One

Introduction

Over the past three decades, an animated conversation between experiment and theory has brought us to a new and radically simple conception of matter. Fundamental particles called quarks and leptons make up the everyday world, and new laws of nature—in the form of theories of the strong, weak, and electromagnetic forces—govern their interactions. Quantum chromodynamics, the theory of the strong interaction among quarks, and the electroweak theory have both been abstracted from experiment, refined within the framework of local gauge symmetries, and validated to an extraordinary degree through confrontation with experiment. What we have learned suggests paths to a more complete picture of nature—perhaps a unified theory of the fundamental particles and interactions.

But the triumph of this new picture is incomplete. We are still searching for a missing piece, the agent of electroweak symmetry breaking. We also need to discover what accounts for the masses of the electron and the other leptons and quarks, without which there would be no atoms, no chemistry, no liquids or solids—no stable structures. In the standard electroweak theory, both tasks are the work of the Higgs mechanism. Moreover, we have reason to believe that the electroweak theory is imperfect and that new symmetries or new dynamical principles are required to make it fully robust.

To extend our understanding, particle physicists from around the world have launched remarkable experiments using the Large Hadron Collider in Geneva, Switzerland, a superconducting synchrotron 27 km in circumference, in which counterrotating proton beams collide at c.m. energies planned to reach 14 TeV. We do not know what the new wave of exploration will find, but the discoveries we make and the new puzzles we encounter are certain to change the face of particle physics and echo through neighboring sciences. Decisive results on the agent of electroweak symmetry breaking appear imminent.

This book is devoted to an exposition of the logic, structure, and phenomenology of our "standard model" of particle physics, from the experimental systematics and theoretical constructs that underlie it to the highly successful form that joins quantum chromodynamics to the electroweak theory. We shall see how the idea of gauge theories—interactions derived from symmetries observed in nature—makes it possible to capture the regularities embodied in earlier theoretical descriptions in an economical and richly predictive framework that gives new understanding and suggests new consequences. The standard model displays in full measure the attributes of an exemplary theory expressed in Heinrich Hertz's celebration of classical electrodynamics:

> One cannot study Maxwell's marvelous electromagnetic theory of light without sometimes having the feeling that these mathematical formulae have an independent existence and an intelligence of their own, that they are wiser than we are, wiser even than their inventor, that they give back to us more than was originally put into them [1].

The utility of the quark model as a classification tool that provides a systematic basis for hadron spectroscopy has long been appreciated. The quark language also provides an apt description of the dynamics of hadronic interactions. The quark-parton model, refined by quantum chromodynamics, underlies a quantitative phenomenology of deeply inelastic lepton–hadron scattering, electron–positron annihilation into hadrons, hard scattering of hadrons, and decays of hadrons, especially those containing heavy quarks.

An elementary particle, in the time-honored sense of the term, is structureless and indivisible. Although history cautions that the physicist's list of elementary particles is dependent upon experimental resolution—and thus subject to revision with the passage of time—it has also rewarded the hope that interactions among the elementary particles of the moment would be simpler and more fundamental than those among composite systems. Neither quarks nor leptons exhibit any structure on a scale of about 10^{-16} cm, the currently attained resolution. We thus have no experimental reason but tradition to suspect that they are not the ultimate elementary particles. Accordingly, we idealize the quarks and leptons as pointlike particles, remembering that elementarity is subject to experimental test.

Analyses of collision phenomena suggest that quarks behave as free particles within hadrons, and yet the nonobservation of isolated free quarks encourages the idealization that quarks must be permanently confined within the hadrons. This apparently paradoxical state of affairs requires that the strong interaction among quarks be of a rather particular sort. No rigorous theoretical demonstration of the confinement hypothesis has yet been given, but it is widely held that quantum chromodynamics (QCD) contains the necessary elements. In common with other non-Abelian gauge theories, QCD exhibits an effective interaction strength that decreases at short distances and grows at large distances. This property—asymptotic (ultraviolet) freedom vs. infrared slavery—suggests a resolution of the parton-model conundrum. Monte Carlo simulations of the gauge-theory vacuum provide strong numerical evidence for quark confinement.

The appeal of a unified theory of the weak and electromagnetic interactions is at once aesthetic and practical. The effective weak-interaction Lagrangian that evolved from Fermi's description of nuclear β-decay and provided a serviceable low-energy phenomenology is now seen to be the limiting form of a renormalizable field theory. At the same time, neutral-current interactions predicted by the new electroweak theory have been found to occur at approximately the strength of the long-studied charged-current interactions. The observed neutral currents are neutral not only with respect to electric charge, but with respect to all other additive quantum numbers as well. To accommodate this property in the theory requires the introduction of a new quark species, bearing a new additive quantum number known as charm. This, too, has subsequently been observed in experiments. The price for this neat picture includes the prediction of several hypothetical particles: the intermediate vector bosons, W^+, W^-, and Z^0, that carry the weak interactions. Definite predictions for the masses and properties of the intermediate bosons have been confirmed by experiment.

Electromagnetism is a force of infinite range, whereas the influence of the charged-current weak interaction responsible for radioactive beta decay spans only distances shorter than about 10^{-15} cm, less than 1% of the proton radius. If these two interactions, so different in their range and apparent strength, originate in a common gauge symmetry, that symmetry must be spontaneously broken. That is to say, the vacuum state of the universe must not respect the full symmetry. How the electroweak gauge symmetry is spontaneously broken is one of the most urgent and challenging questions before particle physics. The standard-model answer is an elementary scalar field whose self-interactions select a vacuum state in which the full electroweak symmetry is hidden. Experiments in 2012 have discovered a new particle that—at first look—fits the profile of the Higgs boson, as the elementary scalar is known. We do not yet know whether this observation means that a fundamental Higgs field exists or a different agent breaks electroweak symmetry. General arguments imply that the Higgs boson or other new physics is required on the TeV energy scale. Indirect constraints from global analyses of electroweak measurements suggest that the mass of the standard-model Higgs boson is less than 200 GeV. Once its mass is assumed, the properties of the Higgs boson follow from the electroweak theory. Finding the Higgs boson or its replacement is one of the great campaigns now under way in both experimental and theoretical particle physics. The answer—expected soon—will steer the future development of the electroweak theory.

One measure of the electroweak theory's sweep is that its predictions hold over a prodigious range of distances, from about 10^{-18} m to more than 10^8 m. The origins of the theory lie in the discovery of Coulomb's law in tabletop experiments by Cavendish and Coulomb. It was stretched to longer and shorter distances by the progress of experiment. In the long-distance limit, the classical electrodynamics of a massless photon suffices. At shorter distances than the human scale, classical electrodynamics was superseded by quantum electrodynamics (QED), which is now subsumed in the electroweak theory, tested at energies up to a few hundred GeV.

