1 Introduction

1.1 General Remarks

The theoretical structure of electric and magnetic fields is presented in the standard textbooks, and one may ask why further conversation on the subject is useful or interesting. What is new that has not already been said many times before? The reply is that the emphasis in the usual formulation of electromagnetism is directed toward static electric and magnetic fields and then to electromagnetic radiation, whereas we are interested here in the electromagnetism of the cosmos—the large-scale magnetic fields that are transported bodily in the swirling ionized gases (plasmas) of planetary magnetospheres, stars, and galaxies, and, indeed, throughout intergalactic space. The plasma and the magnetic fields appear to be everywhere throughout the universe. The essential feature is that no significant electric field can arise in the frame of reference of the moving plasma. Hence, the large-scale dynamics of the magnetic field is tied to the hydrodynamics (HD) of the swirling plasma in the manner described by theoretical magnetohydrodynamics (MHD). So we shall have a fresh look at the theoretical foundations of both HD and MHD. The conventional derivations of the basic equations of HD and MHD are correct, of course, but the derivations ignore some fundamental questions, allowing a variety of misconceptions to flourish in the scientific community. We work out a minimal physical derivation, laying bare the simplicity of the necessary and sufficient conditions for the validity of HD and MHD to describe the large-scale bulk motion of plasmas and their magnetic fields. The essential condition for HD is that there be enough particles to give a statistically precise definition of the local plasma density; the essential condition for MHD is that there be enough free electrons and ions that the plasma cannot support any significant electric field in its own moving frame of reference. Both of these requirements are satisfied almost everywhere throughout the cosmos, with the result that HD and MHD accurately describe the large-scale bulk dynamics of the plasmas and fields. The magnetic field is transported bodily with the bulk motion of the plasma, and the dynamics is basically the mechanical interaction between the stresses in the magnetic field $\mathbf{B}$ and the pressure p_{ij} and bulk momentum density $NM\mathbf{v}$ of the plasma velocity $\mathbf{v}$. The associated electric current $\mathbf{j}$ and the electric field $\mathbf{E}$ in the laboratory frame of reference play no direct role in the dynamics. They are created and driven by the varying $\mathbf{B}$ and $\mathbf{v}$. If needed for some purpose, they are readily computed once the dynamics has provided $\mathbf{B}$ and $\mathbf{v}$.

It is here that a fundamental misunderstanding has become widely accepted, mistaking the electric current **j** and the electric current **E** (the **E**, **j** paradigm) (Parker 1996a) to be the fundamental physical entities. Steady conditions often can be treated using the **E**, **j** paradigm, but the dynamics of time-dependent systems becomes difficult, if not impossible, because of the inability to express Newton's equation in terms of **E** and **j** in a tractable form. That is to say, **E** and **j** are proxies for **B** and **v**, but too remote from **B** and **v** to handle the momentum equation. So it is not possible to construct a workable set of dynamical field equations in terms of **j** and **E** from the equations of Newton and Maxwell. The generalized Ohm's law is often employed, but Ohm's law does not control the large-scale dynamics. The tail does not wag the dog. This inadequacy has led to fantasy to complement the limited equations available in the **E**, **j** paradigm, attributing the leading dynamical role to an electric field **E** with unphysical properties. Magnetospheric physics has suffered severely from this misdirection, and we will come back to the specific aspects of the misunderstanding at appropriate places in these conversations.

The essential point is that we live in a magnetohydrodynamic universe in which the magnetic field **B** is responsible for the remarkable behavior of the gas velocity **v**, and vice versa. Then we must recognize that the large-scale magnetic stresses in the interlaced field line topologies created by the plasma motions have the peculiar property of causing the field gradients to increase without bound. The resulting thin layers of intense field shear and high current density "eat up" the magnetic fields at prodigious rates. The effect is commonly called *rapid reconnection* of the magnetic field because the field lines are cut and rejoined across the intense shear layer, and it is a universal consequence of the large-scale field line topology. Rapid reconnection is evidently responsible for such phenomena as the solar flare, the million degree temperature of the solar X-ray corona, and the terrestrial aurora. So the MHD universe is far more active and interesting than a purely HD universe, with the magnetic activity of the Sun an outstanding example. R. W. Leighton remarked many years ago that if it were not for magnetic fields, the Sun would be as uninteresting as most astronomers seem to think it is. The activity of the Sun is the model, then, for the unresolved activity of other stars.

