APPENDIX 1

1. We can translate the information given about the typical sleep cycle into values for A, B, C for the function $f(t) = A\cos(Bt)+C$. From Appendix A we know that $B = 2\pi/T$, so that from $T = 1.5$ we get $B = (4/3)\pi$. Since the highest sleep stage is 0 and the lowest is -4, the midline is the middle value $C = -2$. The amplitude A is then the maximum value minus the midline: $A = 0 - (-2) = 2$. Using these values in our function above leads to the $f(t)$ equation given in the chapter.
2. We can simplify the equation $f(t) = -1$ to

$$2\cos\left(\frac{4\pi}{3}t\right) = 1, \qquad \text{or} \qquad \cos\left(\frac{4\pi}{3}t\right) = \frac{1}{2}.$$

By taking the inverse cosine of both sides we get

$$\frac{4\pi}{3}t = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{11\pi}{3}, \frac{13\pi}{3}, \ldots,$$

where the negative values have been omitted since we're interpreting t as time. Solving for t yields $t = 0.25, 1.25, 1.75, 2.75, 3.25, \ldots$. With these numbers in mind, we can picture the t-values for which $f(t) \geq -1$ as the intervals between the gray dots in Figure A1.1 that contain peaks of $f(t)$.
3. We can rearrange $L = 20\log_{10}(50{,}000p)$ to read $\log_{10}(50{,}000p) = L/20$. Using the fact that $10^{\log_{10} x} = x$ for $x > 0$, we then have

$$50{,}000p = 10^{L/20}, \quad \text{or} \quad p(L) = \frac{1}{50{,}000}10^{L/20}.$$

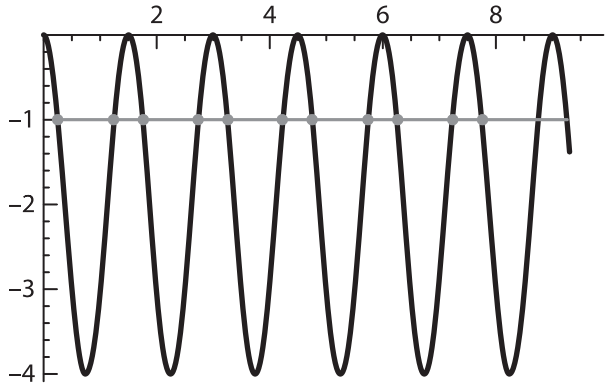

Figure A1.1. The intersection of the line $y(t) = -1$ with the graph of $f(t)$.

4. Intuitively, we know that objects that are accelerating have speeds that change with time (think of an airplane accelerating from rest to takeoff). If we measure the accelerating object's speed at times t_a and t_b and get $v(t_a)$ and $v(t_b)$, then we say that the object's acceleration a over that time interval was

$$a = \frac{v(t_b) - v(t_a)}{t_b - t_a}.$$

For our water molecule with acceleration $a = -g$ we are measuring its speed over the time interval $[0, t]$. Therefore,

$$-g = \frac{v(t) - v_y}{t - 0}, \qquad \text{or} \qquad v(t) = v_y - gt.$$

I should note that this derivation depends on the fact that a is constant.

5. From $x(t) = v_x t$ we have that $t = x/v_x$. Substituting this into $y(t) = 6.5 + v_y t - (g/2)t^2$ yields

$$y(x) = 6.5 + \frac{v_y}{v_x}x - \frac{g}{2v_x^2}x^2.$$