Because the charged-current weak interactions are purely left-handed, it is not possible to construct a self-consistent theory of the weak and electromagnetic interactions based solely on leptons or solely on quarks. They must come in matched sets. This fact suggests a deep connection between the quarks (which experience the strong interactions) and the leptons (which do not). That observation, in turn, motivates a description that gathers both quarks and leptons into extended families—a unified theory of the strong, weak, and electromagnetic interactions. Such a theory can give an understanding of the low-energy strengths of the individual interactions. Another consequence of the unification of forces is the implication of new forces that can transform quarks into leptons.

For all its triumphs, the standard model is not entirely satisfying. The electroweak theory does not make specific predictions for the masses of the quarks and leptons or for the mixing among different flavors. It leaves unexplained how an elementary Higgs-boson mass could remain below 1 TeV in the face of quantum corrections that tend to lift it toward the Planck scale or a unification scale. The Higgs field thought to pervade all of space to hide the electroweak symmetry contributes a vacuum energy density far in excess of what is observed. And the standard model, even when extended to a unified theory of the strong, weak, and electromagnetic interactions, responds inadequately to challenges raised by astronomical observations, including the dark-matter problem and the predominance of matter over antimatter in the universe. These shortcomings argue for physics beyond the standard model.

The remainder of this chapter is a concise review of some of the primitive concepts of particle phenomenology that serve as a basis for our development of the standard model. In succeeding chapters, we shall assemble a detailed description of the electroweak theory and quantum chromodynamics, the two pillars of the standard model, with close attention to the experimental foundations. Our goal is not only to exhibit the successes of the two theories and to establish their utility for reliable calculations, but also to highlight unfinished business and point to the need for future developments. We shall also see how QCD and the electroweak theory might be joined into a unified theory of the strong, weak, and electromagnetic interactions. The text closes by posing essential questions for theory and experiment.

1.1 ELEMENTS OF THE STANDARD MODEL OF PARTICLE PHYSICS

Our picture of matter is based on the identification of a set of pointlike spin-$\frac{1}{2}$ constituents: the (up, down, charm, strange, top, and bottom) quarks,

$$\begin{pmatrix} u \\ d \end{pmatrix}_{\mathrm{L}}, \qquad \begin{pmatrix} c \\ s \end{pmatrix}_{\mathrm{L}}, \qquad \begin{pmatrix} t \\ b \end{pmatrix}_{\mathrm{L}}, \tag{1.1.1}$$

and the leptons (electron, muon, and tau, plus three neutrinos),

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_{\mathrm{L}}, \qquad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_{\mathrm{L}}, \qquad \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_{\mathrm{L}}, \tag{1.1.2}$$

where the subscript L denotes the left-handed components, plus a few fundamental forces derived from gauge symmetries. The quarks are influenced by the strong interaction and so carry *color*, the strong-interaction charge, whereas the leptons do not feel the strong interaction and are colorless. Each of the six quark flavors comes in three distinct colors: red, green, and blue. The right-handed fermions are weak-interaction singlets. By pointlike, we understand that the quarks and leptons show no evidence of internal structure at the current limit of our resolution, ($r \lesssim 10^{-18}$ m) [2].

The notion that the quarks and leptons are elementary—structureless and indivisible—is necessarily provisional. *Elementarity* is one of the aspects of our picture of matter that we test ever more stringently as we improve the resolution

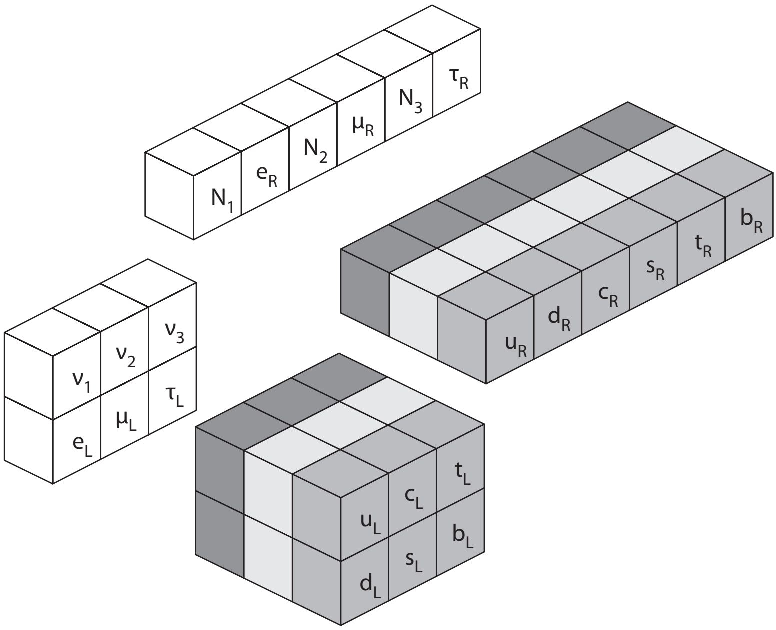


Figure 1.1. The left-handed $SU(2)_L$ doublets and right-handed $SU(2)_L$ singlets of color-triplet quarks and color-singlet leptons from which the standard model of particle physics is constructed.

with which we can examine the quarks and leptons. For the moment, the world's most powerful microscope is the Large Hadron Collider at CERN, where the ATLAS and CMS Collaborations have studied pp collisions at c.m. energy $\sqrt{s} = 8$ TeV. For the production of hadron jets at transverse energy $E_\perp$, we may roughly estimate the resolution as $r \approx (\hbar c)/E_\perp \approx 2 \times 10^{-19}$ TeV m$/E_\perp$ [3].

The left-handed and right-handed fermions behave very differently under the influence of the charged-current weak interactions. In 1956, Wu and collaborators [4] studied the β-decay $^{60}\text{Co} \to {}^{60}\text{Ni}\, e^- \bar{\nu}_e$ and observed a correlation between the direction $\hat{p}_e$ of the outgoing electron and the spin vector $\vec{J}$ of the polarized ^{60}Co nucleus. Spatial reflection, or parity, leaves the (axial vector) spin unchanged, $\mathsf{P} : \vec{J} \to \vec{J}$, but reverses the electron direction, $\mathsf{P} : \hat{p}_e \to -\hat{p}_e$. Accordingly, the correlation $\vec{J} \cdot \hat{p}_e$ is manifestly *parity violating*. Experiments in the late 1950s (cf. §6.3) established that (charged-current) weak interactions are left-handed, and motivated the construction of a manifestly parity-violating theory of the weak interactions with only a left-handed neutrino ν_L.