The conversation is intended to complement, rather than replace, the familiar textbook development of electromagnetic theory and of HD and MHD. It is assumed that the reader is already familiar with the conventional development of electromagnetic theory, and it is to be hoped that the reader has the patience to follow the conversation when it briefly reiterates some of that familiar boilerplate, because the basics are necessarily the same, even as we provide a different emphasis.

There will be some new twists to the development along with the boilerplate. For instance, we show that the Biot-Savart integral form of Ampere's law implies Maxwell's equation. This will-o'-the-wisp is rediscovered every decade or so, but never seems to get into the standard textbooks. It has amusing implications for the early controversy over Maxwell's equation. Then we point out the singular properties of the Maxwell stress tensor in arbitrary equilibrium field topologies.

We show that the familiar equations of hydrodynamics are required by the principles of conservation of particles, momentum, and energy in the large-scale bulk flow of the plasma. These are valid principles regardless of the presence or absence of interparticle collisions and magnetic fields. As already noted, HD is valid so long as there are enough particles to provide a statistically well-defined fluid density, contrary to what one sometimes reads in the literature about the relatively collisionless plasma. We show, too, that the familiar equations of magnetohydrodynamics are inescapable unless there are so few free electrons and ions that the gas is an effective electrical insulator. The air that we breathe is an example, and only upon reaching the ionosphere does MHD become effective.

In particular, the conversation emphasizes the principle—Occam's razor—that the theoretical concepts should contain no unnecessary embellishments. So we prune away concepts and notation that are not vital to the experimental physics, and we note in particular that physical reality is made up of the manner in which things are experimentally perceived to be. This seemingly trivial point is commonly violated by the vocabulary of magnetic induction, and it leads us into conflict with a variety of customs and popular opinions.

The reader will soon see that the conversation enters into numerous digressions, examining and commenting on the scenery as we pass along the minimum theory road. The writing of minimal theory is not obligated to provide only the minimum conversation.

1.2 Electromagnetic Field Equations

Our cosmos exhibits some remarkable electromagnetic symmetries and some remarkable electromagnetic asymmetries, and it is interesting to have a look at both. We begin by noting the well-known fact that, on the one hand, the electric and magnetic fields, **E** and **B**, respectively, are equal partners in their interactions, described by Maxwell's equations

$$\frac{\partial \mathbf{B}}{\partial t} = -c\nabla \times \mathbf{E} \qquad (1.1)$$

$$\frac{\partial \mathbf{E}}{\partial t} = +c\nabla \times \mathbf{B} \tag{1.2}$$

in a vacuum. These two equations, symmetric in **E** and **B**, state simply that any change in either field with the passage of time is accompanied by a proportionate curl of the other, and vice versa. That is the nature of the electromagnetic wave, so it should be no surprise that the proportionality constant, having dimensions of velocity, turns out to be the speed of light c.

Now the **E**, **B** symmetry of the field equations is in contrast with the fact that the universe is itself unsymmetric with respect to electric and magnetic charges. The matter throughout the universe is found to consist only of electrically charged particles, i.e., electrons and nucleons, with no indication of magnetic charges. Obviously, the cosmos was not created by gravity and electromagnetic forces alone.

The reader can see that the conversation employs cgs units, or their equivalent, rather than SI or mks units. The motive is to exhibit the basic dynamical symmetry between **E** and **B**, so thoroughly obscured by SI units in which **E** and **B** are assigned different dimensions! As discussed in section 6.4, the SI treatment, insisting upon the coulomb as the unit of charge, introduces superfluous concepts, contrary to the principle of minimum theoretical complexity.

Now the fact that most of the gases in the universe are at least partially ionized means an abundance of free electrons and ions. Hence, the electric current density **j** is created by a very weak electric field, quickly reducing any large-scale electric field $\mathbf{E}'$ in the frame of reference of the moving plasma to negligible values.

