# 1 The Book Franklin Never Wrote

It seems to me, that if statesmen had a little more arithmetic, or were more accustomed to calculation, wars would be much less frequent.

—Benjamin Franklin (1787) [1]

The American author Ernest Hemingway never composed a guide for writers. Indeed, the very idea was anathema to him, in part because of a superstitious fear that any such discussion of his art would destroy the thing itself, just as dissecting a flower dissolves the very essence of its beauty. Yet there are enough fragments scattered through his private correspondence, in interviews, and in the opinions of his fictional characters, to piece together exactly what he would have opposed: a book called *Ernest Hemingway on Writing*.[2] Likewise Benjamin Franklin said little regarding his magic squares, revealing few results and no methods, but on mathematical matters there is enough surviving material to fill a book on this unexamined side of Franklin's otherwise meticulously documented life. Hence, the present account of Franklin's mathematical experiences and his miraculous numerical creations.

There is a danger here that we might simply be indulging an artist who is working outside his usual field of true expertise and talent, as when today's celebrity actors and musicians tout their novels, poetry, or paintings.[3] However, Franklin's case is quite different, for it is impossible to pin him down to a single area of distinction. He is the poster child for all-around genius, the last true renaissance man: jack of all trades, and master of many. It is hard to believe that so gifted a man as this would find his abilities lacking in any respect.

Nevertheless this is what the experts would have us believe. The editors of *The Papers of Benjamin Franklin* observe that Franklin "was not the mathematician that his friend was," comparing him with the philosopher and clergyman Richard Price, who (like Franklin) speculated on population statistics.[4] A scholar of another eighteenth-century American scientist, Cadwallader Colden, avers that "Franklin could not always follow Colden's reasoning especially in mathematics. . . ."[5] One recent biographer refers to "math, a scholastic deficit he never truly remedied."[6] We find that he "was not sufficiently furnished with a knowledge of mathematics," according to an earlier editor of his papers.[7] Similarly, a Franklin Medal winner described him—in an acceptance speech at the Franklin Institute, no less—as "a *polymath* [a person of greatly varied learning] who excelled at everything *except* mathematics."[8]

If there was an Enlightenment superman, this was Benjamin Franklin: printer, scientist, inventor, author, philosopher, diplomat, and more. As any survivor of the American primary school curriculum can tell you, here was the conqueror of all areas of human achievement. Through hard work and no small share of ingenuity, he managed to overcome a lack of formal education and define the American Dream. And yet, to hear the experts tell it, there remained a gap in Franklin's self-training. The allegation is easy to accept at face value, even comforting. Who among us has never encountered an impediment, an occasional difficulty or even outright failure, in math class? We need our heroes to have flaws, and this one seems plausible enough.

Surely there were gaps in his knowledge, no matter how all-encompassing that polymathic genius may have seemed, yet it is the central thesis of this book that Ben Franklin possessed a mathematical mind. His numerical creations were few, but those that survive

Fig. 1.1. Benjamin Franklin, engraving by A. H. Ritchie (after Charles Nicholas Cochin), no date. American Philosophical Society Library.

demonstrate a feel for number patterns that is unmatched even among many who dedicate their professional lives to mathematics. How much more wonderful, then, that someone who could have devoted only a small portion of his life to the subject would achieve so much in that same pursuit.

A legion of Franklin biographers has misrepresented or misunderstood his fantastic work with magic squares, when not simply ignoring it outright. An exception was Carl Van Doren, whose Pulitzer Prize–winning 1938 biography devoted a few pages to the subject, most of it in Franklin's own words.[9] For his trouble, Van Doren was skewered in a review in *Isis*, the journal of the history of science. The unkind reviewer, I. Bernard Cohen, would go on to become the preeminent science historian of the twentieth century; his articles and books were largely responsible for resuscitating Franklin's scientific reputation in America. The review dismisses Van Doren's biography as "hopelessly inadequate" and claims that the magic squares are given too much attention. Not only are they "of no importance in the development of mathematics," but moreover they represent "no indication of mathematical ability on Franklin's part."[10]