Perhaps our familiarity with parity violation in the weak interactions has dulled our senses a bit. It seems to me that nature's broken mirror—the distinction between left-handed and right-handed fermions—qualifies as one of the great mysteries. Even if we will not get to the bottom of this mystery next week or next year, it should be prominent in our consciousness—and among the goals we present to others as the aspirations of our science.

The family relationships among the quarks and the leptons are depicted in figure 1.1. To excellent approximation, the observed charged-current weak interactions connect up quarks with down, charm with strange, and top with bottom. At the current limits of experimental sensitivity, the charged-current interactions conserve electron number, muon number, and tau number. Although the right-handed quarks and charged leptons do not participate in charged-current weak interactions, their existence is implied by charged-fermion masses and the characteristics of the strong and electromagnetic interactions. We take the weak-isospin doublets as evidence for $\mathrm{SU}(2)_\mathrm{L}$ gauge symmetry and infer $\mathrm{SU}(3)_\mathrm{c}$ gauge symmetry from the three quark colors, interpreted as a continuous symmetry. As we shall see, the successful electroweak theory incorporates a $\mathrm{U}(1)_\mathrm{Y}$ weak-hypercharge phase symmetry, along with $\mathrm{SU}(2)_\mathrm{L}$. We have already commented that the electroweak gauge symmetry must be hidden, $\mathrm{SU}(2)_\mathrm{L} \otimes \mathrm{U}(1)_\mathrm{Y} \to \mathrm{U}(1)_\mathrm{EM}$, with the phase symmetry of electromagnetism the residual symmetry.

The discovery of neutrino oscillations (cf. §6.7), which implies that neutrinos have mass, motivates the inclusion of the right-handed neutrinos, N_i, in figure 1.1. The right-handed neutrinos would be sterile—inert with respect to the known $\mathrm{SU}(3)_\mathrm{c} \otimes \mathrm{SU}(2)_\mathrm{L} \otimes \mathrm{U}(1)_\mathrm{Y}$ interactions.

The representation of the quarks and leptons in figure 1.1 invites not only speculations about the symmetries that lead to the strong, weak, and electromagnetic interactions, but also questions about possible relations between quarks and leptons or between the left-handed and right-handed particles.

Let us now look in slightly greater detail at each of the ingredients of the standard model of particle physics.

1.2 LEPTONS

The leptons can exist as free particles and, therefore, can be studied directly. Three charged leptons—the electron, muon, and tau—are firmly established, the electron and muon by direct observation and the short-lived tau ($c\tau = 87\,\mu$m) through its decay products. The gyromagnetic ratios of the electron and muon have been measured with remarkable precision (cf. problem 1.6). They differ from the value of 2 expected for Dirac particles only by tiny fractions, which have been calculated in QED. This strongly supports their identification as point particles. The electron neutrino and muon neutrino are, likewise, well known (cf. problem 1.9). Decay characteristics of the τ-lepton imply the existence of a distinct tau neutrino, and the first examples of ν_τ interactions have been observed. On short baselines, neutrino flavor is strictly correlated with the flavor of the charged lepton to which it couples. Some properties of the known leptons are summarized in table 6.4.

The traditional names of the neutrinos follow from the structure of the charged weak current, which is represented by the weak-isospin doublets

$$\psi_1 = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_\mathrm{L}, \quad \psi_2 = \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_\mathrm{L}, \quad \psi_3 = \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_\mathrm{L}, \tag{1.2.1}$$

The leptonic charged current thus has the (vector minus axial vector) form [5]

$$J_\lambda^{(\pm)} = \sum_i \bar{\psi}_i \tau_\pm \gamma_\lambda (1 - \gamma_5) \psi_i, \tag{1.2.2}$$

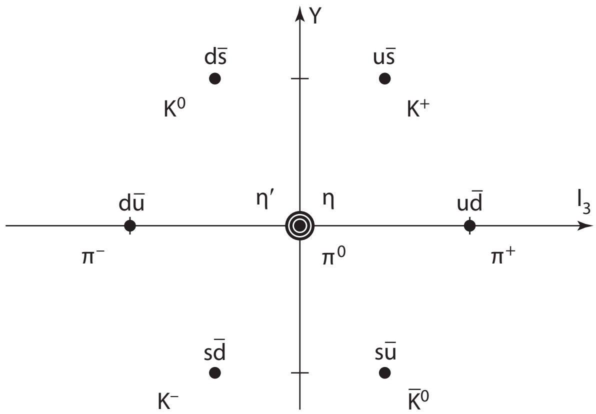


Figure 1.2. SU(3) weight diagram for the pseudoscalar nonet = **1** ⊕ **8**.

where $\tau_\pm \equiv \frac{1}{2}(\tau_1 \pm i\tau_2)$ are the Pauli isospin matrices

$$\tau_+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad \tau_- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}. \tag{1.2.3}$$

The implied universality of weak-interaction transition matrix elements holds to high accuracy for the electronic and muonic transitions and has been verified within 1.5% for the tau family. We shall see that the identification of weak-isospin doublets leads, in addition, to an understanding of the structure of the neutral weak current.

We have not penetrated the origin of the simple and orderly family pattern exhibited by the leptons. Indeed, many apparent facts and regularities are inadequately comprehended; therefore, many questions present themselves. Why are there three doublets of leptons? Might more be found? Is the difference between the number of leptons and antileptons in the universe a conserved quantity? What is the pattern of lepton masses? Is the separate conservation of an additive electron number, muon number, and tau number—apart from the effect of neutrino mixing—an exact, or only approximate, statement?

1.3 QUARKS

Quarks are the fundamental constituents of the strongly interacting particles, the hadrons. They experience all the known interactions: strong, weak and electromagnetic, and gravitational. Quarks have much in common with the leptons, but there is a crucial distinction. Free quarks have never been observed, so it has been necessary to adduce indirect evidence for their existence and their properties.