There can be no magnetic current **J** because there are no magnetic charges—magnetic *monopoles*—so far as anyone can tell. However, it is not without interest to look briefly into the physical consequences of an abundance of monopoles—a monopole plasma. Maxwell's equations would be written

$$4\pi\mathbf{J} + \frac{\partial \mathbf{B}}{\partial t} = -c\nabla \times \mathbf{E} \qquad \nabla\cdot\mathbf{B} = 4\pi\Delta \tag{1.3}$$

$$4\pi\mathbf{j} + \frac{\partial \mathbf{E}}{\partial t} = +c\nabla \times \mathbf{B} \qquad \nabla\cdot\mathbf{E} = 4\pi\delta \tag{1.4}$$

where Δ is the magnetic charge density and δ is the electric charge density. Both the electric current density **j** and the magnetic current density **J** have been introduced into the left-hand side of the vacuum equations. Presumably, the cosmos would have no total magnetic charge, just as we commonly suppose it has no total electric charge, although we will come back to this point later to consider the speculations of Lyttleton and Bondi (1959, 1960; Hoyle 1960).

Assuming that the individual magnetic monopoles are as mobile as the free electrons and ions, the magnetic monopole plasma would reduce the magnetic field in its own moving frame of reference to negligible values, just as the ion–electron plasma eliminates an electric field in its own frame of reference. The nonrelativistic Lorentz transformations between the electric and magnetic fields **E** and **B** in the laboratory and **E**′ and **B**′ in the reference frame moving with velocity **v** relative to the laboratory are

$$\mathbf{E}' = \mathbf{E} + \frac{\mathbf{v} \times \mathbf{B}}{c} \qquad \mathbf{B}' = \mathbf{B} - \frac{\mathbf{v} \times \mathbf{B}}{c}$$

Suppose, then, that the ion–electron plasma has a velocity **v** relative to the laboratory. The free electrons reduce **E**′ in that plasma to zero, from which it follows that

$$\mathbf{E} = \frac{-\mathbf{v} \times \mathbf{B}}{c}$$

in the laboratory. Similarly, suppose that the magnetic monopole plasma has a velocity **V** relative to the laboratory, and the free monopoles reduce **B**′ in that plasma to zero. Then

$$\mathbf{B} = \frac{+\mathbf{V} \times \mathbf{E}}{c}$$

in the laboratory. When **B** is eliminated between these two equations, it follows that

$$\begin{aligned} \mathbf{E} &= \frac{-\mathbf{v} \times (\mathbf{V} \times \mathbf{E})}{c^2} \\ &= -\frac{\mathbf{V}(\mathbf{v}\cdot\mathbf{E})/c^2}{1 - \mathbf{V}\cdot\mathbf{v}/c^2} \end{aligned}$$

Hence, **E** is parallel to **V**, and it follows that **B** = 0. Eliminating **E** between the two equations yields

$$\mathbf{B} = -\frac{\mathbf{v}(\mathbf{V}\cdot\mathbf{B})/c^2}{1 - \mathbf{V}\cdot\mathbf{v}/c^2}$$

So **B** is parallel to **v**, from which it follows that **E** = 0. So that universe would be hydrodynamic (HD) rather than magnetohydrodynamic

(MHD), and the Sun would not show anything comparable to its present MHD activity.

There would, of course, be some interesting things to do with magnetic monopoles. For instance, a toothpick with an electric charge attached to one end and a magnetic charge to the other forms a system with net electromagnetic angular momentum, given the electromagnetic momentum density $\mathbf{E} \times \mathbf{B}/4\pi c$. The toothpick would represent a gyroscope with no moving parts. However, that would be small compensation for the absence of relativistic jets, double radio sources, synchrotron emission, cosmic rays, etc., that are to be found in our own cosmos. There would be no sunspots, no prominences, no flares, no corona, no coronal mass ejections, no solar wind, no geomagnetic field, and no magnetic compass, to name but a few of the things missing from that universe.

In fact, given the central role of magnetic fields in determining the nature of the accretion disks involved in the formation of stars, one may ask to what extent there would be stars and planets? And the possibility of life? It is fashionable these days to conjecture on the existence of parallel universes. So we remark that there might be another universe "somewhere" with both mobile electric charges and mobile magnetic charges with no one in that cosmos to contemplate it.

Another question that springs to mind is whether there might exist a universe in which only magnetic charges exist, so that it would be a replica of our own cosmos except that the atoms would consist of light magnetically charged particles clustered around oppositely charged massive magnetic particles. Communicating by radio between the two universes, would it be possible to determine the difference?