Yet even that distinguished critic would undergo a change of heart. Cohen's own book *Benjamin Franklin's Science* devotes a long passage to the same topic, even going so far as to include a lengthy quote from the same source as Van Doren.[11] This time he sees fit to admit the mathematical importance of magic squares: we must not focus on "obviously practical" goals alone. Magic squares "provide a means of perfecting one's skill in arithmetic." Franklin saw them as "a kind of game or puzzle," which is significant because, as Cohen explains: "The pursuit of mathematics is in any case, according to the German mathematician David Hilbert, like playing a game in which one sets up the rules or operations and sees what results arise from the proper manipulation of the meaningless entities represented by the symbols."[12]

Our object is not to show that Franklin would have identified himself as a mathematician, only that he was adept at the systematic and creative ways of thinking about numbers, arrangements, and relationships that characterize mathematical thought. He was skilled in logical argument, taught himself mathematics as a teenager, and even learned some of the art of navigation on his own. He was a zealous advocate for widespread education in basic accounting skills, repeatedly extolling the virtues of such training for both men and women. His reputation as a universal-genius-*sans*-mathematics is undeserved, as if such a creature were not already an impossibility.

His inner mathematician manifested itself in varied ways. The printing trade, his primary vocation, has mathematical aspects (as we will see in chapter 8). He developed a systematic decision-making technique related to modern *utility theory*, where difficult situations are resolved by means of an algebra for everyday living. For twenty-five years he produced an almanac, a wildly popular pamphlet in a genre that was more typically authored by astronomers and mathematicians. He conceived the most devious magic squares, odd little amusements that must have required considerable facility with number relationships, and these experiments occupied his thoughts periodically for more than half of his long life, as the present book will prove for the first time.

Those magic squares indicate a skill in solving basic algebraic equations, as well as a general comfort with abstract symbols. The latter trait is apparent in other ways, too, such as his use of coded messages and his alphabetic recreations. During the Revolutionary War, Franklin employed simple numerical codes for sensitive communications, though these reveal little of the mathematical sophistication that has come to characterize encryption in more recent times. He attempted to reform the English alphabet, and he corresponded with Noah Webster and Erasmus Darwin on the topic. Several letters from Franklin to his landlady's daughter, and her replies, are even composed in a particular alphabet of his own invention, so it appears that Franklin had no difficulty thinking in abstract, symbolic terms. For what it's worth, his linguistic talents were considerable; he learned languages easily—German, Latin, French, Spanish, and Italian—though he found reading easier than speaking.

It is often said that mathematical and musical proficiency are closely allied; Franklin mechanized the "musical glasses" in his invention of the glass armonica, for which both Mozart and Beethoven composed, and he performed on this instrument. Its very design required knowledge of the relationships between music, geometry, and physics. He created successful lotteries. To describe electrical charge, he appropriated the arithmetic terms *positive* and *negative*, still used for that purpose today. Some say that even the Declaration of Independence bears the mark of Franklin's mathematical side. Thomas Jefferson's original draft asserts, "We hold these truths to be sacred & undeniable, that all men are created equal," and so on. But after incorporating changes from Franklin and John Adams, "sacred & undeniable" was replaced by "self-evident."[13] Like the axioms of Newton or Euclid, each truth is so obvious as to be unprovable, beyond the reach of logical argument. (Among the books Franklin bequeathed to his grandson Ben was a French translation of Euclid's *Elements*, after two millennia the most successful textbook of all time.[14]) It may be no coincidence that the first four of Euclid's five "common" notions also concern equality, such as "Things which equal the same thing also equal one another," though the objects in this case are magnitudes (lengths, areas, or volumes) and not human beings.

While he tended to keep the arguments simple and commonsensical, Franklin had a knack for applying mathematics to areas of scientific and philosophic inquiry where such machinery was as yet rarely used or else completely unknown. His *Observations Concerning the Increase of Mankind and the Peopling of Countries*, an essay composed in 1751 and published four years later, was a landmark in the nascent field of demography, the study of human population statistics. Based on a multitude of factors (such as the heartbreakingly realistic assumption that around half of the children born would not survive to adulthood), he predicted that the population of the colonies would "at least be doubled every twenty years."[15] After some further analysis he allows for the more conservative estimate that it may take twenty-five years. His prognostications were remarkably accurate, especially when one considers that they were made in a time of great social upheaval, and that they belonged to a science that didn't properly exist yet; based on census data from 1790 to 1850, it appears that every twenty years the population increased by 80%, while a complete doubling occurs approximately every twenty-three years, which falls neatly between his two estimates.[16] Franklin's prediction that the population of the colonies would soon outstrip that of England was also borne out, though by then they were colonies no more.[17]

His appears to be a largely intuitive argument, as Franklin refers to the existence of supporting data without actually citing specific quantitative information. Yet careful readers of his almanacs may recognize that, only a year or two earlier, Franklin's *Poor Richard* included population data from three colonies and one European city (broken down in some instances by age, race, and county of residence), and that mortality and doubling-time questions were addressed by him there.[18] Seemingly out of place in a popular almanac, Franklin's ramblings on such topics illuminate some of the mathematical underpinnings of his little excursion into population statistics. As with the magic squares, his mathematical rigor is hidden, but no less real.