Gell-Mann and Zweig proposed quarks in 1964 by as a means for understanding the SU(3) classification of the hadrons known to them [6]. The light mesons occur only in SU(3) singlets and octets. For example, the nine pseudoscalars are shown in the familiar (strangeness versus third component of isospin) hexagonal array in figure 1.2. Similarly, the light baryons are restricted to singlets, octets, and decimets of SU(3). The lowest-lying baryons are displayed in figures 1.3 and 1.4. The observation that no higher representations are indicated is far more restrictive

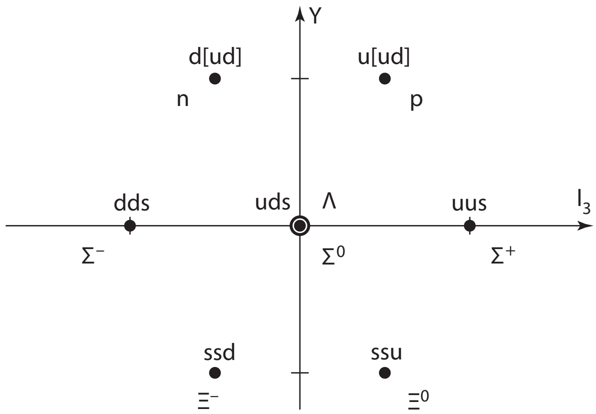


Figure 1.3. The $J^P = \frac{1}{2}^+$ baryon octet.

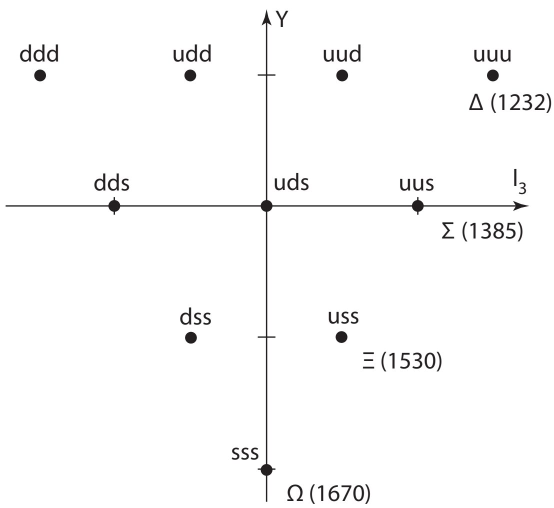


Figure 1.4. The $J^P = \frac{3}{2}^+$ baryon decimet.

than the mere fact that SU(3) is a good classification symmetry and requires explanation.

The observed patterns can be understood in terms of the hypothesis that hadrons are composite structures built up from an elementary triplet of spin-$\frac{1}{2}$ quarks, corresponding to the fundamental representation of SU(3). The three "flavors" of quarks, commonly named *u*p, *d*own, and *s*trange, have the properties summarized in table 1.1, where I is (strong) isospin, S is strangeness, B is baryon number, Y is (strong) hypercharge, and Q is electric charge. The quark masses are indicated in figure 9.6. A meson composed of a quark and antiquark ($q\bar{q}$) then lies in the SU(3) representations

$$3 \otimes 3^* = \mathbf{1} \oplus \mathbf{8}, \tag{1.3.1}$$

TABLE 1.1
Properties of the Light Quarks

Quark	I	I_3	S	B	$Y = B + S$	Q
u	$\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{2}{3}$
d	$\frac{1}{2}$	$-\frac{1}{2}$	0	$\frac{1}{3}$	$\frac{1}{3}$	$-\frac{1}{3}$
s	0	0	-1	$\frac{1}{3}$	$-\frac{2}{3}$	$-\frac{1}{3}$

TABLE 1.2
Some Properties of the Heavy Quarks

Quark	I	Q	Charm	Beauty	Truth	Mass (GeV)
c	0	$\frac{2}{3}$	1	0	0	~ 1.3
b	0	$-\frac{1}{3}$	0	-1	0	~ 4.2
t	0	$\frac{2}{3}$	0	0	1	173

and a baryon, composed of three quarks (qqq), must be contained in

$$\mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} = \mathbf{1} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{10}. \tag{1.3.2}$$

The quark content of the hadrons is indicated in figures 1.2–1.4. This simple model reproduces the representations seen prominently in experiments. It remains, of course, to understand why only these combinations of quarks and antiquarks are observed or to discover the circumstances under which more-complicated configurations (such as $q\bar{q}q\bar{q}$ or $qqqq\bar{q}$ or $6q$) might arise.

The existence of three flavors of quarks has thus been inferred from the quantum numbers of the light hadrons. Let us check the consistency of the properties attributed to the quarks.

If baryons, which are fermions, are to be made of three identical constituents, the constituents must themselves be fermions. The observed hadron spectrum corresponds to objects that can be formed as $(q\bar{q})$ or (qqq) composites of spin-$\frac{1}{2}$ quarks. It is straightforward to work out the level structure in the meson sector:

$$(q\bar{q}) \rightarrow J^{PC} = \underbrace{0^{-+}, 1^{--}}_{L=0};\ \underbrace{0^{++}, 1^{++}, 1^{+-}, 2^{++}}_{L=1};\ \cdots \tag{1.3.3}$$

This conforms closely to the observed ordering of light-meson levels. Moving to the present, we now have identified three additional heavy quarks (*c*harm, *b*ottom, and *t*op), with properties summarized in table 1.2. The expected order of levels is clearly shown in the spectrum of bound states of a charmed quark and antiquark depicted in figure 1.5 [7]. Combinations of spin, parity, and charge conjugation such as $J^{PC} = 0^{--}, 0^{+-}, 1^{-+}$, which cannot be formed from pairs of spin-$\frac{1}{2}$ quarks and antiquarks, have not been observed. The analysis of baryon multiplets is similar but

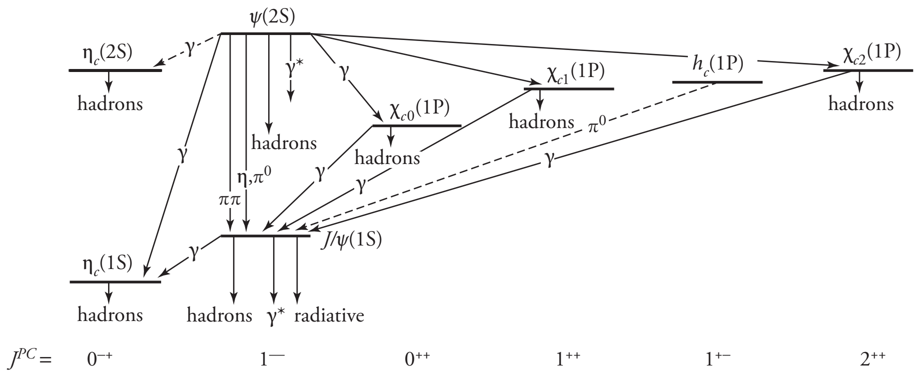


Figure 1.5. Spectrum of charm-anticharm bound states below the threshold for dissociation into charmed-particle pairs. (Adapted from Ref. [2].)