Whether such a universe exists depends on the properties of the fundamental particles in other universes. Contemporary particle theory in our own universe suggests that, if they exist, magnetic monopoles have a mass μ at least as large as 10^{16} GeV, or about 10^{-8} g. There are living organisms with substantially less mass than that. With such a mass, the monopole, with a magnetic charge $g = 137e/2$, is not what we would call mobile. The acceleration in a large-scale magnetic field $\mathbf{B}$ is equal to $g\mathbf{B}/\mu$. In the large-scale magnetic field $B = 4 \times 10^{-6}$ G in a galaxy it would require 4×10^4 years to accelerate the monopole to 100 km/s. That opens up the possibility that a universe with such massive monopoles might be subject to magnetic monopole plasma oscillations, with a monopole plasma frequency $\Omega = (4\pi n g^2/\mu)^{1/2}$, where n is the monopole number density (Turner et al. 1982). This question comes up again in section 1.4, where we review the upper limits on n imposed in our universe by the existence of the magnetic field of the Galaxy.

On the other hand, if one imagines that the other universe forms the magnetic analog of our universe of electrically charged particles, then one would have magnetic monopoles with a magnetic charge e and the small mass m of the electron, and there would be oppositely charged monopoles with the mass M of a proton. Theory in that universe would suggest the possibility of electrically charged particles with masses of the order of 10^{16} GeV, etc.

Large-scale magnetic fields in that magnetic monopole universe would be quickly neutralized in the frame of reference of the swirling monopole plasma, in direct analogy to the obliteration of electric fields in the frame of reference of the plasma in our own universe. The monopole universe would be filled with large-scale electric fields tied to the hydrodynamics of the swirling monopole plasma, in the manner described by theoretical "electrohydrodynamics" (EHD)—the exact analog of MHD.

Returning to the realities of our own cosmos, it appears that we may neglect the occasional magnetic monopole, if there are any at all, and Maxwell's equations are written as

$$\frac{\partial \mathbf{B}}{\partial t} = -c\nabla \times \mathbf{E} \qquad \nabla \cdot \mathbf{B} = 0 \tag{1.5}$$

$$4\pi \mathbf{j} + \frac{\partial \mathbf{E}}{\partial t} = +c\nabla \times \mathbf{B} \qquad \nabla \cdot \mathbf{E} = 4\pi\delta \tag{1.6}$$

Note, then, that these electromagnetic equations make contact with Newtonian mechanics through the electric charge density δ and the mechanical motion of the charges associated with the current density $\mathbf{j}$, on which more will be said in chapter 6.

1.3 Electrical Neutrality

It is important to appreciate the large charge to mass ratio of the electron ($e/m = 5.3 \times 10^{17}$ cgs, compared to $g/M = 2$cgs for the theoretical magnetic monopole). Thus, for instance, one volt of potential difference accelerates an electron to 600 km/s. Note then that one mole of free electrons (6×10^{23} electrons) has a mass of about half a milligram and a charge of 3×10^{14} cgs (10^5 C), enough to supply a current of 1 A for a day. An electric charge of this amount would raise the Sun (radius 7×10^{10} cm) to about 1.3×10^6 V. If the half-milligram of electrons were then released, they would accelerate away from the Sun to relativistic velocities (the rest mass of an electron is 0.5×10^6 eV). Noting again that a potential difference of 1 V is sufficient to accelerate an electron to 600 km/s, it is evident that even a very weak electric field applied to a plasma would produce an immense current. That is to say, the electric field is limited to very small values by the highly charged and freely moving electrons.

As an illustrative example, consider the simple case of a plasma sufficiently dense that the scalar Ohm's law applies, so that

$$\mathbf{j} = \sigma \mathbf{E}' \tag{1.7}$$

where $\mathbf{E}'$ is the electric field in the frame of reference moving with the local plasma. For ionized hydrogen the electrical conductivity is $\sigma = 2 \times 10^7 T^{3/2}$/s (cf. Spitzer 1956). Starting with Ampere's law $4\pi\mathbf{j} = c\nabla \times \mathbf{B}$, it follows that $4\pi j \approx cB/l$, in order of magnitude, where l is the characteristic scale of variation of the magnetic field $\mathbf{B}$. Then with $\mathbf{E}' = \mathbf{j}/\sigma$, it follows that the field magnitudes are related by