That Franklin qualifies as a founder of modern demography can be seen by his influence on Richard Price and Thomas Malthus. Price's analysis of population growth took the form of a personal letter to Franklin, before it appeared in the *Philosophical Transactions of the Royal Society* for 1769. Meanwhile Malthus specifically cites Franklin by name, and his work is acknowledged, in later editions of *An Essay on the Principle of Population*, one of the most important works of social science in all of human history. The Malthusian notion that population may increase exponentially had been hinted at in Poor Richard's almanac, and stated outright in Franklin's *Observations*.[19]

The claim that the number of inhabitants in the colonies would "in another century be more than the people of England" was initially presented, in 1751, in the context of border disputes with the French:

> How important an affair then to Britain is the present treaty for settling the bounds between her colonies and the French, and how careful should she be to secure room enough, since on the room depends so much the increase of her people.

These clashes would soon erupt into the French and Indian War, also called the Seven Years' War, in which both Franklin and a young Colonel Washington served. That same prediction appeared later on in a very different context. An anonymous letter co-written by

Franklin to the London *Public Advertiser* in 1770 used the idea to argue against taxation without representation:

> The British subjects on the west side of the Atlantic see no reason why they must not have the power of giving away their own money, while those on the eastern side claim that privilege. They imagine, it would sound very unmelodious in the ear of an Englishman, to tell him that by the rapidity of population in our colonies, the time will quickly come when the majority of the subjects will be in America; and that in those days there will be no House of Commons in England, but that Britain will be taxed by an American Parliament. . . .[20]

Applying basic mathematics to situations where most of us would not think to do so, he likewise addressed the twin evils of war and slavery. Franklin, a businessman who knew the value of a careful balance sheet, argued in economic terms, circumventing his compatriots' moral ambivalence. Whereas one's views on either issue might be held with a religious zeal, impervious to debate—as in the archaic view that slavery somehow benefited its captives, or in the still popular view that war often serves a greater good—advocates of either position might yield before a purely mathematical argument. To Benjamin Vaughan, the economist and diplomat, Franklin once wrote:

> When will princes learn arithmetic enough to calculate, if they want pieces of one another's territory, how much cheaper it would be to buy them, than to make war for them, even though they were to give a hundred years' purchase? But if glory cannot be valued, and therefore the wars for it cannot be subject to arithmetical calculation so as to show their advantage or disadvantage, at least wars for trade, which have gain for their object, may be proper subjects for such computation; and a trading nation, as well as a single trader, ought to calculate the probabilities of profit and loss, before engaging in any considerable adventure. This however nations seldom do, and we have frequent instances of their spending more money in wars for acquiring or securing branches of commerce, than a hundred years' profit or the full employment of them can compensate.[21]

In a letter to his sister Jane Mecom, he pursues the same line of reasoning. Franklin, who had secured foreign loans to support the Revolution and had extensive personal knowledge of its financial aspects, easily enumerates the specific costs associated with war, adding: "you have all the additional knavish charges of the numerous tribe of contractors to defray, with those of every other dealer who furnishes the articles wanted for your army, and takes advantage of that want to demand exorbitant prices."[22] War simply does not stand up to cost-benefit analysis, according to this philosopher-accountant.[23]

Franklin also argued against slavery using quantitative reasoning. According to his essay on population,

> It is an ill-grounded opinion that, by the labor of slaves, America may possibly vie in cheapness of manufactures with Britain. The labor of slaves can never be so cheap here as the labor of working men is in Britain. Anyone can compute it. Interest of money is in the colonies from 6 to 10 per cent. Slaves, one with another, cost £30 per head. Reckon then the interest of the purchase of the first slave, the insurance or risk on his life, his clothing and diet, expenses in his sickness. . . .[24]