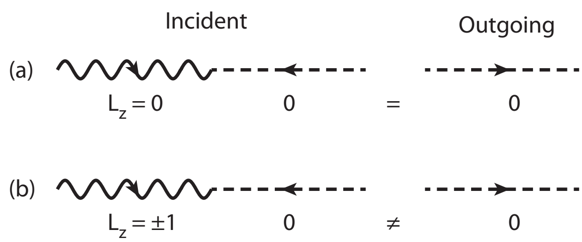


Figure 1.6. Photoabsorption by a scalar particle in the reference frame in which the three-momentum of the outgoing scalar is minus the incoming momentum. (a) Absorption of a longitudinal photon by a spinless particle is allowed by angular momentum conservation. (b) Absorption of a transverse photon is forbidden.

more tedious. Again, the observed spectrum is in agreement with the quark-model pattern.

In addition to this successful classification scheme, there are dynamical tests of the quark spin. Consider the cross sections for absorption of longitudinal or transverse virtual photons on point particles. In the Breit frame of the struck particle, illustrated in figure 1.6, it is easy to see that a spinless "quark" can absorb only a longitudinal (helicity $= 0$) photon, because angular momentum conservation forbids the absorption of a transverse (helicity $= \pm 1$) photon. Similarly (cf. problem 1.3), a spin-$\frac{1}{2}$ quark can absorb a transverse (helicity $= \pm 1$) photon but not a longitudinal photon. Within the parton model (cf. §7.4), deeply inelastic scattering of electrons from nucleon targets is analyzed as the scattering of electrons from noninteracting and structureless charged constituents. The relative size of the cross sections for absorption of longitudinal and transverse photons is a diagnostic for the spin of the charged constituents.

A related test is provided by the angular distribution of hadron jets in electron–positron annihilations, which is identical to the production angular distribution of muons in the reaction

$$e^+e^- \to \mu^+\mu^-. \tag{1.3.4}$$

That hadrons are emitted in well-collimated jets (cf. problem 1.5) supports the interpretation of particle production by means of the elementary process

$$e^+e^- \to q\bar{q}, \tag{1.3.5}$$

followed by the hadronization of the noninteracting quarks. The jet angular distribution then reflects the angular distribution of the spin-$\frac{1}{2}$ quarks.

Quarks have baryon number $\frac{1}{3}$; antiquarks have baryon number $-\frac{1}{3}$. This follows from the assertion that three quarks make up a baryon. The quarks also carry fractional electric charge [8]. The Gell-Mann–Nishijima formula [9] for displaced charge multiplets,

$$Q = I_3 + \tfrac{1}{2}Y = I_3 + \tfrac{1}{2}(B + S), \tag{1.3.6}$$

implies the charge assignments shown in table 1.1. The same assignments follow directly from examination of members of the baryon decimet:

$$\begin{aligned} \Delta^{++} &= uuu, \\ \Delta^{+} &= uud, \\ \Delta^{0} &= udd, \\ \Delta^{-} &= ddd, \end{aligned} \qquad \Omega^- = sss. \tag{1.3.7}$$

The characteristics of quarks that we have discussed until now—spin, flavor, baryon number, electric charge—are directly indicated by the experimental observations that originally motivated the quark model. Quarks have still another property, less obvious but of central importance for the strong interactions. This additional property is known as *color*. At first sight, the Pauli principle seems not to be respected by the wave function for the Δ^{++}. This nucleon resonance is a (uuu) state with spin $= \frac{3}{2}$ and isospin $= \frac{3}{2}$, in which all the quark pairs are in relative s-waves. Thus, it is apparently a symmetric state of three identical fermions. Unless we are prepared to suspend the Pauli principle or to forgo the quark model, it is necessary [10] to invoke a new, hidden degree of freedom, which permits the Δ^{++} wave function to be antisymmetrized. In order that a (uuu) wave function can be antisymmetrized, each quark flavor must exist in no fewer than three distinguishable types, called colors. More than three colors would raise the unpleasant possibility of distinguishable (colored) species of protons, which is contrary to common experience. Thus motivated, the introduction of color may appear arbitrary and artificial—a "desperate remedy." However, a number of observables are sensitive to the number of distinct species of quarks, and subsequent measurements of these quantities have given strong support to the color hypothesis.

The inclusive cross section for electron–positron annihilation into hadrons is described, as earlier noted, by the elementary process $e^+e^- \to q\bar{q}$, where the quark and antiquark materialize with unit probability into the observed hadron jets. At a particular collision energy, the ratio

$$R \equiv \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} \tag{1.3.8}$$

is then simply given as

$$R = \sum_{\substack{\text{quark}\\ \text{species}}} e_q^2. \tag{1.3.9}$$

At barycentric energies between approximately 1.5 and 3.6 GeV, pairs of up, down, and strange quarks are kinematically accessible. In the absence of hadronic color, we would, therefore, expect a mean level

$$R_1 = e_u^2 + e_d^2 + e_s^2 = \tfrac{2}{3}, \tag{1.3.10}$$

but if each quark flavor exists in three colors, we should have

$$R_3 = 3(e_u^2 + e_d^2 + e_s^2) = 2. \tag{1.3.11}$$

Experiment decisively favors the color-triplet hypothesis, as shown in the top panel of figure 1.7. At still higher energies the heavier c- and b-quarks may be produced in the semifinal state. Between the $c\bar{c}$ and $b\bar{b}$ thresholds, the color-triplet model thus predicts

$$R = 3(e_u^2 + e_d^2 + e_s^2 + e_c^2) = \tfrac{10}{3}, \tag{1.3.12}$$

to be compared with data in the middle panel of figure 1.7. Above the $b\bar{b}$ threshold, we expect

$$R = 3(e_u^2 + e_d^2 + e_s^2 + e_c^2 + e_b^2) = \tfrac{11}{3}, \tag{1.3.13}$$

which agrees well with the data shown in the lower panel of figure 1.7. (cf. the extension to higher energies shown in figure 8.25).

