CHAPTER ONE

The Collapse of the "Classical" World View

> I remember discussions with Bohr which went through many hours till very late at night and ended almost in despair; and when at the end of the discussion I went alone for a walk in the neighboring park I repeated to myself again and again the question: Can nature possibly be as absurd as it seemed to us in these atomic experiments?
>
> —*Werner Heisenberg*

WE ARE GOING TO follow the fascinating trail that led to the scientific revolution of quantum mechanics in the first quarter of the twentieth century. Together with the theory of relativity, the conceptual structure of quantum mechanics is now the basis of the modern view of the physical world. Rarely in history has a new theory been so hotly contested, or called forth such great energies from such great minds. And even though its power to predict phenomena is unprecedented in the history of science, quantum mechanics has stirred up controversy—no less unprecedented—over its meaning. This will not seem surprising, once we penetrate the "secrets" of the new microcosm revealed by the theory. These secrets—the incredible feats we will discover microscopic systems can perform—are quite revolutionary when set beside the "classical" conceptions formerly elaborated to explain our macroscopic experience. It is only to be expected that the new theory would require so much effort and suffering to be worked out, or that such furious debate would still be raging today about its philosophical implications.

What the physicist Isidor Isaac Rabi said (quoted at the beginning of the preface) lamenting public ignorance of modern advances in physics, applies with particular force to quantum mechanics. In the words of the great Robert Oppenheimer, "As history, its re-creation would call for an art as high as the story of Oedipus or the story of Cromwell, yet in a realm of action so remote from our common experience that it is unlikely to be known to any poet or historian."

The prologue of such a drama would consist in the fundamental incapacity of "classical" conceptual schemes to explain certain fundamental physical phenomena. A complete list would be too lengthy, and an exhaustive analysis would require a discussion of sophisticated effects that would be inappropriate for the nontechnical spirit of the present work. Instead, I will limit myself to listing a few elementary processes that could not be explained within the conceptual system of the "classical" view of the physical world.

But before I begin, I should explain that such expressions as "the classical conception of the world" (or the equivalent) will be used in this work to designate the body of knowledge elaborated over the long course of development of scientific thought, from Galileo's revolution in the early 1600s to about 1800. This knowledge was synthesized into the two pillars of nineteenth-century physics, namely, mechanics and electromagnetism.

Classical mechanics, born in the early seventeenth century from the profound intuitions of Galileo, found its concrete realization in the inspired labors of Isaac Newton. Increasingly refined and generalized formulations were attained in the eighteenth century in the works of Joseph Louis de Lagrange, and in the nineteenth by William Rowan Hamilton. This superb theory, as is well known, treats the movement of material bodies as determined by forces acting upon them; such forces as the mutual attraction (or repulsion) between individual particles govern the motion of bodies down to the tiniest detail. Classical mechanics managed the unification of what appeared to be the most diverse phenomena: for example, the doctrine showed that the movement of the planets in the heavens was governed by exactly the same laws that regulate the movement of any physical object as it falls to the ground. And the theory would reach still farther, for, whereas scientists had formerly thought that the physical process involved in temperature exchange could not be explained in terms of mechanics, by the nineteenth century it was possible to show that even thermal processes had their origin in the disordered movements of material constituents. Classical mechanics saw one of its greatest triumphs in the nineteenth century, when Willard Gibbs, Ludwig Boltzmann, and James Clerk Maxwell realized a more profound unification of physical phenomena through the mechanical explanation of thermodynamic processes.

A parallel story can be told of the other great "classical" theory, electromagnetism. For a long time the phenomena of light seemed to have nothing to do with electrical or magnetic phenomena. The researches of Faraday, Maxwell, and others led to the recognition that these so disparate processes were nothing other than diverse manifestations of a single entity, known as the electromagnetic field: equations were worked out that governed precisely all the phenomena in question. In this way, the concept of *field*—that is, as we shall soon see, of a physical entity continuously distributed in space and time—made its dramatic entry into science, requiring the recognition of an existence just as fundamental as one of the material particles. The electromagnetic field is capable of transporting energy through empty space in the form of light waves, radio waves, x rays, and so forth. This "wavelike" nature then became the fundamental characteristic of all processes governed by the laws of electromagnetism, as formulated by Maxwell.

This was the framework achieved by the end of the nineteenth century: a real existence had to be attributed both to discrete material particles ("corpuscles," from Latin *corpusculum*, "little body"), and to continuous fields. These physical entities were understood to evolve in a precise way in space, under the influence of their mutual interactions, and as codified in the equations of mechanics and electromagnetism. The equations, in turn, would permit the understanding of all other processes of the physical world.

Imagine, then, the crisis that occurred when some simple physical processes were discovered to be absolutely incomprehensible—to resist all attempts to reduce them to a classical understanding!

1.1. The Dependence of the Color of Objects on Temperature

It is common knowledge that a physical object, such as an iron bar, changes its color as its temperature changes. At low temperatures, the iron appears "natural" in color to us, but when its temperature is raised, it begins to give off heat (remember that thermal radiation is a form of electromagnetic radiation). Then the iron looks at first red, then yellow, and finally, white hot. This process involves thermodynamic effects, which bring an increase in thermal agitation of the constituents of the matter. These constituents, of course, are electrically charged particles, and the laws of electromagnetism teach us that charges in nonuniform motion release electromagnetic radiation. If the conditions are suitable, this radiation will appear as light. No matter how complex the process, in accordance with the previous remarks, it should eventually reenter the typical framework of "classical physics" and become perfectly intelligible. Unfortunately, such hope proved groundless. Despite the persistent efforts of the scientific community, even a process so common as this one did not admit of explanation in terms of classical laws of mechanics and electromagnetism. And it remained a mystery until Max Planck advanced an hypothesis of an absolutely revolutionary nature that would upset all classical ideas about light—or, to be more precise, about electromagnetic radiation.