He also sought to turn public opinion based on the sheer size of the slave trade, which was not fully appreciated at that time. In a letter to the *London Chronicle* (1772), he writes that "there are now eight hundred and fifty thousand negroes in the English islands and colonies. . . . [The] yearly importation is about one hundred thousand, of which one third perish" in transit or the "seasoning." He argues by the numbers.[25]

Elsewhere his economic argument is more muted: "Our slaves, Sir, cost us money, and we buy them to make money by their labour. If they are sick, they are not only unprofitable, but expensive."[26] In his later years, Franklin made the transition from small-time slaveholder to outspoken abolitionist, and as president of the Pennsylvania Abolition Society he lobbied Congress on that issue.[27] It would be the last great public act for this former almanac writer who had once intoned: "Nor let me Africa's sable Children see, vended for Slaves though formed by Nature free."[28]

The tendency to think in a precise, rational way about seemingly nonmathematical issues did not fade with age. In his twilight years, Franklin made a rather convincing quantitative argument that the positive qualities of one person do not necessarily translate into similar attributes on the part of their descendants.

In the 1780s, the prospect of establishing a new nobility loomed. American army officers had formed the Society of the Cincinnati, an elite fraternal organization in which membership would automatically pass from father to son. In an era of newly won egalitarianism, such an act was bound to be unpopular. After initial public outcry, membership was to be extended to all who served, not to officers alone. Yet the specter of a hereditary peerage arising so soon after the triumph of democracy over monarchy continued to raise the hackles of a sensitive public and was the subject of much controversy.

Franklin approached the question as an arithmetic problem. Did the sons and grandsons of distinguished veterans deserve to reap the fruits of their fathers' victories? Certainly not, said Franklin, for "descending honours" was a ludicrous notion. While great achievement by an individual may indeed reflect well upon his ancestors, conversely his son shares in only half the honor—as a child is the product of two different families.[29] (The longstanding theory that progeny arose from the seed of one parent alone was by now in its death throes.[30]) Grandchildren share in one-quarter, and so on, until after only nine generations (up to three centuries, he reckons) each descendant will share in "but a 512$^{th}$ part" of that honor. Thus the notion of a hereditary order is not only contrary to the ideals for which the Revolution was fought, it is also contrary to mathematics. (Showing an uncharacteristic absence of tact, Franklin—who amassed several lifetimes' worth of high honors—first introduces this "mathematical demonstration" in a letter to his own daughter.[31]) He opines:

> that all *descending* Honours are wrong and absurd; that the Honour of virtuous Actions appertains only to him that performs them, and is in its nature incommunicable. If it were communicable by

> Descent, it must also be divisible among the Descendants; and the more ancient the Family, the less would be found in any one Branch of it. . . .[32]

He refers here to the fact that one-half of one-half of one-half, and so on, moves ever closer to zero. A more nuanced approach to the question of inherited characteristics would have to wait for Charles Darwin (grandson of Franklin's friend Erasmus), Gregor Mendel, and their scientific descendants. Heritable traits are transmitted in a far more subtle and complex way than Franklin suggests; but the point of this example is not that he foresaw any major revolution in genetics, but rather that he felt a "mathematical demonstration" was the appropriate tactic in what was essentially a social debate.[33]

Another simple mathematical idea was used to great effect when Franklin invented the notion of daylight saving time. In a letter to the *Journal de Paris*, he calculates the hypothetical benefit to the city, were his plan to be adopted for roughly half the year.[34] Start with a value of 183 nights. Multiply by seven hours' candle-burning required each night by a household, which accounts for all rooms of the house; then by 100,000, the number of families in Paris. Next multiply this answer by one-half pound, which is the amount of wax and tallow used in an hour. (Lest anyone object to this ad hoc estimate, please note that Franklin grew up in a candle-maker's household!) The final factor is the cost of each pound of these materials, which is around 30 sols. Therefore the cost of all those candles is 1,921,500,000 sols. Since the livre tournois is worth 20 sols, we can divide by 20 to convert the cost to 96,075,000 livres tournois. "An immense sum! that the city of Paris might save every year, only by the economy of using sunshine instead of candles."[35] (One supposes that, were such an idea first proposed today, its implementation would be prevented out of concern for the wax industry.) There's something absolutely poetic in hearing an appeal from spendthrift Poor Richard's alter ego, urging us to save money—a *sol* instead of a *penny* saved—and tricking us into rising early, in the bargain.

