

understand whether the solutions are unique and strictly positive, or not. Our task for the rest of this chapter will be to understand the properties of these equations and their solutions.

To guide us in this investigation, there are three key questions that we can ask: (1) what is the solution for factor prices? (2) if prices change, how do factor prices change? (3) if endowments change, how do outputs change? Each of these questions is taken up in the sections that follow. The methods we shall use follow the “dual” approach of Woodland (1977, 1982), Mussa (1979), and Dixit and Norman (1980).

DETERMINATION OF FACTOR PRICES

Notice that our four-equation system above can be decomposed into the zero-profit conditions as *two* equations in *two* unknowns—the wage and rental—and then the full-employment conditions, which involve both the factor prices (which affect a_{iL} and a_{iK}) and the outputs. It would be especially convenient if we could uniquely solve for the factor prices from the zero-profit conditions, and then just substitute these into the full-employment conditions. This will be possible when the hypotheses of the following lemma, are satisfied.

LEMMA (FACTOR PRICE INSENSITIVITY)

So long as both goods are produced, and factor intensity reversals (FIRs) do not occur, then each price vector (p_1, p_2) corresponds to unique factor prices (w, r) .

This is a remarkable result, because it says that the factor endowments (L, K) do not matter for the determination of (w, r) . We can contrast this result with a one-sector economy, with production of $y = f(L, K)$, wages of $w = pf_L$, and diminishing marginal product $f_{LL} < 0$. In this case, any increase in the labor endowments would certainly reduce wages, so that countries with higher labor/capital endowments (L/K) would have lower wages. This is the result we normally expect. In contrast, the above lemma says that in a two-by-two economy, with a fixed product price p , it is possible for the labor force or capital stock to grow *without* affecting their factor prices! Thus, Leamer (1995) refers to this result as “factor price insensitivity.” Our goal in this section is to prove the result and also develop the intuition for why it holds.

Two conditions must hold to obtain this result: first, that both goods are produced; and second, that factor intensity reversals (FIRs) do not occur. To understand FIRs, consider figures 1.5 and 1.6. In the first case, presented in figure 1.5, we have graphed the two zero-profit conditions, and the unit-cost lines intersect only *once*, at point A . This illustrates the lemma: given (p_1, p_2) , there is a *unique* solution for (w, r) . But another case is illustrated in figure 1.6, where the unit-cost lines intersect *twice*, at points A and B . Then there are two possible solutions for (w, r) , and the result stated in the lemma no longer holds.

The case where the unit-cost lines intersect more than once corresponds to “factor intensity reversals.” To see where this name comes from, let us compute the labor and capital requirements in the two industries. We have already shown that a_{iL} and a_{iK} are the derivatives of the unit-cost function with respect to factor prices, so it follows that the vectors (a_{iL}, a_{iK}) are the *gradient vectors* to the iso-cost curves for the two industries

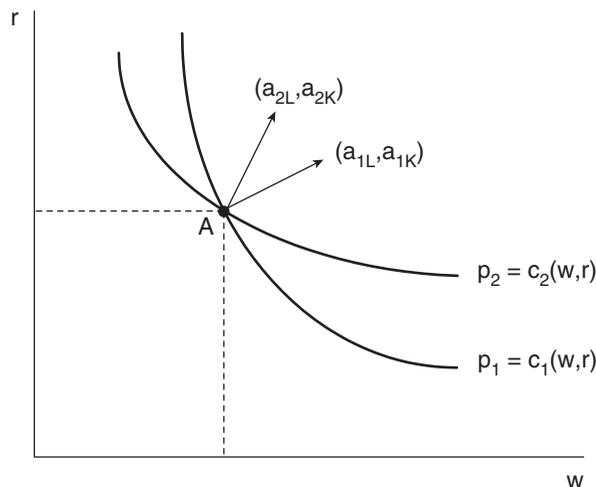


Figure 1.5

in figure 1.5. Recall from calculus that gradient vectors point in the direction of the maximum increase of the function in question. This means that they are *orthogonal* to their respective iso-cost curves, as shown by (a_{1L}, a_{1K}) and (a_{2L}, a_{2K}) at point A. Each of these vectors has slope (a_{iK}/a_{iL}) , or the capital-labor ratio. It is clear from figure 1.5 that (a_{1L}, a_{1K}) has a smaller slope than (a_{2L}, a_{2K}) , which means that *industry 2 is capital intensive*, or equivalently, *industry 1 is labor intensive*.⁷

In figure 1.6, however, the situation is more complicated. Now there are two sets of gradient vectors, which we label by (a_{1L}, a_{1K}) and (a_{2L}, a_{2K}) at point A and by (b_{1L}, b_{1K}) and (b_{2L}, b_{2K}) at point B. A close inspection of the figure will reveal that industry 1 is *labor intensive* ($a_{1K}/a_{1L} < a_{2K}/a_{2L}$) at point A, but is *capital intensive* ($b_{1K}/b_{1L} > b_{2K}/b_{2L}$) at point B. This illustrates a *factor intensity reversal*, whereby the comparison of factor intensities changes at different factor prices.

While FIRs might seem like a theoretical curiosity, they are actually quite realistic. Consider the footwear industry, for example. While much of the footwear in the world is produced in developing nations, the United States retains a small number of plants. For sneakers, New Balance has a plant in Norridgewock, Maine, where employers earn about \$14 per hour.⁸ Some operate computerized equipment with up to twenty sewing machine heads running at once, while others operate automated stitchers guided by cameras, which allow one person to do the work of six. This is a far cry from the plants in Asia that produce shoes for Nike, Reebok, and other U.S. producers, using century-old technology and paying less than \$1 per hour. The technology used to make sneakers in Asia is like that of industry 1 at point A in figure 1.5, using labor-intensive

⁷ Alternatively, we can totally differentiate the zero-profit conditions, holding prices fixed, to obtain $0 = a_{iL}dw + a_{iK}dr$. It follows that the slope of the iso-cost curve equals $dr/dw = -a_{iL}/a_{iK} = -L_i/K_i$. Thus, the slope of each iso-cost curve equals the relative demand for the factor on the horizontal axis, whereas the slope of the gradient vector (which is orthogonal to the iso-cost curve) equals the relative demand for the factor on the vertical axis.

⁸ The material that follows is drawn from Aaron Bernstein, "Low-Skilled Jobs: Do They Have to Move?" *Business Week*, February 26, 2001, pp. 94–95.