1.2. Atoms and Their Properties

At the end of the nineteenth century, then, and at the beginning of the twentieth, a series of researches was undertaken (the most important being those conducted by Sir Ernest Rutherford) which would lead to a model of the atom very similar to the one used today.[1] An atom was conceived as a miniature planetary system, having a positively charged nucleus where almost the entire mass of the atom was concentrated. Around the nucleus, revolving like planets around a tiny sun, were electrons, negative in charge, and of such a number as to neutralize exactly the positive charge of the nucleus. The attraction between the opposing charges (according to Coulomb's law) functioned like solar-planetary gravitation,[2] and thus every electron was attracted by its nucleus (the law of attraction between two opposite charges has the same mathematical form as the attraction between two masses, that is, it decreases by the square of the distance that separates the two charges). But, once again, we will be surprised to learn that this analogy (which seems so natural to us) between atomic structure and a planetary system is, for various reasons, absolutely untenable within the classical scheme. Let us analyze a few of these reasons.

1. The constancy of atomic characteristics. It is only natural to wonder how all the atoms of one element—oxygen, for example—exhibit absolutely the same physical properties, however these atoms may have been produced; or again, we may wonder how such properties can persist unchanged, while the systems in question are subjected to highly invasive procedures, such as fusion or evaporation, and then returned to their initial state. Something analogous would be impossible for a classical system, *qua* planetary system. In fact, the orbits of the planets, especially in a system with many bodies, depend in an absolutely critical way on the initial conditions. Different procedures of "preparation" of a planetary

system would inevitably lead to systems that are appreciably different. Interactions even of the most minute entity with other systems determine important changes in evolution and change the structure of any such system. Consequently, given a fixed nucleus of an atom and a fixed number of electrons to orbit it, the effective physical and chemical properties of the resultant system should show an enormous variety. There would have to be many atoms with a nucleus equal to that of oxygen (a mass sixteen times that of a proton with a positive charge equal to the eight electrons orbiting around it), and the variety of atoms would correspond to the different ways in which the nucleus has, so to speak, "captured" its electrons at the moment of its formation. But the opposite is true: all the physical and chemical phenomenology shows that the properties of an atomic element are absolutely identical, independently of preparatory conditions and any subsequent transformations.

2. These precise, persistent properties that characterize an atom also determine its behavior in physical and chemical processes. Nevertheless, as is well known, even in the case of an atom with many electrons, the characteristic properties change radically as we go from one atom to another with only one electron more or less. For example, we can consider an atom of xenon, a noble gas, which has a nucleus that contains (along with its neutrons) 54 protons, around which revolve an equal number of electrons. This atom is chemically inert, that is to say, it will be extremely difficult for it to form chemical compounds with other atoms. We only need to take away one electron and one proton (the relevant constituents for holding the system together) and a few neutrons to change it into an atom of the nonmetal solid iodine, a system with a very precise and conspicuous electronic affinity, and thus with radically different behavior and properties. This would certainly *not* happen in a planetary system with 54 planets: the passage from one system to another with one fewer planet (and with its sun correspondingly a little lighter in weight) would not, it would seem, bring with it such radical changes in the behavior of the system.

3. Finally, another fundamental fact generates an incurable conflict between the stability of atoms and the "planetary" model within the conceptual structure of classical physics. According to the equations of classical dynamics, a system of electric charges can remain in equilibrium only if the charges are in motion. The fact that the atom has a limited extension requires that the charges present in it should move in circular or elliptical orbits (similar to the planets), and thus have acceleration. Now, according to securely established laws of electromagnetism (Maxwell's

equations), an accelerated charge inevitably emits electromagnetic waves, or radiation. By radiating, the electron would lose energy and its orbit would decrease, causing it to fall into the nucleus within a very short period of time.[3] Exact calculation brings the conclusion that every atom ought to have a very ephemeral life, showing "constant" properties for extremely brief periods (fractions of a second). Of course, this would contradict all the familiar phenomena.

Many other facts too—in particular the specific modes of interaction between atoms and electromagnetic radiation—bring us into an irreconcilable conflict with classical conceptions. This is how a crisis began to prepare the way—as has often happened in the history of science—for a true scientific revolution. Thanks to the combined strength of a remarkable group of geniuses, that revolution arrived in the form of quantum theory, the theme of this book.

1.3. Wave Phenomena

Once having derived the laws of electromagnetism, Maxwell came to the conclusion that an accelerated charge radiates electromagnetic energy. This energy propagates itself in space in waves characterized by a double field, electrical and magnetic. This was verified by Hertz in 1888, and in 1901 Marconi succeeded in transmitting electromagnetic waves across the Atlantic Ocean. This phenomenon—the transmission of radio waves—is probably the most well-known example of the propagation of electromagnetic waves. In this case, the variable current (that is, the accelerated movement of the electrical charges contained in it) which runs through a radio antenna produces the emission of waves (Figure 1.1), which are propagated in space. Afterward, the information that these waves transport can be, as it were, decoded by the receiving apparatus.

We now need to examine this phenomenon more profoundly, in order to grasp the essential points we need to know, for understanding what follows. In fact, this part of the book, including the rest of this chapter and the first sections of the next, is dedicated to the discussion of the phenomenon of *light polarization,* and will appear rather technical, even though the exposition of the important points will stay at an elementary level. I need to ask the reader for a little extra effort to master the few, simple concepts I am going to explain, since their correct understanding will pave the way for the rest.