A similar count of the number of distinguishable quarks of each flavor is provided by the decay branching ratios of the tau lepton. Within the quark model, decays may be described as shown in figure 1.8, namely, by the decay of τ into ν_τ plus a virtual intermediate boson W^-. The intermediate boson may then disintegrate into all kinematically accessible fermion–antifermion pairs: $(e^-\bar{\nu}_e)$, $(\mu^-\bar{\nu}_\mu)$, $(u\bar{d})$. (See the following section and §7.1 for the refinement of Cabibbo mixing, which does not alter the logic of this argument.) The universality of the charged-current weak interactions implies equal rates for each of these decays. Therefore, in the absence of color, we expect

$$B_1 = \frac{\Gamma(\tau \to e^-\bar{\nu}_e\nu_\tau)}{\Gamma(\tau \to \text{all})} = \frac{1}{3}. \tag{1.3.14}$$

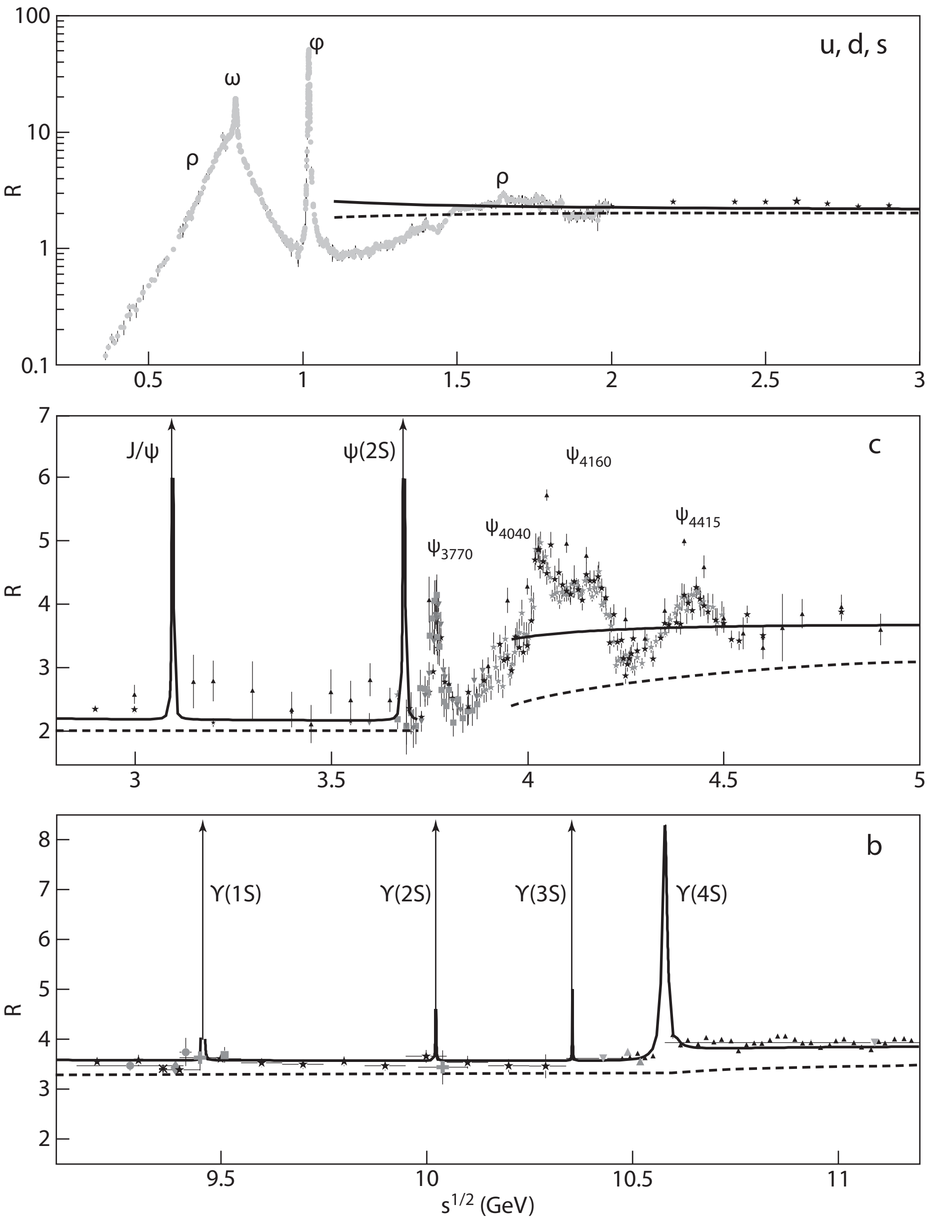


Figure 1.7. The ratio $R \equiv \sigma(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)$ in the light-flavor, charm-threshold, and beauty-threshold regions. The dashed lines represent the parton-model expectations given by (1.3.11), (1.3.12), and (1.3.13). The solid curves show the influence of perturbative QCD corrections. (Adapted from Ref. [2].)

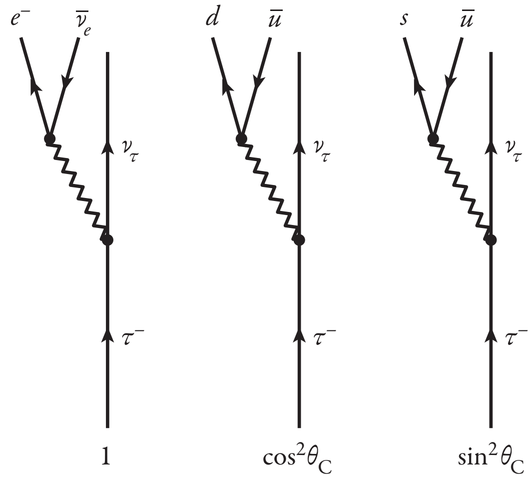


Figure 1.8. Leptonic decays (weight 1) and Cabibbo-favored (weight $\cos^2\theta_C$) and Cabibbo-suppressed (weight $\sin^2\theta_C$) semileptonic decays of the tau lepton.

If the quarks are color triplets, $(u\bar{d})$ is increased to $3(u\bar{d})$, and the leptonic branching ratio becomes [11]

$$B_3 = \tfrac{1}{5}. \tag{1.3.15}$$

The experimentally measured branching ratio,

$$B_{\text{exp}} = (17.44 \pm 0.85)\%, \tag{1.3.16}$$

is in accord with the color hypothesis.

The other important measure of the number of quark colors is the $\pi^0 \to \gamma\gamma$ decay rate [12]. A calculation for π^0-decay via a quark–antiquark loop is straightforward to carry out, although the required justification is somewhat subtle. The result is that

$$\Gamma(\pi^0 \to \gamma\gamma) = \left(\frac{\alpha}{2\pi}\right)^2 [N_c(e_u^2 - e_d^2)]^2 \frac{M_\pi^3}{16\pi f_\pi^2}, \tag{1.3.17}$$

where N_c is the number of colors and $f_\pi \approx 92$ MeV is the pion decay constant, determined from the charged pion lifetime. The predicted rate is then

$$\Gamma(\pi^0 \to \gamma\gamma) = \begin{cases} 0.86 \text{ eV}, & N_c = 1, \\ 7.75 \text{ eV}, & N_c = 3, \end{cases} \tag{1.3.18}$$

to be compared with the measured rate of (7.74 ± 0.37) eV.