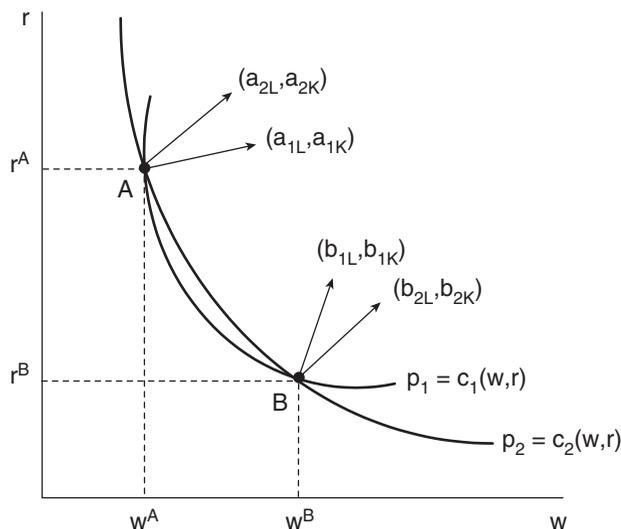


Figure 1.6

technology and paying low wages w^A , while industry 1 in the United States is at point B , paying higher wages w^B and using a capital-intensive technology.

As suggested by this discussion, when there are two possible solutions for the factor prices such as points A and B in figure 1.6, then some countries can be at one equilibrium and others countries at the other. How do we know which country is where? This is a question that we will answer at the end of the chapter, where we will argue that a *labor-abundant* country will likely be at equilibrium A of figure 1.6, with a low wage and high rental on capital, whereas a *capital-abundant* country will be at equilibrium B , with a high wage and low rental. Generally, to determine the factor prices in each country we will need to examine its full-employment conditions in addition to the zero-profit conditions.

Let us conclude this section by returning to the simple case of no FIR, in which the lemma stated above applies. What are the implications of this result for the determination of factor prices under free trade? To answer this question, let us sketch out some of the assumptions of the Heckscher-Ohlin model, which we will study in more detail in the next chapter. We assume that there are two countries, with identical technologies but different factor endowments. We continue to assume that labor and capital are the two factors of production, so that under free trade the equilibrium conditions (1.7) and (1.8) apply in *each* country with the *same* product prices (p_1, p_2) . We can draw figure 1.5 for each country, and in the absence of FIR, this *uniquely* determines the factor prices in each countries. In other words, the wage and rental determined by figure 1.5 are *identical* across the two countries. We have therefore proved the factor price equalization (FPE) theorem, which is stated as follows.

FACTOR PRICE EQUALIZATION THEOREM (SAMUELSON 1949)

Suppose that two countries are engaged in free trade, having identical technologies but different factor endowments. If both countries produce both goods and FIRs do not occur, then the factor prices (w, r) are equalized across the countries.

The FPE theorem is a remarkable result because it says that *trade in goods* has the ability to equalize factor prices: in this sense, trade in goods is a “perfect substitute” for trade in factors. We can again contrast this result with that obtained from a one-sector economy in both countries. In that case, equalization of the product price through trade would certainly not equalize factor prices: the labor-abundant country would be paying a lower wage. Why does this outcome *not occur* when there are two sectors? The answer is that the labor-abundant country can *produce more of, and export*, the labor-intensive good. In that way it can fully employ its labor while still paying the same wages as a capital-abundant country. In the two-by-two model, the opportunity to disproportionately produce more of one good than the other, while exporting the amounts not consumed at home, is what allows factor price equalization to occur. This intuition will become even clearer as we continue to study the Heckscher-Ohlin model in the next chapter.

CHANGE IN PRODUCT PRICES

Let us move on now to the second of our key questions of the two-by-two model: if the product prices change, how will the factor prices change? To answer this, we perform comparative statics on the zero-profit conditions (1.7). Totally differentiating these conditions, we obtain

$$dp_i = a_{iL}dw + a_{iK}dr \Rightarrow \frac{dp_i}{p_i} = \frac{wa_{iL}}{c_i} \frac{dw}{w} + \frac{ra_{iK}}{c_i} \frac{dr}{r}, i = 1, 2. \quad (1.9)$$

The second equation is obtained by multiplying and dividing like terms, and noting that $p_i = c_i(w, r)$. The advantage of this approach is that it allows us to express the variables in terms of *percentage changes*, such as $d \ln w = dw/w$, as well as *cost-shares*. Specifically, let $\theta_{iL} = wa_{iL}/c_i$ denote the cost-share of labor in industry i , while $\theta_{iK} = ra_{iK}/c_i$ denotes the cost-share of capital. The fact that costs equal $c_i = wa_{iL} + ra_{iK}$ ensures that the shares sum to unity, $\theta_{iL} + \theta_{iK} = 1$. In addition, denote the percentage changes by $dw/w = \hat{w}$ and $dr/r = \hat{r}$. Then (1.9) can be re-written as

$$\hat{p}_i = \theta_{iL} \hat{w} + \theta_{iK} \hat{r}, i = 1, 2. \quad (1.9')$$

Expressing the equations using these cost-shares and percentage changes follows Jones (1965) and is referred to as the “Jones algebra.” This system of equations can be written in matrix form and solved as

$$\begin{pmatrix} \hat{p}_1 \\ \hat{p}_2 \end{pmatrix} = \begin{pmatrix} \theta_{1L} & \theta_{1K} \\ \theta_{2L} & \theta_{2K} \end{pmatrix} \begin{pmatrix} \hat{w} \\ \hat{r} \end{pmatrix} \Rightarrow \begin{pmatrix} \hat{w} \\ \hat{r} \end{pmatrix} = \frac{1}{|\theta|} \begin{pmatrix} \theta_{2K} & -\theta_{1K} \\ -\theta_{2L} & \theta_{1L} \end{pmatrix} \begin{pmatrix} \hat{p}_1 \\ \hat{p}_2 \end{pmatrix}, \quad (1.10)$$

where $|\theta|$ denotes the determinant of the two-by-two matrix on the left. This determinant can be expressed as

$$\begin{aligned} |\theta| &= \theta_{1L} \theta_{2K} - \theta_{1K} \theta_{2L} \\ &= \theta_{1L} (1 - \theta_{2L}) - (1 - \theta_{1L}) \theta_{2L} \\ &= \theta_{1L} - \theta_{2L} = \theta_{2K} - \theta_{1K} \end{aligned} \quad (1.11)$$

where we have repeatedly made use of the fact that $\theta_{iL} + \theta_{iK} = 1$.

In order to fix ideas, let us assume henceforth that *industry 1 is labor intensive*. This implies that its labor cost-share in industry 1 exceeds that in industry 2, $\theta_{1L} - \theta_{2L} > 0$, so that $|\theta| > 0$ in (1.11).⁹ Furthermore, suppose that the relative price of good 1 *increases*, so that $\hat{p} = \hat{p}_1 - \hat{p}_2 > 0$. Then we can solve for the change in factor prices from (1.10) and (1.11) as

$$\hat{w} = \frac{\theta_{2K}\hat{p}_1 - \theta_{1K}\hat{p}_2}{|\theta|} = \frac{(\theta_{2K} - \theta_{1K})\hat{p}_1 + \theta_{1K}(\hat{p}_1 - \hat{p}_2)}{(\theta_{2K} - \theta_{1K})} > \hat{p}_1, \quad (1.12a)$$

since $\hat{p}_1 - \hat{p}_2 > 0$, and,

$$\hat{r} = \frac{\theta_{1L}\hat{p}_2 - \theta_{2L}\hat{p}_1}{|\theta|} = \frac{(\theta_{1L} - \theta_{2L})\hat{p}_2 - \theta_{2L}(\hat{p}_1 - \hat{p}_2)}{(\theta_{1L} - \theta_{2L})} < \hat{p}_2, \quad (1.12b)$$

since $\hat{p}_1 - \hat{p}_2 > 0$.