$$\frac{E'}{B} = \frac{c}{4\pi\sigma l} \tag{1.8}$$

$$= \frac{10^{-4}}{l}\left(\frac{10^4}{T}\right)^{3/2} \tag{1.9}$$

in order of magnitude, with l in centimeters. Ionized hydrogen suggests $T \geq 10^4$ K. So with l as small as 1 km it follows that $E'/B \leq 10^{-9}$. On the large scales associated with stars and galaxies, E'/B is very small indeed. It is evident, then, that electric field stresses are insignificant compared to magnetic field stresses, the ratio of the stresses being $(E'/B)^2$. We will have more to say on this later. Only on the very small scales arising in shock fronts and in spontaneous tangential discontinuities and rapid magnetic reconnection can the electric field play a significant role in the dynamics.

Consider, then, the curious fact that one sign of charge—the one we call negative—is associated with the lightweight electron or lepton, while the opposite charge—called positive—is associated with the proton or baryon. Why such different particles? Perhaps the answer lies in the experimental fact that the positive and negative manifestations of charge assigned to the same type of particle represent a particle and its antiparticle. Such particles annihilate when they meet and convert to photons—gamma rays. So there have to be two different classes of stable particle if there is to be long-lived matter. Positronium just does not do it by itself. Our universe possesses the stable leptons and baryons, and here we are to take note of it.

That said, note that *positive* charge is defined as the charge left on a glass rod after rubbing the rod with silk, the silk being charged negatively, by definition, with the electrons rubbed off the rod. With the same rule applied in an anti-universe, where the atoms consist of positrons orbiting antiprotons, the anti-glass rod could just as well be given the positive charge appellation by the anti-people that live there. The question arises as to how a physicist and an anti-physicist in radio communication would determine whether their universes were both made up of matter, or whether one universe was the anti-universe of the other. Needless to say, if they could meet in person somehow, a handshake would quickly settle the matter. Such a cordial meeting would be the ultimate game of Russian roulette.

The issue could be settled via the radio if the two could (a) establish a common direction perpendicular to the direction of propagation of the radio signals employed for their communication and then (b) agree on a positive and negative sense along that direction. Whether this can be achieved depends on the nature of the radio access between the two universes, on which we can say no more. If they cannot achieve (b), then they cannot know the answer to their question. However, if the anti-physicist resides on an anti-matter planet in our own familiar cosmos—Alfvèn (1966) proposed that half the planets and stars in the Galaxy are matter and half are anti-matter—then the communication can decide the issue, at least in principle. The direction (b) could be decided by observing a common object, e.g., a distinctive distant galaxy, in a direction perpendicular to the line of propagation of their radio signal, and assigning a positive direction along that line, referred to as the positive (b) direction in the sequel.

Then, to proceed with the measurements, suppose that at time $t = 0$ our physicist displaces a charge $+q$ with a modest velocity $\mathbf{v}(t)$ ($<< c$). The vector potential of the radiation at the distant location $\mathbf{r}$ would be

$$\mathbf{A}(r, t) = \frac{q\mathbf{v}(t - r/c)}{cr}$$

The associated electric field is

$$\begin{aligned} \mathbf{E} &= \frac{q}{c^2 r}\left[\mathbf{a}\left(t - \frac{r}{c}\right) \times \mathbf{n}\right] \times \mathbf{n} \\ &= -\frac{q}{c^2 r}\mathbf{a}_{\perp}\left(t - \frac{r}{c}\right) \end{aligned}$$

where $\mathbf{a}(t)$ is the acceleration $d\mathbf{v}/dt$, $\mathbf{n}$ is the unit vector in the radial direction ($\mathbf{r} = \mathbf{n}r$), and $\mathbf{a}_{\perp}$ designates the component of $\mathbf{a}$ perpendicular to $\mathbf{r}$. The acceleration $\mathbf{a}$ is described by radio to the distant physicist in terms of the (b) direction, who measures the force $e\mathbf{E}$ on one of his own protons (or anti-protons?) in the arriving pulse of radiation. As a simple example, let **a**(*t*) be zero for $t < 0$, equal to a fixed value a in the positive direction (b) for a short period Δt, and then an equal acceleration in the negative (b) direction for another period Δt, remaining zero thereafter, $t > 2\Delta t$, with the charge at rest. If the distant world of the other physicist is made of matter, he will determine that the initial incoming **E** is in the negative (b) direction. But if his world is an anti-world, his sign convention would have the first force on his anti-proton in the positive (b) direction, and handshakes are out of the question if ever the two physicists should meet. Needless to say, with the large distance **r** between the two physicists, the operational problem would be to achieve enough signal-to-noise to carry out the measurement and sufficiently long-lived physicists to communicate over interstellar distances. It is 8 years round-trip just to and from Alpha Centauri.