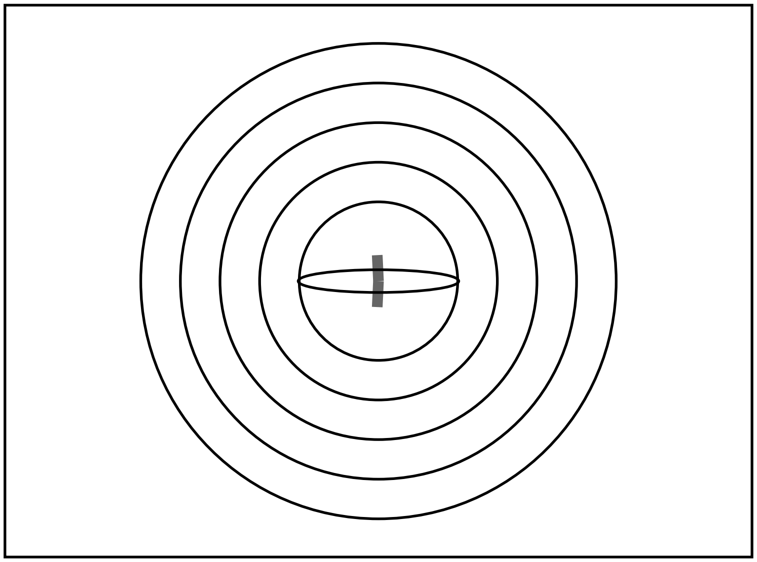

FIGURE 1.1. An electric charge that oscillates periodically along the vertical bar (antenna), being subject to an acceleration, radiates an electromagnetic wave in space that can be received by an appropriate receiver.

Suppose we have an electric charge, moving with a periodic motion along a certain segment, represented by the dark vertical line in Figure 1.1. The charge will radiate electromagnetic waves of the same frequency in the surrounding space. One electromagnetic wave consists of two field vectors, mutually perpendicular, and perpendicular to the direction of the propagation of the wave. These two vectors represent the electric field *E* and the magnetic field *H*, respectively (see Figure 1.2). The velocity of propagation of the wave in a vacuum is the same as the velocity of light, which means that it is traveling at about 300,000 kilometers per second.

Let us study the electric field first. We can observe it from two perspectives: we can study how its magnitude and direction vary at a given point in space with the passage of time, or, alternatively, we can study what values it assumes at various points of space at one given instant of time. In the first case we have a sinusoid or "foldlike" shape like that in Figure 1.3, which tells us, for example, that at the considered point, and at time t_1 the field points upward, and has the value E_1. The time T that it takes the field to return to exactly the same value at the same point in space, is the period of oscillation, and coincides with the oscillation of the charge that generates it. In the second case (Figure 1.4) we have an analogous

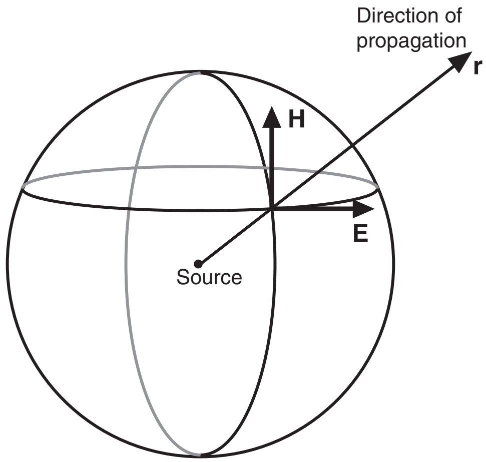

FIGURE 1.2. The electric and magnetic fields associated with the propagation of a spherical wave from a point source. The vectors are perpendicular to each other and perpendicular to the direction *r* of the propagation of the wave at the point under consideration.

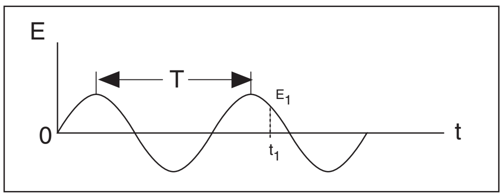

FIGURE 1.3. Variations of the electric field E in a fixed point of space, depending on change of time. The time interval that occurs, for example, between two maximal peaks represents the period of the radiation.

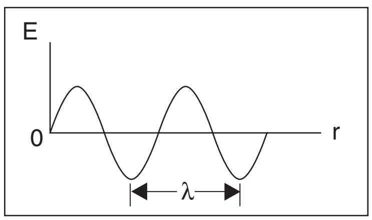

FIGURE 1.4. The variation of the electric field E at a fixed moment of time, depending on its spatial position. The spatial interval which intervenes, for example, between two troughs or betwen two crests represents the wavelength λ of the radiation.

representation, but giving us an image of the field at different points of space, along the direction of wave propagation, at one given instant. This also is sinusoidal in shape. We can define the wavelength λ of the electromagnetic field as the spatial distance between the two successive troughs, or crests, of the wave itself.

In place of the period T, it is also convenient to introduce its inverse, i.e., the frequency ν of the wave:

$$\nu = 1/T \tag{1.1}$$

which represents the number of oscillations the field (or the charge which generates the field) performs in one second.

It is true in general of wave phenomena (whether electromagnetic, liquid, or sound waves) that the distance a wave is propagated in one second (its velocity) is equal to the distance λ covered by a single oscillation, times the number of oscillations per second (the frequency ν). If the velocity of propagation is equal to c, then

$$\lambda\nu = c, \tag{1.2}$$

a formula that represents the relation between wavelength, frequency, and velocity of light.

In principle, electromagnetic waves can have any frequency between zero and infinity. The classification of radiations according to their frequency is known as the electromagnetic spectrum, as shown in Figure 1.5 below.

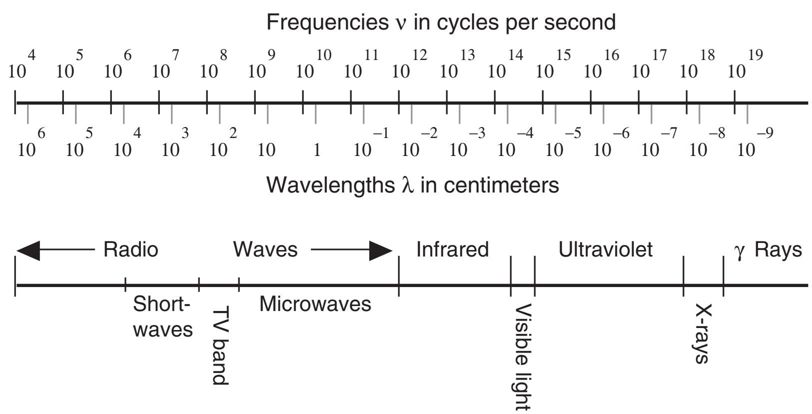

FIGURE 1.5. The electromagnetic spectrum. In the upper part of the graph are given the frequencies; below them, the corresponding wavelengths. Underneath both are shown the types of radiation that correspond to the various frequencies and wavelengths.