From the result in (1.12a), we see that the wage increases *by more* than the price of good 1, $\hat{w} > \hat{p}_1 > \hat{p}_2$. This means that workers can afford to buy more of good 1 (w/p_1 has gone up), as well as more of good 2 (w/p_2 has gone up). When labor can buy more of *both goods* in this fashion, we say that the *real wage* has increased. Looking at the rental on capital in (1.12b), we see that the rental r changes by *less than* the price of good 2. It follows that capital-owner can afford less of good 2 (r/p_2 has gone down), and also less of good 1 (r/p_1 has gone down). Thus the *real return to capital* has fallen. We can summarize these results with the following theorem.

STOLPER-SAMUELSON (1941) THEOREM

An increase in the relative price of a good will increase the real return to the factor used intensively in that good, and reduce the real return to the other factor.

To develop the intuition for this result, let us go back to the differentiated zero-profit conditions in (1.9'). Since the cost-shares add up to unity in each industry, we see from equation (1.9') that \hat{p}_i is a *weighted average* of the factor price changes \hat{w} and \hat{r} . This implies that \hat{p}_i necessarily lies in between \hat{w} and \hat{r} . Putting these together with our assumption that $\hat{p}_1 - \hat{p}_2 > 0$, it is therefore clear that

$$\hat{w} > \hat{p}_1 > \hat{p}_2 > \hat{r}. \quad (1.13)$$

Jones (1965) has called this set of inequalities the “magnification effect”: they show that any change in the product price has a *magnified effect* on the factor prices. This is an extremely important result. Whether we think of the product price change as due to export opportunities for a country (the export price goes up), or due to lowering import tariffs (so the import price goes down), the magnification effect says that there will be both gainers and losers due to this change. Even though we will argue in chapter 6 that there are gains from trade in some overall sense, it is still the case that trade opportunities have strong *distributional* consequences, making some people worse off and some better off!

⁹As an exercise, show that $L_1/K_1 > L_2/K_2 \Leftrightarrow \theta_{1L} > \theta_{2L}$. This is done by multiplying the numerator and denominator on both sides of the first inequality by like terms, so as to convert it into cost-shares.

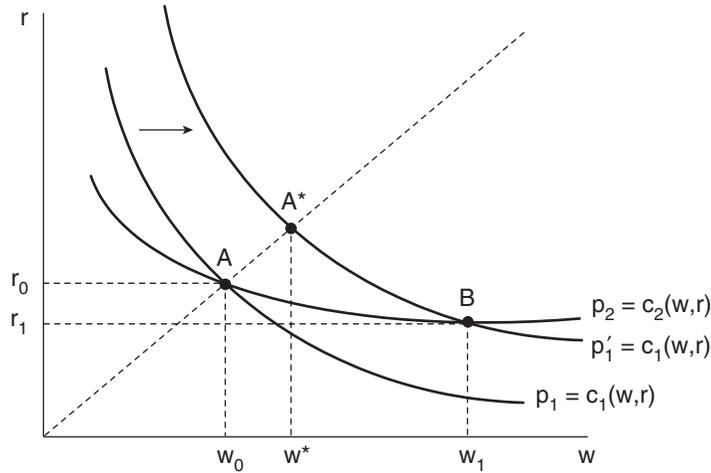


Figure 1.7

We conclude this section by illustrating the Stolper-Samuelson theorem in figure 1.7. We begin with an initial factor price equilibrium given by point A, where industry 1 is labor intensive. An increase in the price of that industry will shift out the iso-cost curve, and as illustrated, move the equilibrium to point B. It is clear that the wage has gone up, from w_0 to w_1 , and the rental has declined, from r_0 to r_1 . Can we be sure that the wage has increased in percentage terms *by more* than the relative price of good 1? The answer is yes, as can be seen by drawing a ray from the origin through the point A. Because the unit-cost functions are homogeneous of degree one in factor prices, moving along this ray increases p and (w, r) in the same proportion. Thus, at the point A^* , the increase in the wage exactly matched the percentage change in the price p_1 . But it is clear that the equilibrium wage increases by *more*, $w_1 > w^*$, so the percentage increase in the wage *exceeds* that of the product price, which is the Stolper-Samuelson result.

CHANGES IN ENDOWMENTS

We turn now to the third key question: if endowments change, how do the industry outputs change? To answer this, we hold the product prices *fixed* and totally differentiate the full-employment conditions (1.8) to obtain

$$\begin{aligned} a_{1L} dy_1 + a_{2L} dy_2 &= dL, \\ a_{1K} dy_1 + a_{2K} dy_2 &= dK. \end{aligned} \tag{1.14}$$

Notice that the a_{ij} coefficients *do not* change, despite the fact that they are functions of the factor prices (w, r) . These coefficients are fixed because p_1 and p_2 do not change, so from our earlier lemma, the factor prices are also fixed.

By rewriting the equations in (1.14) using the “Jones algebra,” we obtain

$$\begin{aligned} \frac{y_1 a_{1L}}{L} \frac{dy_1}{y_1} + \frac{y_2 a_{2L}}{L} \frac{dy_2}{y_2} &= \frac{dL}{L} \\ \frac{y_1 a_{1K}}{K} \frac{dy_1}{y_1} + \frac{y_2 a_{2K}}{K} \frac{dy_2}{y_2} &= \frac{dK}{K} \end{aligned} \Rightarrow \begin{aligned} \lambda_{1L} \hat{y}_1 + \lambda_{2L} \hat{y}_2 &= \hat{L} \\ \lambda_{1K} \hat{y}_1 + \lambda_{2K} \hat{y}_2 &= \hat{K}. \end{aligned} \tag{1.14'}$$

To move from the first set of equations to the second, we denote the percentage changes $dy_i/y_i = \hat{y}_i$, and likewise for all the other variables. In addition, we define $\lambda_{iL} \equiv (y_i a_{iL}/L) = (L_i/L)$, which measures the *fraction of the labor force employed in industry i* , where $\lambda_{iL} + \lambda_{iK} = 1$. We define λ_{iK} analogously as the fraction of the capital stock employed in industry i .

