their writings. This sometimes makes it difficult to distinguish reliably between human and allegedly divine authors, a difficulty further compounded by the Indian custom of bestowing on children the Sanskrit names or epithets of gods or sages.

Given this background, we should be prepared to find some substantial differences between mathematics in the Indian tradition and its counterparts

elsewhere. To take one example, there are few personal chronicles in Sanskrit literature comparable to the doxographical or biographical accounts of Hellenistic or Islamic scientists. Consequently, several medieval writers whose mathematical works were widely known in India—contemporaries of Theon of Alexandria, Zu Chongzhi, or Thābit ibn Qurra, about whose careers and families at least some evidence survives—are less distinct as historical personages than even the ancient Greek mathematicians Euclid and Antiphon, or Ahmes the scribe of the Rhind Mathematical Papyrus. Educational and professional institutions, libraries, and patrons are also frequently obscure. Consequently, it is hardly surprising that some popular histories filled the resulting void with many pseudobiographical legends about Indian mathematicians.

Another and more fundamental difference is that the Sanskrit tradition does not regard mathematical knowledge as providing a unique standard of epistemic certainty. For many Greek philosophers and their Islamic and European successors, a central concept was the abstraction of universal forms from their sensible manifestations in the same way that numbers and geometrical figures are abstracted from physical quantities and shapes. Hence the validity of mathematical knowledge has had profound implications for the nature of reality in western philosophical thought, from the Pythagoreans on down. It has been suggested that the corresponding role of “paradigmatic science” in Indian thought was filled instead by grammar (*vyākaraṇa*). In Sanskrit philosophy and logic, ideas about reasoning and reality are explicitly linked to the understanding of linguistic statements. What philosophers need to probe in such statements, therefore, is their grammatical interpretation rather than their analogies with mathematical entities.

Mathematics, not being an epistemologically privileged discipline in Sanskrit learning, was generally subject to the same truth criteria as other forms of knowledge. In Sanskrit epistemology, valid ways of knowing include direct perception, inference, analogy, and authoritative testimony. This means that the idea of mathematical proof is somewhat different from the formal chains of explicit deduction mandated in Greek geometry. Mathematical assertions in Sanskrit can be justified in a number of different ways according to philosophical truth criteria, and sometimes they are not explicitly justified at all. This is not to say that rigorous demonstration and formal logic were unknown to Indian mathematicians, nor that Indian mathematicians generally permitted arguments from authority to overrule demonstration. But there was no conventional structure of proof consistently invoked as essential to the validation of mathematical statements. True perception, reasoning, and authority were expected to harmonize with one another, and each had a part in supporting the truth of mathematics.