This is a good time to explain that the example considered in Figures 1.3 and 1.4 represents a very simple case, especially because a field with a definite frequency is assumed (so that, for example, if light waves were being considered, they would be the waves of a single color of light). Furthermore, the oscillation is taking place in a well-defined plane (that is, as we shall explain more fully in the next chapter, it exhibits plane polarization). In the most typical instance, however, the field can change in an arbitrary fashion: while still remaining perpendicular to the direction of the waves propagation, it can assume a variety of orientations and amplitudes from time to time, and from one point to the next. Without going into great detail, it is crucial that we understand at least one thing about wave processes, and that is the *linear* nature of the equations that govern them.

Suppose we have a field (let us say, an electrical field) $\mathbf{E}_1(\mathbf{r},t)$, which represents a solution of the equations of wave propagation, under opportune conditions characterizing the production mode of the wave in question, for example, as emitted from a specific antenna (situated in New York City); then say we have a second field $\mathbf{E}_2(\mathbf{r},t)$, which also represents a solution of the equations, relative to another production mode (as emitted from an antenna situated in Philadelphia). Now, the field which at every instant t of time and in every point *r* of space is the vector sum of the two fields (according to the familiar parallelogram rule: see Figure 1.6),

$$\mathbf{E}(\mathbf{r},t) = \mathbf{E}_1\ (\mathbf{r},t) + \mathbf{E}_2\ (\mathbf{r},t), \tag{1.3}$$

is the solution of the equations that define the real state of the electric field, when both sources are operative.

Before concluding this analysis we should recall that what interests us most in the physics of electromagnetic wave propagation is the fact that the wave carries energy. This energy is behind the various processes leading to the very interesting effects of, say, chlorophyllic functions, photographic phenomena, or the stimulation of our sense of sight. Now, in the case of an electromagnetic wave, the density of energy at a given point in space and in a given instant of time is proportional to the sum of the squares of the electric and magnetic fields at that point and in that instant. However, in vacuum, the electric and magnetic fields (with the appropriate units of measurement) are of equal intensity. This means that the electromagnetic energy in a certain volume of space at a certain time is proportional to the product of the square of the electric field and the volume—supposing, for the sake of simplicity, that the volume is small enough to eliminate any appreciable variations of the field within it

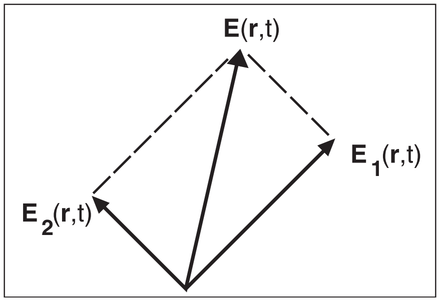

FIGURE 1.6. The sum of the values of the two electrical fields at a certain instant t of time and at the same point *r* of space. In the example, if the fields indicated would both be solutions of the electromagnetic wave-propagation equation, with their characteristic dependence on *r* and t, then their sum at each point of space and at each instant of time would also represent one solution. This fundamental fact derives from the linear nature of the evolution equations.

(technically, in the case of fields that vary rapidly, the quantity that really matters is the integral of the square of the field, extended to the volume). Recalling Figure 1.4, then, where we represented the field at various points in space of a light ray, we now can graph (in Figure 1.7) the density of energy, simply taking its square (E^2). It follows that the area taken up under the curve in a certain interval Δr represents the energy concentrated at that interval. The figure shows how in those regions of space where the field is more intense, a significant concentration of energy will be found, while the weaker fields correspond to low energy densities. However, one of the cardinal points of Maxwell's theory of light, as already mentioned, is that the field, and consequently the energy trans-

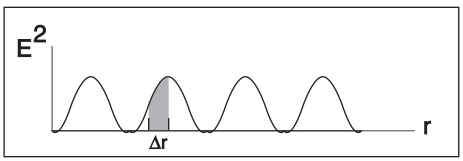

FIGURE 1.7. The square of the field at various points of space represents the volume density of energy transported by the field itself. From this it follows that the shadowed area of the curve represents the total energy of the field in the interval Δr. It should be noted that according to the graph, by varying from point to point, the energy is continuously distributed through the space.

ported by the field, is continuously distributed in space, as the field varies from point to point.

1.4. Diffraction and Interference

We began by saying that at the beginning of the twentieth century, the scientific community was in agreement in setting two kinds of process at the basis of the physical theory of the universe: waves and particles. In the foregoing section, much attention has been paid to the description of wave phenomena. Now, for the full comprehension of the argument to be developed, we will need to linger a little longer on this topic. And in order to underline the radical difference—really, the incompatibility—between the two aspects, it would seem particularly fitting to analyze in detail two phenomena (in reality, two facets of one and the same phenomenon) of great conceptual relevance, which are of major conceptual importance and are a direct consequence of the linear nature of wave equations: diffraction and interference.

The first is the phenomenon we can all observe, for example, on a surface of water in which a point reached by a wave disturbance becomes itself the origin of new waves. In this way, the wave is not compelled to propagate itself only in a rectilinear fashion,[4] but can, in a certain sense, "get around the obstacles." The second phenomenon can also be easily observed on a surface of water. If a certain point (or area) of the surface is reached by two waves (generated, for instance, by the passing of two motorboats or the tossing of two rocks into the water), the wave phenomena on every point of the surface will be governed by the "sum" of the two disturbances, so that it may happen that a certain point will remain still, if it is being lifted by one movement and lowered by the other. Thus the two waves at some points will get in the way of each other, in some places nullifying each other (destructive interference), in other places reinforcing each other (constructive interference).