This system of equations is written in matrix form and solved as

$$\begin{bmatrix} \lambda_{1L} & \lambda_{2L} \\ \lambda_{1K} & \lambda_{2K} \end{bmatrix} \begin{pmatrix} \hat{y}_1 \\ \hat{y}_2 \end{pmatrix} = \begin{pmatrix} \hat{L} \\ \hat{K} \end{pmatrix} \Rightarrow \begin{pmatrix} \hat{y}_1 \\ \hat{y}_2 \end{pmatrix} = \frac{1}{|\lambda|} \begin{bmatrix} \lambda_{2K} & -\lambda_{2L} \\ -\lambda_{1K} & \lambda_{1L} \end{bmatrix} \begin{pmatrix} \hat{L} \\ \hat{K} \end{pmatrix}, \quad (1.15)$$

where $|\lambda|$ denotes the determinant of the two-by-two matrix on the left, which is simplified as

$$\begin{aligned} |\lambda| &= \lambda_{1L}\lambda_{2K} - \lambda_{2L}\lambda_{1K} \\ &= \lambda_{1L}(1 - \lambda_{1K}) - (1 - \lambda_{1L})\lambda_{1K} \\ &= \lambda_{1L} - \lambda_{1K} = \lambda_{2K} - \lambda_{2L}, \end{aligned} \quad (1.16)$$

where we have repeatedly made use of the fact that $\lambda_{iL} + \lambda_{iK} = 1$ and $\lambda_{1K} + \lambda_{2K} = 1$.

Recall that we assumed *industry 1 to be labor intensive*. This implies that the share of the labor force employed in industry 1 exceeds the share of the capital stock used there, $\lambda_{1L} - \lambda_{1K} > 0$, so that $|\lambda| > 0$ in (1.16).¹⁰ Suppose further that the endowments of labor is increasing, while the endowments of capital remains fixed such that $\hat{L} > 0$, and $\hat{K} = 0$. Then we can solve for the change in outputs from (1.15)–(1.16) as

$$\hat{y}_1 = \frac{\lambda_{2K}}{(\lambda_{2K} - \lambda_{2L})} \hat{L} > \hat{L} > 0 \quad \text{and} \quad \hat{y}_2 = \frac{-\lambda_{1K}}{|\lambda|} \hat{L} < 0. \quad (1.17)$$

From (1.17), we see that the output of the labor-intensive industry 1 expands, whereas the output of industry 2 contracts. We have therefore established the Rybczynski theorem.

RYBCZYNSKI (1955) THEOREM

An increase in a factor endowment will increase the output of the industry using it intensively, and decrease the output of the other industry.

To develop the intuition for this result, let us write the full-employment conditions in vector notation as:

$$\begin{pmatrix} a_{1L} \\ a_{1K} \end{pmatrix} y_1 + \begin{pmatrix} a_{2L} \\ a_{2K} \end{pmatrix} y_2 = \begin{pmatrix} L \\ K \end{pmatrix}. \quad (1.8')$$

We have already illustrated the gradient vectors (a_{iL}, a_{iK}) to the iso-cost curves in figure 1.5 (with not FIR). Now let us take these vectors and regraph them, in figure

¹⁰ As an exercise, show that $L_1/K_1 > L_2/K_2 \Leftrightarrow \lambda_{1L} > \lambda_{1K}$ and $\lambda_{2K} > \lambda_{2L}$.

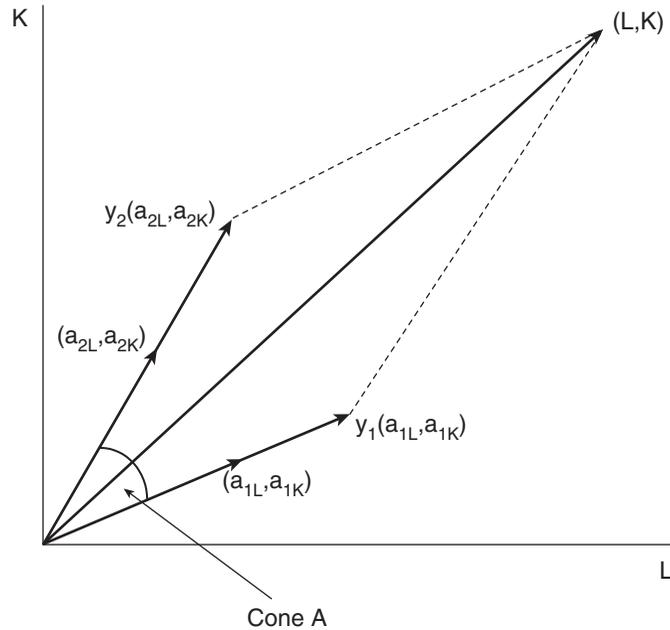


Figure 1.8

1.8. Multiplying each of these by the output of their respective industries, we obtain the total labor and capital demands $y_1(a_{1L}, a_{1K})$ and $y_2(a_{2L}, a_{2K})$. Summing these as in (1.8') we obtain the labor and capital endowments (L, K) . But this exercise can also be performed in reverse: for any endowment vector (L, K) , there will be a *unique* value for the outputs (y_1, y_2) such that when (a_{1L}, a_{1K}) and (a_{2L}, a_{2K}) are multiplied by these amounts, they will sum to the endowments.

How can we be sure that the outputs obtained from (1.8') are positive? It is clear from figure 1.8 that the outputs in both industries will be positive if and only if the endowment vector (L, K) lies *in between* the factor requirement vectors (a_{1L}, a_{1K}) and (a_{2L}, a_{2K}) . For this reason, the space spanned by these two vectors is called a “cone of diversification,” which we label by cone A in figure 1.8. In contrast, if the endowment vector (L, K) lies *outside* of this cone, then it is *impossible* to add together any positive multiples of the vectors (a_{1L}, a_{1K}) and (a_{2L}, a_{2K}) and arrive at the endowment vector. So if (L, K) lies outside of the cone of diversification, then it must be that only *one good* is produced. At the end of the chapter, we will show how to determine which good it is.¹¹ For now, we should just recognize that when only one good is produced, the factor prices are determined by the marginal products of labor and capital as in the one-sector model, and will certainly depend on the factor endowments.

Now suppose that the labor endowment increases to $L' > L$, with no change in the capital endowment, as shown in figure 1.9. Starting from the endowments (L', K) , the *only* way to add up multiples of (a_{1L}, a_{1K}) and (a_{2L}, a_{2K}) and obtain the endowments is to *reduce* the output of industry 2 to y_2' , and *increase* the output of industry 1 to y_1' . This means that not only does industry 1 absorb the entire amount of the extra labor

¹¹ See problem 1.5.