In fact, it is apparent from the observed very low level of the expected gamma rays from electron–positron and proton–antiproton annihilation that the universe is filled with matter, to the exclusion of antimatter. Anti-matter is created only as individual anti-particles by collisions of cosmic ray protons, etc., with the nuclei of interstellar matter, and by the creation of electron–positron pairs by gamma rays. So anti-particles are rare, and the behavior of individual anti-particles is very interesting on its own, of course.

With these remarks on positive versus negative charges, consider the point already mentioned that the net electrical charge of the universe is identically zero. One presumes that the equal numbers of electrons and protons were established through creation and annihilation of particles in the initial Big Bang. We may ask, then, what are the experimental and observational upper limits on the net electric charge of the cosmos? Some years ago Lyttleton and Bondi (1959, 1960; Hoyle 1960) considered the consequences of a net electric charge, with the suggestion that a fractional excess charge in the amount of one part in 2×10^{18} would account for the expansion of the universe on the basis of electrostatic repulsion and Newtonian mechanics. It would appear, then, that one part in 2×10^{18} is an upper limit on the positive–negative electric charge asymmetry. For, if the asymmetry were greater, the expansion of the universe would exceed the Hubble constant $R^{-1}dR/dt \approx 80\,\text{km/s}$ per megaparsec inferred from observation. Indeed, with the long-term acceleration of the expansion, presently inferred from observations of distant type Ia supernovae, one might ask if the continuing acceleration could be a consequence of a net electric charge?

Lyttleton and Bondi noted that there may be equal numbers of electrons and protons but the magnitude of the charges may differ slightly, by perhaps one part in 2×10^{18}. Alternatively, the charges may be equal but there is a different total number of electrons and protons. If the former, then the neutron would have a total charge of $\pm e/(2 \times 10^{18})$. If the latter, there would be about 3×10^5 more electrons than protons, or more protons than electrons, in a mole of matter. The mobility of excess electrons might provide electrical conductivity in a cold gas where none exists otherwise. Excess protons represent positive ions of some sort, with possible doping effects.

The Newtonian approach to this electrostatic cosmology considers a sphere of arbitrary radius R at an arbitrary location in an infinite space. The universe outside this sphere is divided into concentric spherical shells extending to unlimited radii. The individual spherical shell contributes neither gravitational nor electrostatic fields to its interior, from which it follows that the cosmos outside the radius R has no effect on the dynamics of R itself. The reader is referred to appendix A for a brief discussion of the mechanics of the sphere with Lagrangian radius $R(t)$. In particular, it follows that the acceleration from electrostatic propulsion is confined to the very early universe, when $R(t)$ was small. So, electrostatic repulsion cannot be responsible for the long-term increase in the Hubble constant.

The Lyttleton-Bondi conjecture challenged experimentalists to look for a slight difference between the magnitudes of the electron and proton charges. For instance, one may contemplate what is implied by the decay of a free neutron into an electron and a proton if the difference in charge is nonzero. One part in 2×10^{18} is a very small difference (see discussion in appendix A), and it was not until Dylla and King (1973) that the experimental upper limit was pushed down to one part in 10^{19}, showing that any difference is negligible so far as cosmology is concerned. Of course, this does not rule out precisely equal charges with a difference in the total number of electrons and protons.