It is important to realize that the possibility of demonstrating these characteristic aspects of wave phenomena depends, in an absolutely crucial way, on the relationship between the wavelength that characterizes the phenomenon and the dimensions of the obstacles encountered by the wave in its propagation. To illustrate the point it will be useful to move beyond the propagation of waves on a water surface, as analyzed above, and consider light phenomena and the familiar process of shadow formation. If a screen is illuminated, in which has been made a hole of macroscopic

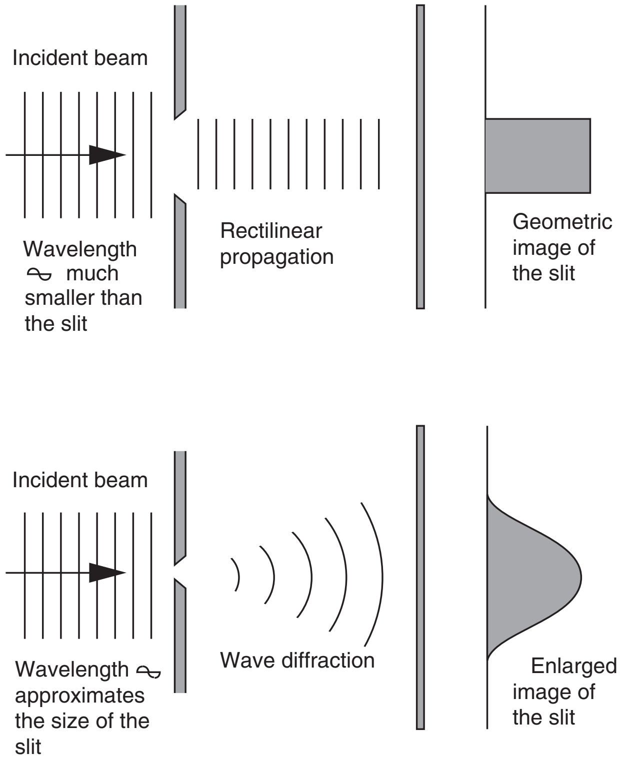

FIGURE 1.8. The phenomenon of the diffraction of light. While the wavelength is much smaller than the obstacles it encounters, the propagation is governed (in a very accurate way) by the laws of geometrical optics which predict rectilinear trajectories for luminous rays and distinct shadows of objects. When the objects encountered by the waves have dimensions approximating the wavelengths, diffraction phenomena occur, and the result is the enlargement of the image of the opening. In the illustration, the shaded areas to the right of the screen represent the square of the intensity of the field, and thereby the density of the electromagnetic energy at any single point.

size (one centimeter square, for example), behind the screen will be seen a geometrically exact image of the hole. The light does not diffract, but propagates in a rectilinear fashion.[5] However, if we make the hole so small that its dimensions become comparable to the wavelength, we then see the light "angle out," so that the shadow becomes larger. We all experience a similar diffraction when we look directly at a beam of light by squinting. The divergent behaviors of the two cases are pictured in Figure 1.8.

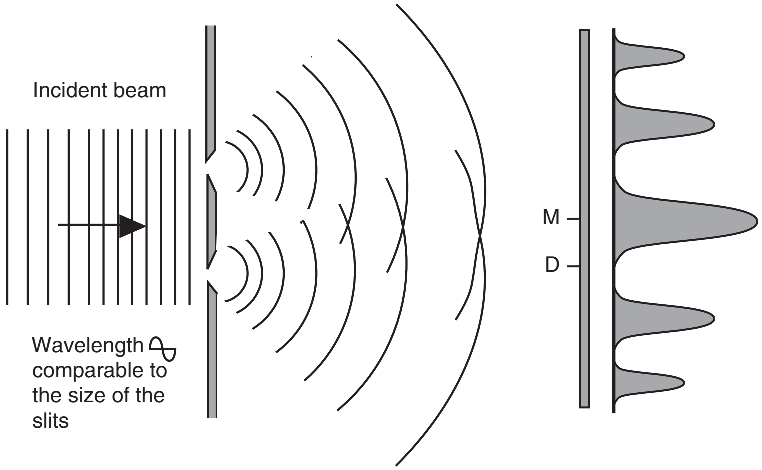

FIGURE 1.9. The phenomenon of interference. The two waves which are generated by the two holes interfere with each other in the region of the screen. At some places, as at M, the contributions of the two waves mutually reinforce each other, while in other places, as at D, they nullify each other. In this case, too, the amplitude of the shape on the right represents the density of energy at the various points of the screen, and therefore, if the screen were a photographic film, it would have an alternation of lighter and darker areas.

Analogously, Figure 1.9 shows the phenomenon of interference. The beam of light entering the two apertures gives rise to two waves, which propagate to the right of the screen. At every point in the space, the electrical field is thus the sum of the fields associated with the waves that emerge from the two holes.[6] It is intuitively apparent that at one point between the openings (such as at M) on the screen to the right, the two electrical fields, which have had equal paths to travel from their origins, arrive, so to speak, "in phase," and they add together, so that they reinforce each other. But at point D, for which the difference between the two paths from the openings is half a wavelength, the field coming from the first slit points upward, while the one coming from the second points downward, and thus their sum will be zero (it is assumed that since the screen is very far from the openings with respect to the extension of the wavelength, the two rays are in fact in line with each other). In both figures, beyond the screen are shown shadowed areas, representing the density of electromagnetic energy at the various points. Where these shapes have a larger amplitude, the energy is greater, so that if the screen were a photographic film those spaces would be more heavily exposed. With reference to Figure 1.9, we can say more exactly that it shows the interference fringes so characteristic of the process.

After these prefatory points, we can face the problem of the emission of electromagnetic energy on the part of a body when its temperature is varied, and thereby illustrate the revolutionary hypothesis that led Planck to its first theoretical explanation.