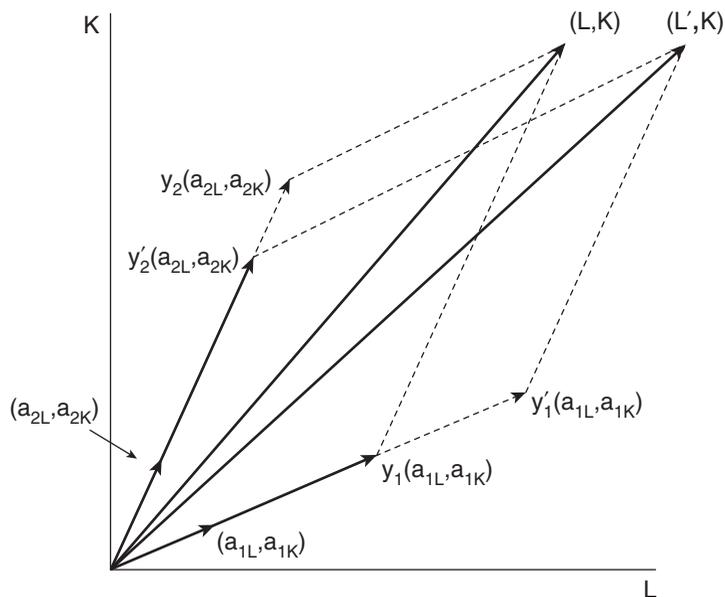


Figure 1.9

endowment, but it also absorbs further labor and capital from industry 2 so that its ultimate labor/capital ratio is unchanged from before. The labor/capital ratio in industry 2 is also unchanged, and this is what permits both industries to pay exactly the same factor prices as they did before the change in endowments.

There are many examples of the Rybczynski theorem in practice, but perhaps the most commonly cited is what is called the “Dutch Disease.”¹² This refers to the discovery of oil off the coast of the Netherlands, which led to an increase in industries making use of this resource. (Shell Oil, one of the world’s largest producers of petroleum products, is a Dutch company.) At the same time, however, other “traditional” export industries of the Netherlands contracted. This occurred because resources were attracted away from these industries and into those that were intensive in oil, as the Rybczynski theorem would predict.

We have now answered the three questions raised earlier in the chapter: how are factor prices determined; how do changes in product prices affect factor prices; and how do changes in endowments affect outputs? But in answering all of these, we have relied on the assumptions that *both goods are produced*, and also that factor intensity reversals do not occur, as was stated explicitly in the FPE theorem. In the remainder of this chapter we need to investigate both of these assumptions, to understand either when they will hold or the consequences of their not holding.

We begin by tracing through the changes in the outputs induced by changes in endowments, along the equilibrium of the production possibility frontier. As the labor endowment grows in figure 1.9, the PPF will shift out. This is shown in figure 1.10, where the outputs will shift from point A to point A’ with an increase of good 1 and reduction of good 2, at the unchanged price p . As the endowment of labor rises, we

¹²See, for example, Corden and Neary (1982) and Jones, Neary, and Ruane (1987).

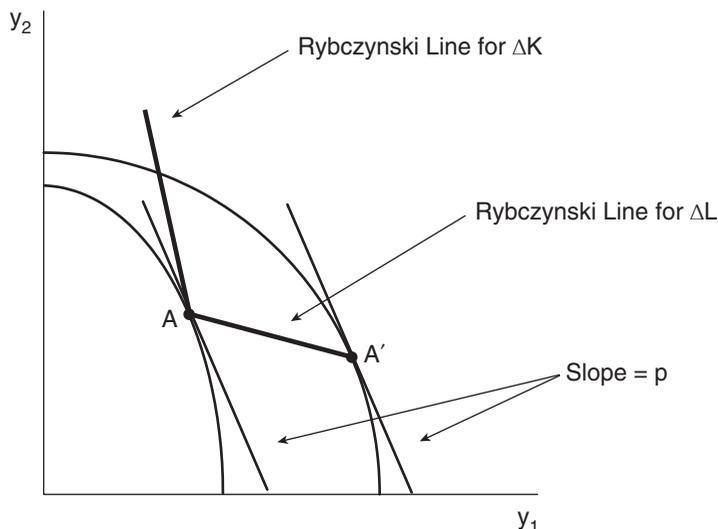


Figure 1.10

can join up all points such as A and A' where the slopes of the PPFs are equal. These form a downward-sloping line, which we will call the Rybczynski line for changes in labor (ΔL). The Rybczynski line for ΔL indicates how outputs change as labor endowment expands.

Of course, there is also a Rybczynski line for ΔK , which indicates how the outputs change as the capital endowment grows: this would lead to an increase in the output of good 2, and reduction in the output of good 1. As drawn, both of the Rybczynski lines are illustrated as *straight* lines: can we be sure that this is the case? The answer is yes: the fact that the product prices are fixed along a Rybczynski line, implying that factor prices are also fixed, ensures that these are straight lines. To see this, we can easily calculate their slopes by differentiating the full-employment conditions (1.8). To compute the slope of the Rybczynski line for ΔL , it is convenient to work with the full-employment condition for *capital*, since that endowment does not change. Total differentiating (1.8) for capital gives

$$a_{1K}y_1 + a_{2K}y_2 = K \Rightarrow a_{1K}dy_1 + a_{2K}dy_2 = 0 \Rightarrow \frac{dy_2}{dy_1} = -\frac{a_{1K}}{a_{2K}}. \quad (1.18)$$

Thus, the slope of the Rybczynski line for ΔL is the negative of the ratio of capital/output in the two industries, which is constant for fixed prices. This proves that the Rybczynski lines are indeed straight.

If we continue to increase the labor endowment, outputs will move downward on the Rybczynski line for ΔL in figure 1.10, until this line hits the y_1 axis. At this point the economy is fully specialized in good 1. In terms of figure 1.9, the vector of endowments (L, K) is coincident with the vector of factor requirements (a_{1L}, a_{1K}) in industry 1. For further increases in the labor endowment, the Rybczynski line for ΔL then *moves right along the y_1 axis* in figure 1.10, indicating that the economy remains specialized in good 1. This corresponds to the vector of endowments (L, K)

lying outside and below the cone of diversification in figure 1.9. With the economy fully specialized in good 1, factor prices are determined by the marginal products of labor and capital in that good, and the earlier “factor price insensitivity” lemma no longer applies.

FACTOR PRICE EQUALIZATION REVISITED

Our finding that the economy produces both goods whenever the factor endowments remain *inside* the cone of diversification allows us to investigate the FPE theorem more carefully. Let us continue to assume that there are no FIRs, but now rather than *assuming* that both goods are produced in both countries, we will instead derive this as an *outcome* from the factor endowments in each country. To do so, we engage in a thought experiment posed by Samuelson (1949) and further developed by Dixit and Norman (1980).