For the present, then, we adopt the simple view that the universe as a whole is electrically neutral, and turn our attention to local neutrality. We are interested in scales of, say, a kiloparsec or less, over which any cosmological net charge has no significant effect. Local electrical neutrality is enforced by the electric field $\mathbf{E}$ ($\nabla \cdot \mathbf{E} = 4\pi\delta$) associated with a net charge density δ. The restoration of charge neutrality ($\delta = 0$) takes place in a characteristic time of the order of the Landau damping time of any plasma oscillations (at the plasma frequency $\omega_p = (4\pi Ne^2/m)^{1/2}$) and of the order of the characteristic resistive damping time $1/4\pi\sigma$, where σ is the electrical conductivity (see appendix B). Thus, there are no surviving electrostatic fields on a macroscopic scale even if somehow a local net charge density were momentarily created. There are, of course, the fluctuating electric fields on the microscopic scale of the Debye radius $(kT/4\pi Ne^2)^{1/2}$, with no implications for the large-scale electric and magnetic fields.

The next question is what happens in an electrically neutral collisionless plasma when a large-scale electric field $\mathbf{E}$ perpendicular to $\mathbf{B}$ is applied by external sources, as one might do in the laboratory? The freely moving electrons and ions are prevented by **B** from streaming in the direction of **E**, which is to say that the conventional concept of large electrical conductivity σ is not applicable. The well-known fact is that the electrons and ions are accelerated by **E** and end up moving around **B** in circles with the appropriate cyclotron frequency, while the circle moves with the steady electric drift velocity $c\mathbf{E} \times \mathbf{B}/B^2$ in the direction perpendicular to both **E** and **B**. In that drifting reference frame there is no electric field (see appendix C) and no further acceleration of the electrons and ions.

So one way or another, there is no significant persistent large-scale electric field in a plasma (collisionless or collision dominated). One might say that a plasma abhors electric fields and invariably finds a means to avoid them. Only by reducing the degree of ionization of a gas to negligible values, e.g., the lower terrestrial atmosphere where we reside, is there a possibility for interesting large-scale electric field effects.

1.4 Electric Charge and Magnetic Field Dominance

In contrast with the electric field, magnetic fields are not erased, because there are no magnetic charges and currents to neutralize them. We have already commented on the absence of magnetic monopoles, and we turn now to the magnetic field of the Galaxy, whose existence places a very low upper limit on the abundance of monopoles (Parker 1970; Turner et al. 1982). If there were n free monopoles per unit volume, each of mass μ, magnetic charge g, and mean conduction velocity $\mathbf{u}$, then the magnetic current density $\mathbf{J}$ would be $gn\mathbf{u}$. The magnetic field of the Galaxy, lying along the spiral arms, has a typical strength $B \approx 4 \times 10^{-6}$ G over a scale Λ of at least one kiloparsec (3×10^{21} cm), indicating magnetic potential differences $\Lambda B \geq 10^{16}$ G cm. The kinetic energy imparted to a monopole by this potential difference implies a monopole velocity of the order of 10^8 cm/s, so we anticipate that the magnetic conduction velocity $\mathbf{u}$ may be a substantial part of the motion of the individual magnetic monopole.

The rate at which the magnetic field $\mathbf{B}$ does work on $\mathbf{J}$ is $\mathbf{B} \cdot \mathbf{J} = gn\mathbf{B} \cdot \mathbf{u}$ per unit volume, providing a decline in the magnetic energy density $B^2/8\pi$ at the rate

$$\frac{d}{dt}\frac{B^2}{8\pi} = -gn\mathbf{B} \cdot \mathbf{u} \tag{1.10}$$

The characteristic magnetic dissipation time τ is then

$$\frac{1}{\tau} = -\frac{1}{B}\frac{dB}{dt} = \frac{4\pi ngu}{B} \tag{1.11}$$

That is to say, the conduction flux density nu is related to τ by

$$nu = \frac{B}{4\pi g\tau} \tag{1.12}$$

The continued existence of the galactic magnetic field implies that the dissipation time of the field exceeds the time over which the magnetic field is generated. The magnetic field of the Galaxy exists today, after some 10^{10} years, and it could be argued, therefore, that $\tau > 10^{10}$ years. Put $g = 137e/2$ cgs $= 3.3 \times 10^{-8}$ cgs, with $\tau \geq 3 \times 10^{17}$ s and $B = 4 \times 10^{-6}$ G. The result is $nu < 3 \times 10^{-17}$ monopoles/cm^2 s, i.e., not more than three monopoles intersect a football field (3×10^7 cm^2) in a century, and, of course, perhaps none at all.