The essential idea here is the *multiplication principle*, also known as the *product principle*: if there are 183 days and nights in which the new scheme is to be used, and seven hours of candle-burning to be saved each night, then this amounts to $183 \times 7 =$ 1,281 hours for each family. If we combine the benefits for all 100,000 families, then $183 \times 7 \times 100{,}000 = 128{,}100{,}000$ hours are at issue, and the calculation continues in this way. Analogous illustrations were employed for entirely different purposes in the pages of *Poor Richard*.[36]

Franklin's proposal is framed as a discovery, not an invention; while anyone who consults an almanac can verify that the sun rises "still earlier every day till towards the end of June," they seem unaware "that he gives light as soon as he rises." Though his suggestion was made in a less than serious manner, this letter to the *Journal* marks the origin of the daylight-saving schemes used today in most of the United States and in other parts of the world. Nothing but the simplest arithmetic, put to serious use.

But the most obvious way in which Franklin embraced mathematical thinking was in his love for the matrix known as the "magic square." That numerical puzzle occupied his thoughts periodically from the early 1730s through the late 1770s, that is, for nearly half a century. As a pastime enjoyed for the better part of a lifetime, by one of the greatest minds of that era, it is surely worth our attention. For the uninitiated, here is a brief introduction to the magic square.

First, a *matrix* (plural *matrices*) is a rectangular array of numbers, letters, words, or other objects. This could be a bookkeeping record, a chart of the tides, or any other arrangement of items, especially abstract symbols or data, into rows and columns. Whenever I teach a course in matrix theory, I wait for the inevitable question: Isn't the definition redundant? Isn't every "array" automatically rectangular? But one can certainly envision arrangements into other shapes. As you'll see in chapter 7, for instance, Franklin constructed a rather ingenious *circular* array. A more familiar example is the infinite *triangular* array called *Pascal's triangle*, named for the French mathematician and religious philosopher

Blaise Pascal (though he was not the first to discover it).[37] The first few rows are

$$\begin{array}{ccccccccccccc}
&&&&&&1&&&&&& \\
&&&&&1&&1&&&&& \\
&&&&1&&2&&1&&&& \\
&&&1&&3&&3&&1&&& \\
&&1&&4&&6&&4&&1&& \\
&1&&5&&10&&10&&5&&1& \\
1&&6&&15&&20&&15&&6&&1
\end{array}$$

Each entry is equal to the sum of the two entries immediately above it to the left and right. For example, $6 = 1 + 5$ and $15 = 5 + 10$. You could use this rule to work out as many rows as you like, so it really is an infinite triangular array.

In mathematics, though, the term *matrix* does refer specifically to a rectangular arrangement of objects that can be thought of as lying in a grid of smaller squares or rectangles. Matrices defined many aspects of Franklin's life: from the technical aspects of his trade; to the chessboard that he loved; to the weekly record where he kept track of each transgression committed against virtue, in his personal quest for moral perfection (figure 1.2); to the tables of calendars, currency, and kings that filled Poor Richard's almanacs between 1733 and 1758; to his magical squares. The word "matrix" had not yet acquired its modern meaning at that time—in the printing trade, for instance, a matrix (or *matrice*) referred to the mold in which a letter of type is cast—yet it is clear that the concept itself was a motif in Franklin's life. As one commentator puts it in another context, "it is not too much to say that he saw the world through the grid of a case of type."[38]

A *magic square* is a type of square matrix. That means it is the same number of units wide as it is long. We write in the spaces as you would fill in the cells of a crossword puzzle, except that this crossword has no blacked-out cells. In the lingo of puzzlers, it is

| | TEMPERANCE. | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | *Eat not to Dulness.*<br>*Drink not to Elevation.* | | | | | | | |
| | | S | M | T | W | T | F | S |
| Temperance | T | | | | | | | |
| Silence | S | * | * | | * | | * | |
| Order | O | ** | * | * | | * | * | * |
| Resolution | R | | | * | | | * | |
| Frugality | F | | * | | | * | | |
| Industry | I | | | * | | | | |
| Sincerity | S | | | | | | | |
| Justice | J | | | | | | | |
| Moderation | M | | | | | | | |
| Cleanliness | Cl. | | | | | | | |
| Tranquillity | T | | | | | | | |
| Chastity | Ch. | | | | | | | |
| Humility | H | | | | | | | |

Fig. 1.2. Sample report card from the *Autobiography*.

more appropriate to refer to it as a cross-*number* puzzle, as we will usually fill the spaces with numerals instead of letters. The goal is to write them so that each line of numbers across, down, or diagonally always totals the same value. For example, in a $5 \times 5$ grid there are five rows across, five columns down, and two diagonals (joining opposite corners and passing through the center), or twelve patterns to satisfy in all.