Initially, suppose that labor and capital are *free to move* between the two countries until their factor prices are equalized. Then all that matters for factor prices are the *world* endowments of labor and capital, and these are shown as the length of the horizontal and vertical axis in figure 1.11. The amounts of labor and capital choosing to reside at home are measured relative to the origin O , while the amounts choosing to reside in the foreign country are measured relative to the origin O^* ; suppose that this allocation is at point B . Given the world endowments, we establish equilibrium prices for goods and factors in this “integrated world equilibrium.” The factor prices determine the demand for labor and capital in each industry (assuming no FIR), and using

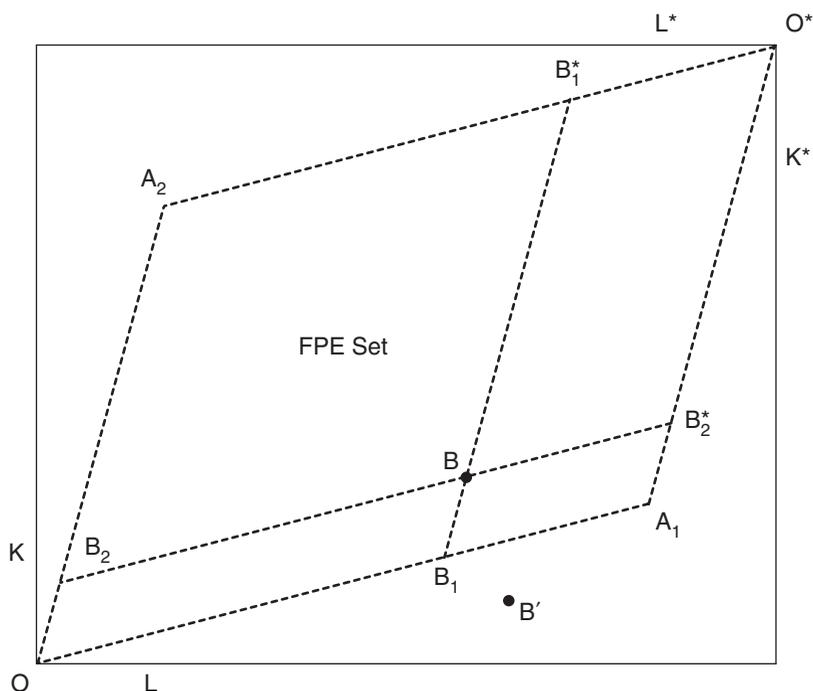


Figure 1.11

these, we can construct the diversification cone (since factor prices are the same across countries, then the diversification cone is also the same). Let us plot the diversification cone relative to the home origin O , and again relative to the foreign origin O^* . These cones form the parallelogram $OA_1O^*A_2$.

For later purposes, it is useful to identify precisely the points A_1 and A_2 on the vertices of this parallelogram. The vectors OA_i and O^*A_i are proportional to (a_{iL}, a_{iK}) , the amount of labor and capital used to produce one unit of good i in each country. Multiplying (a_{iL}, a_{iK}) by world demand for good i , D_i^w , we then obtain the *total* labor and capital used to produce that good, so that $A_i = (a_{iL}, a_{iK})D_i^w$. Summing these gives the total labor and capital used in world demand, which equals the labor and capital used in world production, or world endowments.

Now we ask whether we can achieve exactly the same world production and equilibrium prices as in this “integrated world equilibrium,” but *without* labor and capital mobility. Suppose there is some allocation of labor and capital endowments across the countries, such as point B . Then can we produce the same amount of each good as in the “integrated world equilibrium”? The answer is clearly yes: with labor and capital in each country at point B , we could devote OB_1 of resources to good 1 and OB_2 to good 2 at home, while devoting $O^*B_1^*$ to good 1 and $O^*B_2^*$ toward good 2 abroad. This will ensure that the same amount of labor and capital worldwide is devoted to each good as in the “integrated world equilibrium,” so that production and equilibrium prices must be the same as before. Thus, we have achieved the same equilibrium but without factor mobility. It will become clear in the next chapter that there is still *trade in goods* going on to satisfy the demands in each country.

More generally, for *any allocation* of labor and capital within the parallelogram $OA_1O^*A_2$ both countries remain diversified (producing both goods), and we can achieve the same equilibrium prices as in the “integrated world economy.” It follows that factor prices *remain equalized across countries* for allocations of labor and capital within the parallelogram $OA_1O^*A_2$, which is referred to as the *factor price equalization (FPE) set*. The FPE set illustrates the range of labor and capital endowments between countries over which both goods are produced in both countries, so that factor price equalization is obtained. In contrast, for endowments *outside* of the FPE set such as point B' , then at least one country would have to be fully specialized in one good and FPE no longer holds.

FACTOR INTENSITY REVERSALS

We conclude this chapter by returning to a question raised earlier: when there are “factor intensity reversals” giving multiple solutions to the zero-profit conditions, how do we know which solution will prevail in each country? To answer this, it is necessary to combine the zero-profit with the full-employment conditions, as follows.

Consider the case in figure 1.6, where the zero-profit conditions allows for two solutions to the factor prices. Each of these determine the labor and capital demands shown orthogonal to the iso-cost curves, labeled as (a_{1L}, a_{1K}) and (a_{2L}, a_{2K}) , and (b_{1L}, b_{1K}) and (b_{2L}, b_{2K}) . We have redrawn these in figure 1.12, after multiplying each of them by the outputs of their respective industries. These vectors create *two* cones of diversification, labeled as cones A and B . Initially, suppose that the factor endowments for each country lie within one cone or the other (then we will consider the case where the endowments are outside both cones).

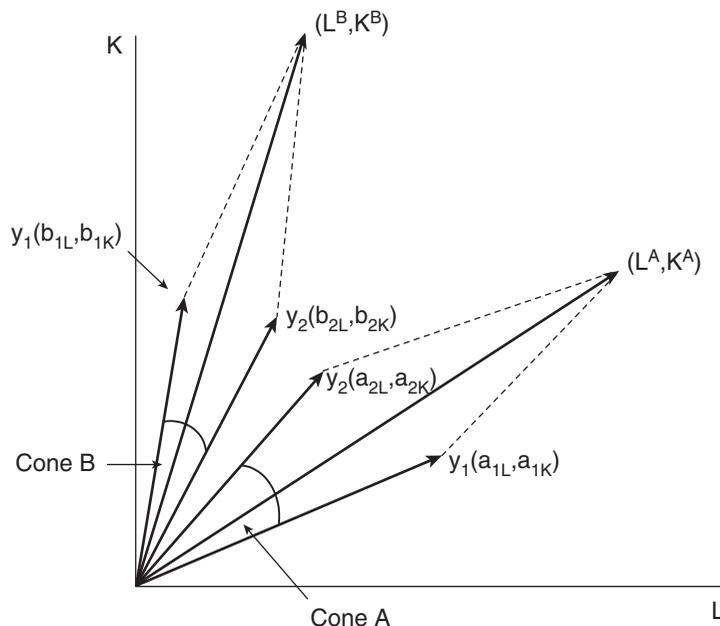


Figure 1.12

Now we can answer the question of which factor prices will apply in each country: a *labor-abundant* economy, with a high ratio of labor/capital endowments, such as (L^A, K^A) in cone A of figure 1.12, will have factor prices given by (w^A, r^A) in figure 1.6, with low wages; whereas a *capital-abundant* economy with a high ratio of capital/labor endowments such as shown by (L^B, K^B) in cone B of figure 1.12, will have factor prices given by (w^B, r^B) in figure 1.6, with high wages. Thus, factor prices depend on the endowments of the economy. A labor-abundant country such as China will pay low wages and a high rental (as in cone A), while a capital-abundant country such as the United States will have high wages and a low rental (as in cone B). Notice that we have now reintroduced a link between factor endowments and factor prices, as we argued earlier in the one-sector model: when there are FIR in the two-by-two model, factor prices vary systematically with endowments *across* the cones of diversification, even though factor prices are independent of endowments *within* each cone.