From all these experimental indications and from a further theoretical argument to be given in section 6.8, we may conclude that the hidden color degree of freedom is indeed present. It is then tempting to identify color as the property that distinguishes quarks from leptons and might play the role of a strong-interaction charge. This insight will eventually lead to the gauge theory of strong interactions, quantum chromodynamics.

The observation of hadrons with various internal quantum numbers, such as isospin, strangeness, and charm, has led to the application of flavor symmetries SU(2), SU(3), ..., to the strong interactions. These internal symmetry groups serve both for the classification of hadrons (whence the inspiration for the quark model) and for dynamical relations among strong-interaction amplitudes. Both isospin and SU(3) are excellent, but not exact, strong-interaction symmetries. Isospin invariance holds within a few percent, and flavor SU(3) is respected at the 10% to 20% level. All the same, the outcome of the preceding discussion has been to minimize the direct importance of flavors in the strong interactions and to emphasize the significance of color. Reasons for dismissing a strong-interaction theory based on flavor symmetry will be developed in chapter 4, but the problem of accounting for the existence and the breaking of the flavor symmetries will remain. According to an evolving view, the breaking of SU(N) flavor symmetry is a consequence of the quark mass differences

$$m_u < m_d < m_s < m_c < m_b < \cdots , \tag{1.3.19}$$

which themselves follow, in a manner not fully understood, from the spontaneous symmetry breaking of the weak and electromagnetic interactions. The goodness of flavor SU(3) symmetry is then owed to the smallness and near degeneracy of the up, down, and strange quark masses.

This line of thought has not yet led to a complete understanding of the strong-interaction symmetries. It is ironic that isospin invariance, the oldest and most exact of strong-interaction symmetries, should seem merely coincidental. We do not know why the pattern of quark masses and mixing angles should be as it is or even why so many "fundamental" fermions should exist. Within the strong interactions, flavor appears not to have any essential dynamical role, its only role being to contribute a richness.

1.4 THE FUNDAMENTAL INTERACTIONS

The elementary interactions of the quarks and leptons can be understood as consequences of gauge symmetries. The notions of local gauge invariance and gauge theories and the construction and application of specific theories will be developed in logical sequence in succeeding chapters. As prologue, let us merely recall here some superficial aspects of the familiar gauge theories and their practical consequences.

The gauge theory known as quantum chromodynamics, in which colored quarks interact by means of massless colored gauge bosons named *gluons*, is now established as a comprehensive theory of the strong interactions among quarks. The three quark colors are regarded as the basis of the color-symmetry group $SU(3)_c$, distinct from the flavor SU(3) group that relates up, down, and strange quarks. An $SU(3)_c$ octet of vector gluons mediate the interactions among all colored objects, including the quarks and the gluons themselves.

It will be found appealing to argue that only color-singlet objects may exist in isolation. Color confinement, as it is called, would then imply quark confinement, which would, in turn, explain the fact that free quarks have never been observed: free quarks and gluons simply could not exist. We will show in chapter 8 that the coupling "constant" of the color interaction decreases at short distances and increases at long distances. This property of QCD, which is called asymptotic freedom, helps us understand why permanently confined quarks behave within hadrons as if they are free particles.

The experimental evidence for the existence of the color gauge bosons, the gluons, is multifaceted. We note two examples here, with more evidence to follow in chapter 8. First, energy-momentum sum rules in lepton–nucleon scattering indicate that the partons that interact electromagnetically or weakly, namely, the quarks, carry only about half the momentum of a nucleon. Something else, electrically neutral and inert with respect to the weak interactions, must carry the remainder. This is a role for which the gluons are ideally suited. Second, at barycentric energies exceeding about 17 GeV, a fraction of hadronic events produced in electron–positron annihilations display a three-jet structure instead of the familiar two-jet ($e^+e^- \to q\bar{q}$) structure [13]. We interpret the three-jet structure as evidence for the process

$$e^+e^- \to q\bar{q} + \text{gluon}, \tag{1.4.1}$$

in which the gluon is radiated from the outgoing quark in a hadronic analogue of electromagnetic bremsstrahlung. This interpretation survives further scrutiny.

The electroweak theory is built upon the weak-isospin symmetry suggested by the $\mathrm{SU}(2)_\mathrm{L}$ doublets abstracted from the systematics of radioactive β-decay and other charged-current interactions, plus a $\mathrm{U}(1)_\mathrm{Y}$ phase symmetry associated with weak hypercharge. A key element is that the $\mathrm{SU}(2)_\mathrm{L} \otimes \mathrm{U}(1)_\mathrm{Y}$ gauge symmetry must be spontaneously broken to $\mathrm{U}(1)_\mathrm{EM}$, the phase symmetry that underlies quantum electrodynamics. We develop the theory and a plausible mechanism for spontaneous symmetry breaking in chapters 6 and 7. It will be useful here to anticipate some elementary features of the charged-current and neutral-current interactions [14]. We will see that, whereas flavor is incidental to the strong interactions, it has an intrinsic importance to the weak interactions.

Consider first a world populated by electron and muon doublets. The electromagnetic current is given by

$$J_\lambda^{(\mathrm{EM})} = -\bar{e}\gamma_\lambda e - \bar{\mu}\gamma_\lambda \mu, \tag{1.4.2}$$

which evidently leaves all additive quantum numbers unchanged. The charged weak current is indicated by the weak-isospin doublets (1.2.1) as

$$J_\lambda^{(+)} = \bar{\nu}_e \gamma_\lambda (1 - \gamma_5) e + \bar{\nu}_\mu \gamma_\lambda (1 - \gamma_5) \mu. \tag{1.4.3}$$

If the idea of weak-isospin symmetry is to be taken seriously, these long-studied leptonic charged currents must be supplemented by another current that completes the weak isovector:

$$\begin{aligned} J_\lambda^{(3)} &= \tfrac{1}{2}\sum_i \bar{\psi}_i \tau_3 \gamma_\lambda (1-\gamma_5)\psi_i \\ &= \tfrac{1}{2}[\bar{\nu}_e \gamma_\lambda (1-\gamma_5)\nu_e - \bar{e}\gamma_\lambda(1-\gamma_5)e \\ &\quad + \bar{\nu}_\mu \gamma_\lambda (1-\gamma_5)\nu_\mu - \bar{\mu}\gamma_\lambda(1-\gamma_5)\mu]. \end{aligned} \tag{1.4.4}$$

Like the electromagnetic current, the third component of weak isospin leaves additive quantum numbers unchanged. The weak neutral current that emerges after spontaneous symmetry breaking may be expressed as the linear combination

$$J_\lambda^{(0)} = J_\lambda^{(3)} - 2\sin^2\theta_W J_\lambda^{(EM)}, \tag{1.4.5}$$

with the relative weights of the two terms governed by the weak mixing parameter $\sin^2\theta_W$. The explicit form is

$$J_\lambda^{(0)} = \tfrac{1}{2}\bar{\nu}_e\gamma_\lambda(1-\gamma_5)\nu_e + L_e\bar{e}\gamma_\lambda(1-\gamma_5)e + R_e\bar{e}\gamma_\lambda(1+\gamma_5)e + (e \to \mu), \tag{1.4.6}$$

where the chiral coupling strengths L_e and R_e depend on $\sin^2\theta_W$. Identical couplings occur in the muon sector because the lepton generations have the same $SU(2)_L \otimes U(1)_Y$ quantum numbers. Notice that the electron and muon sectors remain disconnected, as required by the separate conservation of electron number and muon number. The leptonic neutral current is said to be flavor conserving, or diagonal in flavors.