1.5. Planck's Hypothesis, and Its Later Elaboration by Einstein

As we saw in section 2 of this chapter, all the attempts to explain the radiance of a body as it changes temperature in terms of "classical" theories were miserable failures, even though this problem had engaged some of the most brilliant minds of the time. Planck showed that a solution could be obtained only by granting that exchanges of energy between radiation and matter did not occur continuously. For the classical model required continuity: the electric field associated with a wave acting on a charged particle makes it oscillate by giving it energy; but Planck assumed that a field with frequency ν could exchange energy with matter only through discrete "quanta," that is to say, only in quantities that would be whole number multiples of the quantity $h\nu$. In this relationship, the constant h is the universal constant that now bears Planck's name, and whose value is extremely small, that is to say, equal to 6.55×10^{-27} erg-seconds. From the first moment of its appearance, this constant, also known as the "quantum of action," has played an increasingly important role in the interpretation of the most diverse physical phenomena, especially at the microscopic level. But Planck's hypothesis, which was surely a decisive step in the development of modern scientific thinking, did not immediately win the consensus it deserved, precisely because the quantum of action introduced an uncomfortable rupture into the continuity of natural phenomena universally accepted at the time. The hypothesis appeared to Planck himself as a mathematical artifice, an abstract construction that did not directly require a profound alteration in the understanding of reality. In a way, Planck saw himself constrained to introduce the quantum of action as soon as he realized that there was no other way to provide a theoretical basis for the phenomena of radiation.

But this state of affairs soon changed. In 1905 Einstein used Planck's discovery to explain the photoelectric effect (that is, the process by which it is possible, through the illumination of a metal surface under the right conditions, to pull electrons from the metal), at the same time extending the idea and remarkably increasing its significance: "the observations ... connected with the emission and transformation of light, are easily understood when we assume that the energy of the light is discontinuously distributed in space." It is interesting to note that Einstein did not receive the Nobel Prize for his monumental constructions (the theories of general and special relativity) but for his interpretation of the photoelectric effect.[7]

From that day forward, a beam of light was likened to a flow of quanta, of energy granules or *photons*, rather than to a continuous distribution of energy. This revolutionary hypothesis which, among other things, attributed corpuscular or particle properties to electromagnetic radiation, led eventually to quantum mechanics, the conceptual scheme to which this book is dedicated.

We can get an idea of the very small size of the quantum of action by trying to estimate, for example, how many photons pass over an opening of one square centimeter, situated one meter away from a 100 watt lamp. For the sake of simplicity we will disregard the electromagnetic emission produced by the lamp in the form of heat, and we suppose that the radiation has a frequency typical of the visible spectrum, such that $\nu = 5 \times 10^{14}$ cycles per second. We turn on the lamp for one second. This provides an energy of 100 joules, or 10^9 ergs. Since one photon carries an energy equal to

$$h\nu = (6.55 \times 10^{-27}) \times (5 \times 10^{14}) = 32.75 \times 10^{-13} \text{ ergs}, \qquad (1.4)$$

the number of photons that, in one second, cross the sphere (with a diameter of one meter) surrounding the lamp will be equal to the ratio between the total emitted energy and the amount carried by one photon, that is, $[10^9 : (32.75 \times 10^{-13})] = 3 \times 10^{21}$. A sphere like this has a surface of $4\pi r^2$, or an area equal to 12.56×10^4 cm^2. From this it follows that the number of photons that passes in one second through an opening one centimeter square would be 2.4×10^{16}, which is to say 24 million billions (or 24 quadrillions) of photons. It should not be surprising that at this level the difference between a continuous distribution and a discrete, quantized distribution of energy will be difficult to discern, and this is why the experience with electromagnetic fields at the macroscopic level suggests a continuous distribution of energy in space. But, as we shall see, modern technology makes possible situations where, over a surface such as the one in question, in each second only a few photons, or just a single photon passes. In this case it will be possible to "see" them (that is, to detect them) one by one, and this should convince us that the hypothesis of Planck and Einstein is correct: the electromagnetic waves, right alongside the aspects that characterize them as undulatory or wavelike, possess other aspects as well, which in the classical understanding would be corpuscular or particlelike.

1.6. Bohr's Atom and Quantization

We have seen how Planck was driven to put forward a strange hypothesis in conflict with the conceptual scheme of classical physics in order to overcome the difficulties of the problem of radiation. We will now see how, in an analogous fashion, Bohr overcame a great part of the difficulties of the planetary model of the atom through a similarly "heretical" hypothesis: the quantization of material systems. The idea is rather simple: we assume, for whatever reason, that the classical vision, according to which all the "planetary" movements of the electrons are possible, is not in fact correct, but that there are some orbits that are "privileged" and that these and only these can be traveled by an electron. It may be remarked that this hypothesis no less than Planck's negates the fundamental continuity of physical processes, that is, the deeply rooted idea that "nature does not make leaps" (*natura non facit saltus*). While, according to the classical vision, and as the above discussion has maintained, continuous variations of the initial conditions of the "capture" of a planet by the sun would make for continuous variations in the orbit, according to Bohr's hypothesis this would not happen: an electron, by becoming connected to an atom, has to be "captured" in one of the peculiar conditions that are uniquely compatible with the formation of that system. Distinct values of energy correspond to these discrete orbits, and this energy will be quantized, and no longer continuously variable. The fact that only some orbits are possible explains above all the regularity of the atoms and their relative insensibility (at the appropriate scale) to external disturbances. For, however an electron connects itself to an atom, it must do so in such a way as to wind up on one of the orbits permitted to it, and not on just any arbitrary orbit. Furthermore, no small disturbance resulting from interaction with external systems will be able to change the atom, insofar as its own state can change only when the electron is refitted into another one of its permitted orbits. But the energy of the permitted orbit closest to the original orbit is separated from the latter by a finite amount. Unless this small amount of energy can be provided, the system remains unchanged.