What if the endowment vector of a country does not lie in either cone? Then the country will be fully specialized in one good or the other. Generally, we can determine which good it is by tracing through how the outputs change as we move through the cones of diversification, and it turns out that outputs depend *non-monotonically* on the factor endowments.¹³ For example, textiles in South Korea or Taiwan expanded during the 1960s and 1970s, but contracted later as capital continued to grow. Despite the complexity involved, many trade economists feel that countries do in fact produce in different cones of diversification, and taking this possibility into account is a topic of research.¹⁴

¹³ See problem 1.5.

¹⁴ Empirical evidence on whether developed countries fit into the same cone is presented by Debaere and Demiroğlu (2003), and the presence of multiple cones is explored by Leamer (1987), Harrigan and Zakrajšek (2000), and Schott (2003).

CONCLUSIONS

In this chapter we have reviewed several two-sector models: the Ricardian model, with just one factor, and the two-by-two model, with two factors, both of which are fully mobile between industries. There are other two-sector models, of course: if we add a third factor, treating capital as specific to each sector but labor as mobile, then we obtain the Ricardo-Viner or “specific-factors” model, as will be discussed in chapter 3. We will have an opportunity to make use of the two-by-two model throughout this book, and a thorough understanding of its properties—both the equations and the diagrams *labor-abundant* economy—is essential for all the material that follows.

One special feature of this chapter is the dual determination of factor prices, using the unit-cost function in the two industries. This follows the dual approach of Woodland (1977, 1982), Mussa (1979), and Dixit and Norman (1980). Samuelson (1949) uses a quite different diagrammatic approach to prove the FPE theorem. Another method that is quite commonly used is the so-called Lerner (1952) diagram, which relies on the production rather than cost functions.¹⁵ We will not use the Lerner diagram in this book, but it will be useful to understand some articles, for example, Findlay and Grubert (1959) and Deardorff (1979), so we include a discussion of it in the appendix to this chapter.

This is the only chapter where we do not present any accompanying empirical evidence. The reader should not infer from this that the two-by-two model is unrealistic: while it is usually necessary to add more goods or factors to this model before confronting it with data, the relationships between prices, outputs, and endowments that we have identified in this chapter will carry over in some form to more general settings. Evidence on the pattern of trade is presented in the next chapter, where we extend the two-by-two model by adding another country, and then many countries, trading with each other. We also allow for many goods and factors, but for the most part restrict attention to situations where factor price equalization holds. In chapter 3, we examine the case of many goods and factors in greater detail, to determine whether the Stolper-Samuelson and Rybczynski theorems generalize and also how to estimate these effects. In chapter 4, evidence on the relationship between product prices and wages is examined in detail, using a model that allows for trade in intermediate inputs. The reader is already well prepared for the chapters that follow, based on the tools and intuition we have developed from the two-by-two model. Before moving on, you are encouraged to complete the problems at the end of this chapter.

APPENDIX: THE LERNER DIAGRAM AND FACTOR PRICES

The Lerner (1952) diagram for the two-by-two model can be explained as follows: With perfect competition and constant returns to scale, we have that revenue = costs in both industries. So let us choose a special isoquant in each industry such that revenue = 1. In each industry, we therefore choose the isoquant $p_i y_i = 1$, or

$$Y = f_i(L_i, K_i) = 1/p_i \Rightarrow wL_i + rK_i = 1.$$

¹⁵This diagram was used in a seminar presented by Abba Lerner at the London School of Economics in 1933, but not published until 1952. The history of this diagram is described at the “Origins of Terms in International Economics,” maintained by Alan Deardorff at <http://www-personal.umich.edu/~alandear/glossary/orig.html>. See also Samuelson (1949, 181 n.1).

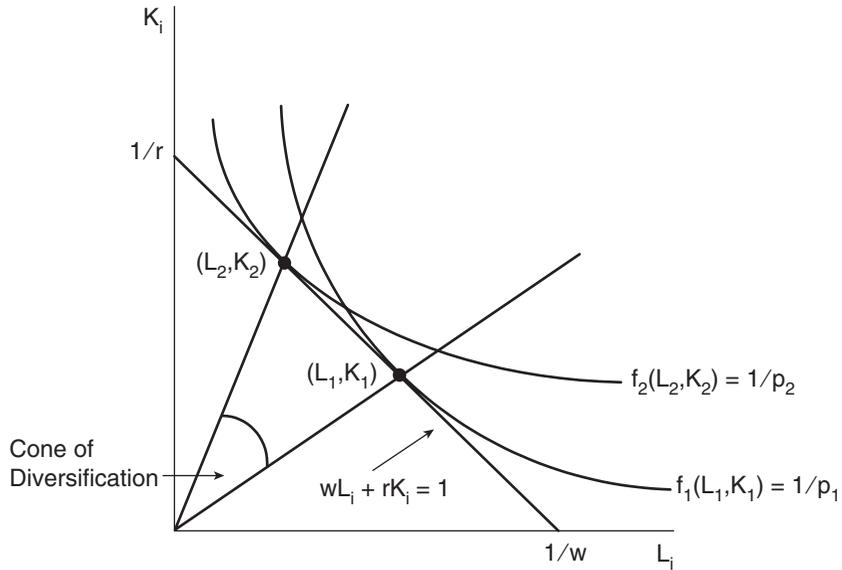


Figure 1.13

Therefore, from cost minimization, the $1/p_i$ isoquant in each industry will be *tangent* to the line $wL_i + rK_i = 1$. This is the same line for both industries, as shown in figure 1.13.

Drawing the rays from the origin through the points of tangency, we obtain the cone of diversification, as labeled in figure 1.13. Furthermore, we can determine the factor prices by computing where $wL_i + rK_i = 1$ intersects the two axis: $L_i = 0 \Rightarrow K_i = 1/r$, and $K_i = 0 \Rightarrow L_i = 1/w$. Therefore, given the prices p , we determine the two isoquants in figure 1.13, and drawing the (unique) line tangent to both of these, we determine the factor prices as the intercepts of this line. Notice that these equilibrium factor prices do not depend on the factor endowments, provided that the endowment vector lies within the cone of diversification (so that both goods are produced). We have thus obtained an alternative proof of the “factor price insensitivity” lemma, using a primal rather than dual approach. Furthermore, with two countries having the same prices (through free trade) and technologies, then figure 1.13 holds in both of them. Therefore, their factor prices will be equalized.