If we suppose, on the other hand, that the magnetic field of the Galaxy is regenerated by a galactic dynamo in a time comparable to the rotation period of the Galaxy (2.5×10^8 years or 8×10^{15} s) (Parker 1971a–c,1979), then nu might be 40 times larger, i.e., $nu < 1 \times 10^{-15}$ monopoles/cm^2 s, and the football field might be hit once in a year. (A more detailed analysis can be found in Turner et al. 1982.) It appears that experimental search for monopoles would be a daunting undertaking at best.

Now one might try to avoid the upper limit on monopoles by noting (Turner et al. 1982) that a universe strewn with equal numbers of positive and negative monopoles could experience magnetic monopole Langmuir-type plasma oscillations, with the observed galactic and intergalactic magnetic fields representing the magnetic fields associated with the opposite displacements of positive and negative monopoles. The magnetic monopole plasma frequency would be $\Omega = (4\pi ng^2/\mu)^{1/2}$. The number density n can be estimated from the upper limit on nu, providing an upper limit on Ω and a lower limit on the period of oscillation. If we suppose that $u = 10^{-3}$ $c = 300$ km/s, then for $nu < 10^{-15}$ monopoles/cm^2 s, it follows that $n < 3 \times 10^{-23}$/cm^3. The period $2\pi/\Omega$ is then 4×10^7 years. On the other hand, if we suppose that the monopoles have been accelerated by the galactic magnetic field B over a distance $\Lambda = 1$ kpc, the result is $u = 2.3 \times 10^8$ cm/s, so that $n < 4 \times 10^{-24}$/cm^3 and the period of oscillation is 1.1×10^8 years. The monopole oscillations are slow, and one could imagine how the associated magnetic fields would be seen as permanent or static by the transient human observer. However, further investigation shows that the magnetic fields would behave in a curious way, rather contrary to what we seem to see in the cosmos. We imagine the plasma of ionized gas moving with velocity $\mathbf{v}$ relative to the massive background monopole plasma. It is readily shown (Parker 1984, 1987) that the magnetic fields are transported bodily with the velocity **v**/2, i.e., with a velocity halfway between the ionized gas and the monopole plasma. The magnetic field of a galaxy moving relative to the monopole plasma would slip out the trailing side of the galaxy. This seems to be contrary to the observed bulk transport of galactic fields with the motion of the ionized gases contained in the galaxy, and we conclude that the existing large-scale magnetic fields have no association with magnetic monopoles.

The essential point is simply that magnetic fields are not strongly dissipated, whereas large-scale electric fields in the frame of reference of the ionized gases are quickly neutralized. Observations show that nature has not missed this opportunity for proliferation of magnetic fields. The polarization of starlight reddened by passage through interstellar dust, the Faraday rotation effect in radio waves from distant sources, and the synchrotron radiation from energetic electrons show that the plasma filling the universe is everywhere encumbered with magnetic fields. Even solid planets sport magnetic fields, as a consequence of the high electrical conductivity of their rotating convecting liquid interiors. The essential feature for the production and existence of magnetic field is the high electrical conductivity, i.e., the inability to support an electric field in the moving frame of reference of plasma or liquid planetary interior.

This is all so foreign to the situation in the lower terrestrial atmosphere where we reside, the air being an excellent electrical insulator. Here we see none of the magnetic effects, the atmospheric winds blowing freely through the geomagnetic field. Instead, we see such powerful electrostatic phenomena as lightning, driven by potential differences of millions of volts. The tropical thunderstorms charge Earth to some $3–4 \times 10^5$ V negative with respect to the ionosphere and the space beyond. So there is a downward directed electric field of the order of 1 V/cm here in the lower atmosphere, diminishing upward to the ionosphere at about 100 km altitude. The high density and low temperature of the atmosphere create this unique situation. Indeed, it would appear that the formation of life is possible only in such a situation of low temperature and, hence, negligible electrical conductivity. So, living things can discover the general magnetic character of the cosmos only by remote observation. Only in the physics laboratory can the magnetic plasma conditions be duplicated to some degree.

The fact is that we can understand the remarkable hydrodynamic activity of the astronomical universe only if we have a proper understanding of the dynamical effects of magnetic fields. So, with this in mind, consider the experimental basis for the dynamical theory of electric and magnetic fields.