Bohr found a precise mathematical formula for determining these privileged orbits, that is, the only ones that are possible for electrons circling the nucleus of an atom. For an understanding of the arguments from this point forward it will not be necessary to enter into the details of the formula, or make explicit the rules of quantization. Instead, it will be worthwhile to observe how, respecting the principle of the conservation of energy, and recalling the hypothesis of Planck, Bohr also succeeded in explaining another mystery of the behavior of atoms: the fact that they emit and absorb radiation at only very precise frequencies. The reason is fairly simple. If an atom can stand only in one of a series of states of precise energy, it will be able to emit or absorb electromagnetic radiation only by passing from one state to another. In a corresponding way, it will emit a quantum, or photon, whose energy will be such as to guarantee the conservation of total energy. Let us suppose therefore that an atom passes from an orbit with the energy E_2 to an orbit with less energy, E_1, and thereby emits a light quantum. The energy that the electron loses becomes transferred to the photon, and since the energy of this latter is proportional to its frequency $\nu_{(2\rightarrow 1)}$ we will have

$$E_2 - E_1 = h\nu_{(2\rightarrow 1)}. \tag{1.5}$$

If we keep in mind that only certain specific energy states are permitted, it will follow that only precise frequencies can be emitted (or absorbed) by an atom; and this is just what explains the nature of atomic emission spectra (Figure 1.10).

Bohr's theory, based on a single, simple hypothesis involving a unique and precise rule for identifying permitted orbits, rendered intelligible an incredible quantity of data about the emission and absorption of the various elements studied by the spectroscopists of the time. Then it suddenly became evident to everyone that this peculiar hypothesis had a very profound significance. It has to be stressed that Bohr had the courage to present a model that was patently inconsistent. In fact, after using the classical laws to determine the orbits of the electrons, he introduced the contradictory hypothesis that not all orbits are possible! Not only that: he also violated the laws of classical electromagnetism, by assuming that the electrons, although moving in circular orbits (and thus possessing acceleration), did not radiate, so long as they stood in one of their permitted orbits.

FIGURE 1.10. Whereas the white light that comes from the sun contains, so to speak, all the visible colors and thus when analyzed by a crystal prism yields the familiar spectrum (i.e., the rainbow), the light emitted by an atom contains only precise wavelengths and thus, when analyzed by a spectroscope (an instrument like a prism except more sophisticated), gives us a series of "spectral lines." In the figure are represented the lines characteristic of the emissions of an atom of hydrogen heated to high temperature, in an interval of wavelength between 500 Å and 7000 Å. Reminder: an angstrom (Å) represents a unit of measurement used by spectroscopists and is equivalent to 10^{-8} cm.

1.7. De Broglie's Hypothesis

For our purposes, it is not necessary to slow down our pace to describe the discomfort of the scientific community during those passionate years. We will summarize, mentioning only a few salient facts.

In 1924 Louis Victor de Broglie presented his doctoral thesis (which would earn him the Nobel Prize) in which he set forth an hypothesis that showed how we could advance our understanding of disconcerting aspects of reality through "increasing" the confusion. Not only do we have to recognize, with Planck and Einstein, that waves present some particle aspects; it is also useful to suppose that particles present some wave aspects! His idea could be expressed quite simply: a wave of a certain wavelength can be associated with any particle with a mass m and velocity v:

$$\lambda = h/mv, \tag{1.6}$$

where h is the familiar and ever-present Planck's constant. Two comments are in order.

1. The reason why increasing confusion brings greater understanding is that once the de Broglie hypothesis is accepted, it will follow that the orbits permitted according to Bohr's rule of quantization are precisely and solely those orbits where the velocity of the electron is such that a *whole number* of wavelengths stay in exactly one orbit. If it is allowed that an electron can in a certain sense be effectively associated with a wave, this condition is in fact the only one which permits (see Figure

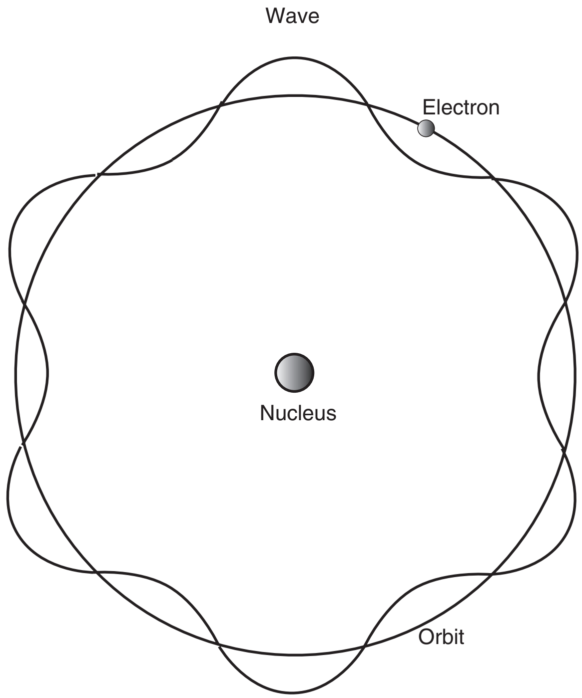

FIGURE 1.11. The orbit of an electron in an atom and the wavelength of the wave associated with it, according to de Broglie's hypothesis. The stationary condition of the process requires that an integral number of wavelengths be located perfectly within the circumference. This condition exactly identifies the same orbits as Bohr's theory.

1.11) a stationary situation on an orbit (something like a violin string, always vibrating at the same frequency)—that is, without variation over time. In turn, the stationary nature of the process is needed to explain how properties of atoms do not change.

2. The hypothesis now discussed immediately poses a problem: why the wave aspects postulated by de Broglie never emerge in experiments on material bodies. If de Broglie's hypothesis is true, why has nobody ever witnessed (for example) the diffraction of a particle? The answer is fairly simple, as long as we keep in mind that diffraction occurs only when the dimensions of the obstacles which the wave encounters are comparable to the wavelength in question. The solution to the problem is to evaluate the wavelengths of various particles at various velocities. I do not wish to slow us down with tedious calculations, even though, on

the basis of the preceding discussion, it would not be difficult to determine "de Broglie's wavelength" of a material particle moving at a certain velocity. I will confine myself to the consideration of a few simple examples. An atom of hydrogen that moves at the velocity characterizing its thermal agitation at normal temperature is associated with a wave whose wavelength is about 10^{-8} cm. As seen in Figure 1.5, this corresponds to the length of x rays. Since the length of the wave decreases with the increase of the mass, it will be apparent that all the particles of the "classical" world will be associated with waves whose wavelength is so small that, to reveal the effects of diffraction, they would have to encounter obstacles of dimensions well below the scale of the processes they involve. And this is why nobody has ever encountered the wave aspects of material bodies. But if the preceding calculation is carried out for an electron moving in an atom, it should be clear (as discussed before with reference to de Broglie's hypothesis, and from Figure 1.11) that the dimensions of the atom are comparable to the wavelengths that characterize the electrons themselves. From this it follows that, if de Broglie's hypothesis is true, it must necessarily be taken into account when describing such systems.