Lerner (1952) also showed how figure 1.13 can be extended to the case of factor intensity reversals, in which case the isoquants intersect twice. In that case there will be *two* lines $wL_i + rK_i = 1$ that are tangent to both isoquants, and there are two cones of diversification. This is shown in figure 1.14. To determine which factor prices apply in a particular country, we plot its endowments vector and note which cone of diversification it lies in: the factor prices in this country are those applying to that cone. For example, the endowments (L^A, K^A) will have the factor prices (w^A, r^A) , and the endowments (L^B, K^B) will have the factor prices (w^B, r^B) . Notice that the labor-abundant country with endowments (L^A, K^A) has the low wage and high rental, whereas the capital-abundant country with endowments (L^B, K^B) has the high wage and low rental.

How likely is it that the isoquants of industries 1 and 2 intersect twice, as in figure 1.14? Lerner (1952, 11) correctly suggested that it depends on the elasticity of substitution between labor and capital in each industry. For simplicity, suppose that each

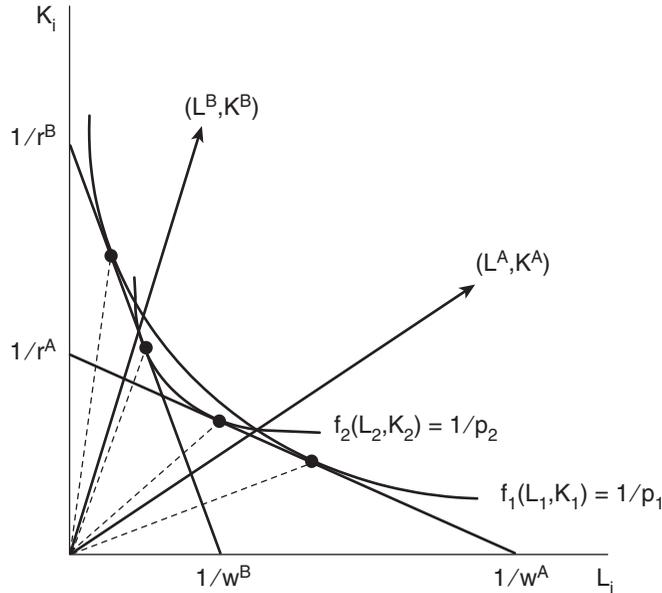


Figure 1.14

industry has a constant elasticity of substitution production function. If the elasticities are the same across industries, then it is impossible for the isoquants to intersect twice. If the elasticities of substitution differ across industries, however, and we choose prices $p_i, i = 1, 2$, such that the $1/p_i$ isoquants intersect at least once, then it is *guaranteed* that they intersect twice. Under exactly the same conditions, the iso-cost lines in figure 1.6 intersect twice. Thus, the occurrence of FIR is very likely once we allow elasticities of substitution to differ across industries. Minhas (1962) confirmed that this was the case empirically, and discussed the implications of FIR for factor prices and trade patterns. This line of empirical research was dropped thereafter, perhaps because FIR seemed too complex to deal with, and has been picked up again more recently (see note 14 of this chapter).

PROBLEMS

- 1.1 Rewrite the production function $y_1 = f_1(L_1, K_1)$ as $y_1 = f_1(v_1)$, and similarly, $y_2 = f_2(v_2)$. Concavity means that given two points $y_1^a = f_1(v_1^a)$ and $y_1^b = f_1(v_1^b)$, and $0 \leq \lambda \leq 1$, then $f_1(\lambda v_1^a + (1-\lambda)v_1^b) \geq \lambda y_1^a + (1-\lambda)y_1^b$. Similarly for the production function $y_2 = f_2(v_2)$. Consider two points $y^a = (y_1^a, y_2^a)$ and $y^b = (y_1^b, y_2^b)$, both of which can be produced while satisfying the full-employment conditions $v_1^a + v_2^a \leq V$ and $v_1^b + v_2^b \leq V$, where V represents the endowments. Consider a production point midway between these, $\lambda y^a + (1-\lambda)y^b$. Then use the concavity of the production functions to show that this point can *also* be produced while satisfying the full-employment conditions. This proves that the production possibilities set is *convex*. (Hint: Rather than showing that $\lambda y^a + (1-\lambda)y^b$ can be produced while satisfying the full-employment

conditions, consider instead allocating $\lambda v_1^a + (1 - \lambda) v_1^b$ of the resources to industry 1, and $\lambda v_2^a + (1 - \lambda) v_2^b$ of the resources to industry 2.)

- 1.2 Any function $y = f(v)$ is homogeneous of degree α if for all $\lambda > 0$, $f(\lambda v) = \lambda^\alpha f(v)$. Consider the production function $y = f(L, K)$, which we assume is homogeneous of degree one, so that $f(\lambda L, \lambda K) = \lambda f(L, K)$. Now differentiate this expression with respect to L , and answer the following: Is the marginal product $f_L(L, K)$ homogeneous, and of what degree? Use the expression you have obtained to show that $f_L(L/K, 1) = f_L(L, K)$.
- 1.3 Consider the problem of maximizing $y_1 = f_1(L_1, K_1)$, subject to the full-employment conditions $L_1 + L_2 \leq L$ and $K_1 + K_2 \leq K$, and the constraint $y_2 = f_2(L_2, K_2)$. Set this up as a Lagrangian, and obtain the first-order conditions. Then use the Lagrangian to solve for dy_1/dy_2 , which is the slope of the production possibilities frontier. How is this slope related to the marginal product of labor and capital?
- 1.4 Consider the problem of maximizing $p_1 f_1(L_1, K_1) + p_2 f_2(L_2, K_2)$, subject to the full-employment constraints $L_1 + L_2 \leq L$ and $K_1 + K_2 \leq K$. Call the result the GDP function $G(p, L, K)$, where $p = (p_1, p_2)$ is the price vector. Then answer the following:
 - (a) What is $\partial G / \partial p_i$? (Hint: we solved for this in the chapter.)
 - (b) Give an economic interpretation to $\partial G / \partial L$ and $\partial G / \partial K$.
 - (c) Give an economic interpretation to $\partial^2 G / \partial p_i \partial L = \partial^2 G / \partial L \partial p_i$, and $\partial^2 G / \partial p_i \partial K = \partial^2 G / \partial K \partial p_i$.
- 1.5 Trace through changes in outputs when there are factor intensity reversals. That is, construct a graph with the capital endowment on the horizontal axis, and the output of goods 1 and 2 on the vertical axis. Starting at a point of diversification (where both goods are produced) in cone A of figure 1.12, draw the changes in output of goods 1 and 2 as the capital endowment grows outside of cone A, into cone B, and beyond this.