It's easiest to begin with a $3 \times 3$ array, like a blank tic-tac-toe board (see figure 1.3). Now you could take the easy way, and just write the number 1 in every space, but most people wouldn't find that to be a very impressive solution. It makes more sense to fill these nine spaces with the first nine counting numbers 1,2,3,4,5,6,7,8,9 in some order. Feel free to put the book down for a few minutes and experiment before reading on.

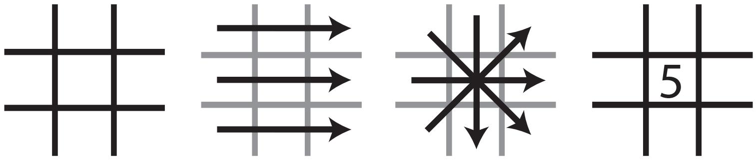


Fig. 1.3. Deriving a $3 \times 3$ magic square.

Since every row, column, and diagonal must have the same total, it would be helpful to know in advance just what that total should be. The value is readily determined without even knowing which number goes where, as follows. If you were to add up all nine numbers in this little $3 \times 3$ matrix, the sum would be (in some order) $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 = 45$. Therefore, the three equal rows, taken together, add up to 45. That means each row alone sums to 15, so we now know what the "magic sum" will be. (If the puzzle has stumped you till now, try it again with the aid of this clue.)

Once you know that the rows, columns, and diagonals each add up to 15, it's possible to determine what number is placed in the middle of the grid. The key is to look at the middle row across, the middle column down, and both diagonals, all at once. That corresponds to four copies of the "magic sum," so the total of these 12 numbers is equal to four 15's, or 60. But the middle value was included multiple times (four times, to be exact), whilst the other values in our $3 \times 3$ grid were each included just once. That explains why we got a larger total than 45 this time. Overcounting the middle value three times increased the whole total by 15, so the middle value is equal to 5. There! We finally have one particular entry in place. (That's your last hint before we finish the puzzle!)

Now you need to fill in the middle row, middle column, and both diagonals. Each of these configurations should add up to 15, but each already contains a value of 5 in the middle. To fill out the remaining eight spaces, use pairs: 1 and 9, then 2 and 8, then 3 and 7, and finally 4 and 6. (For extra credit: Why can't the number 1 appear in a corner cell?) There are eight different answers, all equally correct, as shown in figure 1.4.

| | | |
|---|---|---|
| 4 | 9 | 2 |
| 3 | 5 | 7 |
| 8 | 1 | 6 |

| | | |
|---|---|---|
| 8 | 3 | 4 |
| 1 | 5 | 9 |
| 6 | 7 | 2 |

| | | |
|---|---|---|
| 6 | 1 | 8 |
| 7 | 5 | 3 |
| 2 | 9 | 4 |

| | | |
|---|---|---|
| 2 | 7 | 6 |
| 9 | 5 | 1 |
| 4 | 3 | 8 |

| | | |
|---|---|---|
| 8 | 1 | 6 |
| 3 | 5 | 7 |
| 4 | 9 | 2 |

| | | |
|---|---|---|
| 6 | 7 | 2 |
| 1 | 5 | 9 |
| 8 | 3 | 4 |

| | | |
|---|---|---|
| 2 | 9 | 4 |
| 7 | 5 | 3 |
| 6 | 1 | 8 |

| | | |
|---|---|---|
| 4 | 3 | 8 |
| 9 | 5 | 1 |
| 2 | 7 | 6 |

Fig. 1.4. Eight solutions.

In fact there is really just one answer, in a sense, because all the other solutions are obtained either by rotating the first solution or else by flipping it over (that is, by a mirror reflection). You can check that all three rows, all three columns, and both diagonals add up to the same total. It's magic!

Clearly some creative skill with arithmetic is required by anyone who deals in such puzzles. For centuries, mystics and mathematicians struggled to create ever more impressive magic squares. By the era of pre-Revolutionary America, it was time for the master of the magic square to unveil his work.