The line of thinking sparked off by the courageous idea of de Broglie would turn out to be extremely fertile, and, in less than a year, would bring Erwin Schrödinger to work out "wave mechanics" (one of the two equivalent formulations of quantum theory). The great importance of de Broglie's hypothesis is revealed in a statement by Albert Einstein in a letter he sent to Hendrick Antoon Lorentz in December, 1924: "A younger brother of . . . de Broglie has undertaken a very interesting attempt to interpret the Bohr-Sommerfeld quantum rules [this was the business of fitting the electron waves around the Bohr orbits] . . . I believe it is the first feeble ray of light on this worst of our physics enigmas. I, too, have found something which speaks for his construction. . . ."[8] Einstein played a key role in securing the cooperation of de Broglie's reluctant thesis supervisor Paul Langevin. Einstein made it clear, when Langevin asked about its merit, that he found it very interesting indeed. Abraham Pais aptly remarked that in this way Einstein was not only one of the fathers of quantum theory, he was also the "grandfather" of wave mechanics!

The important role of Einstein's thought for Schrödinger was fully recognized by the latter, who on various occasions said that "his theory meant nothing other than taking seriously the Einstein-de Broglie wave theory of particles in motion," and Schrödinger would always refer to the assumption I have been discussing in this section as "the de Broglie–Einstein hypothesis."

1.8. Overcoming the Crisis

The profound crisis over the vision of physical processes we have delineated in the preceding pages found its ultimate outcome a quarter century later. During 1925 and 1926 Heisenberg and Schrödinger, working independently and following two quite different lines of thought, succeeded in precisely formulating two theoretical models that signaled the birth of quantum mechanics.

Heisenberg, a disciple of Bohr and profound student of his work, was convinced that the master's insistence on visualizing the atom as a planetary system represented an obstacle to overcoming the contradictions of the theory. He therefore decided to focus his attention not on something that could not be experienced, such as the orbits of electrons, but instead on the radiation emitted by atoms, namely, the spectral lines of which we spoke in section 1.6. In this way, in 1925 Heisenberg arrived at the formulation of a mathematical scheme that would eventually be known as "matrix mechanics." The interpretation of matrix mechanics remained an open problem for a time, and its full understanding required the important contributions of Born, Jordan, and Pauli.

Like Einstein's 1905, Schrödinger's 1926 was a year of glory. By this time he was thirty-eight, with a brilliant although not really outstanding scientific career. He felt conscious of his worth, and was somewhat discouraged that he had not yet made any significant contribution to science. In November, 1925, he wrote to Einstein, speaking with enthusiasm about the idea of the wave nature of particles. Again, at the end of the same month he wrote a letter to Hans Thirring, in which, according to Thirring, one can find a distinct outline of the ideas that would constitute the basis of quantum mechanics. But it still lacked some decisive elements.

Schrödinger was at a curious period in his life: his marriage was foundering and he spoke often about divorce. At the end of the year he decided to spend Christmas vacation at Arosa, an Alpine spot of 1,700 meters altitude, not far from Davos. He wrote to "an old girl friend from Vienna," intending to meet her, while his wife stayed in Zürich. It is not possible to identify the girl friend: as Walter Moore commented in his biography of Schrödinger, this woman, like the "black lady" who inspired the sonnets of Shakespeare, will probably be forever unknown. But according to the explicit admission of the great scientist, she set in motion a period of creative activity unequaled in the history of science.[9] In fact, in those two weeks at Arosa, Schrödinger found his own path, and upon his return to Vienna, taking advantage of the subtle mathematical knowledge of his friend Hermann Weyl, in less than a year he brought to completion a series of six works containing the definitive formulation of wave mechanics.

We can conclude this biographical digression with a return to our theme: the birth of the new conception of physical processes. It is interesting to consider that the two formulations of the theory, that of Heisenberg and that of Schrödinger, do not appear to have any relationship to each other, and in their form they each use a very different language. The theory of Heisenberg, as I mentioned, was formulated in the language of matrices, and hence acquired the name matrix mechanics, while that of Schrödinger came to be formulated in the typical language of wave equations, and came to be known as wave mechanics. It would take some years for it to be rigorously proved, thanks to the further researches of Schrödinger himself and of Paul Adrien Maurice Dirac, that the two theories were really but two different mathematical modes of expressing the same laws.

1.9. The Wave/Particle Dualism

The new theory presents very peculiar aspects but, as the reader will probably have sensed, at the same time brings an important new unification of our conceptions of nature. Phenomena so diverse from the point of view of classical physics—such as waves and particles—become assimilated. Every physical process simultaneously involves these two faces of reality. The quanta of light, or photons, behave in many experiments just like particles (with peculiar properties, of course), and analogously, particles, under opportune conditions, behave like waves. The problem arises, then, of understanding how concepts that seem so irreconcilable and contradictory, can ever be integrated. Quantum formalism required an interpretation, and a new phase began, characterized by great interest and enthusiasm—a feverish time, which would bring the vision I shall illustrate in the following pages.

Of course, apart from the enormous successes that the theory met with quite soon in the explanation of a variety of physical processes, it immediately stirred up an experimental problem of great interest: is it possible to get direct evidence of the wave aspects of particles in the laboratory? Various researchers cooperated in this endeavor and the response was in the affirmative.

I would now like to close this first chapter with a strange and delightful observation. In 1937, George Paget Thompson received the Nobel Prize for having shown that the electron was a wave. Exactly thirty-one years earlier, his father, Joseph John Thompson, had received the Nobel Prize for having shown, through experiments in radioactive processes, that the electron was a particle![10]