## Notes

1. Letter from Franklin to his sister Jane Mecom, Sept. 20, 1787. Albert Henry Smyth, ed., *The Writings of Benjamin Franklin, Collected and Edited with a Life and Introduction by Albert Henry Smyth*, New York: The Macmillan Company, 1905–7, Vol. 9, p. 613.
2. Larry W. Phillips, ed., *Ernest Hemingway on Writing,* New York: Touchstone, 1999, esp. pp. 48–49. Hemingway also states that "having books published . . . is even worse than making love too much" (p. 55).
3. Hopefully this comment does not apply to a mathematician who writes history.
4. Leonard W. Labaree *et al*., eds., *The Papers of Benjamin Franklin*, New Haven and London: Yale University Press, 1972, Vol. 16, p. 81. Henceforth this source will be abbreviated *Papers.*

5. Unsigned preface to *The Letters and Papers of Cadwallader Colden,* New York: Printed for the New York Historical Society, 1918–37, Vol. 1 (1711–1729), p. vi. The assertion might be correct. However, it may be that Franklin claimed ignorance simply to avoid insulting Colden, who had asked for feedback on a mathematical manuscript that suffered from many shortcomings (according to James Logan, whose qualifications are not in dispute).

6. Walter Isaacson, *Benjamin Franklin: An American Life*, New York: Simon & Schuster, 2003.

7. Smyth, *The Writings of Benjamin Franklin,* Vol. 1, p. 73.

8. The speaker was computer scientist and mathematician Donald Knuth, a man of Franklin-level brilliance who won the award in 1988. To his everlasting credit, Knuth has assured me that this assertion will be recanted in future printings of his *Selected Papers on Computer Science*, after having read one of my articles on Franklin.

9. Carl Van Doren, *Benjamin Franklin*, New York: The Viking Press, 1938, pp. 143–146.

10. I. Bernard Cohen, review, *Isis*, Vol. 31, No. 1, 1939, pp. 91–94. Later, Cohen became editor of the journal.

11. Like Van Doren, Cohen allocates approximately one thousand words to the topic.

12. I. Bernard Cohen, *Benjamin Franklin's Science*, Cambridge: Harvard University Press, 1990. The chapter I have quoted was originally published as "How Practical Was Benjamin Franklin's Science?" in *The Pennsylvania Magazine of History and Biography*, Oct. 1945, pp. 284–293. It is worth noting that in 1947, Cohen reviewed another work by Van Doren, the collection *Benjamin Franklin's Autobiographical Writings*, which also included material on the magic squares, largely identical to that in his earlier book; but this review made no mention of the fact, and even praised a "model of scholarly workmanship." *Isis*, Vol. 37, No. 1–2, 1947, pp. 85–86.

13. Carl Becker (*The Declaration of Independence: A Study in the History of Political Ideas*, New York: Vintage Books, 1970) and Walter Isaacson (*Benjamin Franklin: An American Life*, 2003) identify Franklin as the likely author of that essential emendation. Others believe Jefferson himself to be responsible for the change. However, the latter argument is based on the belief that this particular alteration was made in Jefferson's handwriting—which does not change the fact that Jefferson incorporated suggestions of Adams and Franklin, which he may have then written out himself.

14. "List of Books for B. F. Bache," manuscript in the Library Company of Philadelphia. The influence of Newtonian science on Franklin and Jefferson is described in I. Bernard Cohen, *Science and the Founding Fathers*, New York: W. W. Norton and Company, 1997.

15. It is interesting to note that, of Franklin's three children, two made it to adulthood. Whether this personal experience influenced his 50% statistic is unknown, for he does not explain it, other than to say that he assumes eight children to a marriage, of whom four survive.

16. The census figures themselves show an increase varying between 77.04% and 84.28%, but a standard statistical approach leads one to the conclusion that 80.45% is the correct figure. (For the technically minded, I have used an exponential regression model fit to eight data points, with $R^2 > 0.998$.)
17. In the 1830s, the American population did surpass the British.
18. *Poor Richard Improved . . . for the Year of Our Lord 1750*, Franklin and Hall, 1749.
19. That he believed growth to be exponential is clear from the fact that he refers to a constant doubling time: "doubled every twenty years," "doubling . . . once in twenty-five years." Contrast this with *arithmetic* growth, where the same number of inhabitants is added every year, so that it takes longer and longer for the population to double as time passes. (Perhaps Franklin's awareness of exponential growth originated in compound interest calculations.)
20. *Papers*, 1973, Vol. 17, pp. 5–8. There is some controversy over the degree of collaboration between these coauthors: see Carla H. Hay, "Benjamin Franklin, James Burgh, and the Authorship of 'The Colonist's Advocate' Letters," *The William and Mary Quarterly,* 3rd Series, Vol. 32, No. 1, 1975, pp. 111–124. The inclusion in this letter of a detail from Franklin's earlier *Observations* adds some weight to the claim that he is a coauthor.
21. *The Writings of Benjamin Franklin*, Vol. 9, pp. 676–677.
22. *Ibid*, pp. 612–613.
23. Yet another instance where Franklin makes an economic argument against war is in a letter to an unnamed correspondent, in William Temple Franklin, *Memoirs of the Life and Writings of Benjamin Franklin,* London: Henry Colburn, Vol. 2, 1817, pp. 106–107.
24. *Observations Concerning the Increase of Mankind*, 1751 (published 1755).
25. *The London Chronicle*, June 18–20, 1772, as reprinted in the *Papers,* 1976, Vol. 19, pp. 187–188; also pp. 112–115 for his source, the abolitionist Anthony Benezet, explicitly acknowledged on p. 269.
26. Letter dated January 15, 1766, in *Papers*, 1969, Vol. 13, pp. 44–49. The letter is signed "Homespun," but it was attributed to Franklin by his grandson.
27. Joseph Ellis, *Founding Brothers: The Revolutionary Generation*, New York: Alfred A. Knopf, 2000.
28. *Poor Richard Improved* for 1752, paraphrasing Richard Savage's *Of Public Spirit in Regard to Public Works* (1737).
29. He points out that the commandments instruct one to honor one's father and mother, not necessarily one's children. I am reminded of Plutarch: "It is indeed a desirable thing to be well descended, but the glory belongs to our ancestors."
30. This theory (preformationism) was itself divided into two opposing camps: the spermists and the ovists. For example, the former held that the sperm cell contained a tiny but complete person, and within each tiny male *homunculus* were even smaller *homunculi*, and that the entirety of the future human race was stacked like Russian dolls, ad infinitum. The successor theory recognized that both parents contributed essential ingredients.

31. These ideas are described in detail in a letter to Sarah Bache (Jan. 26, 1784), and referred to briefly in another to George Whatley (May 23, 1785). Smyth, *Writings*, Vol. 9, pp. 161–168 and 331–339. For a similar (and earlier) mathematical application, see *Poor Richard Improved* for 1751.

32. Letter to G. Whatley; see note 31.

33. A similar explanation is sometimes given as an objection to the use of DNA testing in order to identify remote ancestors (see, for example, in *Time*, July 11, 2005). If you learn of a tenth-generation ancestor of great importance, that still leaves a thousand others who made equal contribution to your being.

34. *Writings*, Vol. 9, pp. 183–189.

35. A desire to conserve resources remains the driving force behind the use of daylight saving time (DST). During a major oil crisis in the 1970s, the federal government temporarily mandated its use year round. Ironically, oil was not a new issue here; Franklin's thoughts on the subject were initially inspired, not by candles, but by the question of a new oil lamp then in use: "whether the oil it consumed was not in proportion to the light it afforded, in which case there would be no saving in the use of it." The question is ever more relevant in an age in which we are burning the candle at both ends. (A new law mandates four extra weeks of DST, to begin in 2007.)

36. Poor Richard's almanacs include quite a few examples of such repeated multiplication, as we will see in chapter 3.

37. It is true that, by rotating and stretching the configuration, Pascal's triangle can be expressed as an infinite *rectangular* array:

$$\begin{array}{cccc} 1 & 1 & 1 & 1 \ldots \\ 1 & 2 & 3 & 4 \ldots \\ 1 & 3 & 6 & 10 \ldots \\ \vdots & \vdots & \vdots & \vdots \end{array}$$

and it is so written in some of the early European works. However, in a much older Chinese incarnation—and in the version most of us learned in school—it is built on an acute angle (less than ninety degrees) opening downward as we have drawn it. Indeed, Pascal's triangle is usually abbreviated as a *finite* array in classroom settings, and this finite array is always triangular (never rectangular).

38. Christopher Looby, "Phonetics and Politics: Franklin's Alphabet as a Political Design," *Eighteenth-Century Studies*, Vol. 18, No. 1, 1984, pp. 1–34.