What of the hadronic sector? Consider the three quarks u, d, and s that constitute the light hadrons. According to the Cabibbo hypothesis [15] of weak-interaction universality, the hadronic charged current is represented by the weak-isospin doublet

$$\begin{pmatrix} u \\ d_\theta \end{pmatrix}_L = \begin{pmatrix} u \\ d\cos\theta_C + s\sin\theta_C \end{pmatrix}_L . \tag{1.4.7}$$

The charged current for quarks thus has the explicit form

$$J_\lambda^{(+)} = \bar{u}\gamma_\lambda(1-\gamma_5)d\cdot\cos\theta_C + \bar{u}\gamma_\lambda(1-\gamma_5)s\cdot\sin\theta_C. \tag{1.4.8}$$

Even before entering the domain of the electroweak theory, we may ask why the hadron sector should have a superfluous quark, or, in other words, why the orthogonal combination of s- and d-quarks,

$$s_\theta = s\cos\theta_C - d\sin\theta_C, \tag{1.4.9}$$

does not appear in the charged weak current. To pose the same question differently, why do quarks and leptons not enter more symmetrically?

These apparently idle questions become urgent in the framework of the electroweak, theory, where they also find their answers. In the three-quark theory, the weak neutral current would be

$$\begin{aligned} J_\lambda^{(0)} &= J_\lambda^{(3)} - 2\sin^2\theta_W J_\lambda^{(EM)} \\ &= \tfrac{1}{2}[\bar{u}\gamma_\lambda(1-\gamma_5)u - \bar{d}\gamma_\lambda(1-\gamma_5)d\cdot\cos^2\theta_C \\ &\quad -\bar{s}\gamma_\lambda(1-\gamma_5)s\cdot\sin^2\theta_C - \bar{s}\gamma_\lambda(1-\gamma_5)d\cdot\sin\theta_C\cos\theta_C \\ &\quad -\bar{d}\gamma_\lambda(1-\gamma_5)s\cdot\sin\theta_C\cos\theta_C] \\ &\quad -2\sin^2\theta_W(\tfrac{2}{3}\bar{u}\gamma_\lambda u - \tfrac{1}{3}\bar{d}\gamma_\lambda d - \tfrac{1}{3}\bar{s}\gamma_\lambda s). \end{aligned} \tag{1.4.10}$$

Unlike the neutral leptonic current, this hadronic neutral current contains flavor-changing ($d \leftrightarrow s$) terms. This is experimentally unacceptable because of the small rates observed for the decays $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \mu^+\mu^-$. It was shown by Glashow, Iliopoulos, and Maiani [16] that the flavor-changing neutral currents could be eliminated and lepton–quark symmetry could be established by the addition of a second weak-isospin doublet

$$\begin{pmatrix} c \\ s_\theta \end{pmatrix}_L \tag{1.4.11}$$

involving a then-hypothetical charmed quark [17]. The hadronic neutral current would then be

$$\begin{aligned} J_\lambda^{(0)} &= \tfrac{1}{2}[\bar{u}\gamma_\lambda(1-\gamma_5)u + \bar{c}\gamma_\lambda(1-\gamma_5)c - \bar{d}\gamma_\lambda(1-\gamma_5)d \\ &\quad -\bar{s}\gamma_\lambda(1-\gamma_5)s] - 2\sin^2\theta_W J_\lambda^{(EM)}, \end{aligned} \tag{1.4.12}$$

which is manifestly flavor diagonal. The discovery [18] of the family of charmonium bound states and the subsequent observation [19] of charmed particles that decay according to the $(c, s_\theta)_L$ pattern constituted a striking confirmation of the GIM hypothesis and an important validation of weak-electromagnetic unification. We shall have much more to say about the electroweak gauge bosons and the avatar of electroweak symmetry breaking.

This brief survey has presented some evidence that quarks and leptons may properly be regarded as elementary particles and has introduced some of the symmetries and relationships among them. In addition, we have recalled some of the basic ideas and properties of gauge theories and have indicated their connection with the current understanding of the fundamental interactions. With these concepts and aspirations as background, we now turn to the foundations that underlie gauge theories. The first of these is the notion of symmetry in Lagrangian field theory.

PROBLEMS

1.1. Consider bound states composed of a b-quark and a $\bar{b}$-antiquark.
(a) Show that a bound state with orbital angular momentum L must have quantum numbers

$$C = (-1)^{L+s};\ P = (-1)^{L+1},$$

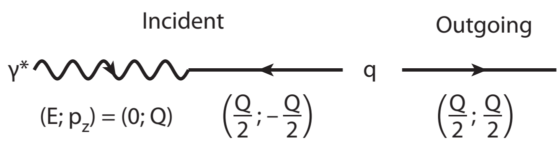
where s is the spin of the composite system.
(b) Allowing for both orbital and radial excitations, construct a schematic mass spectrum of $(b\bar{b})$ bound states. Label each state with its quantum numbers J^{PC}.

1.2. Consider bound states composed of color-triplet scalars (denoted σ) and their antiparticles.
(a) Show that a $(\sigma\bar{\sigma})$ bound state with angular momentum L (i.e., an orbital excitation) must have quantum numbers

$$C = (-1)^L; \; P = (-1)^L.$$

(b) Allowing for both orbital and radial excitation, construct a schematic mass spectrum of $(\sigma\bar{\sigma})$ bound states. Label each state with its quantum numbers J^{PC}. How does the spectrum differ from the quarkonium spectrum constructed in problem 1.1?

1.3. Analyze the absorption of a virtual photon by a spin-$\frac{1}{2}$ quark in the Breit frame (brick-wall frame) of the quark. Kinematics:



(a) Show that the squared matrix element for the absorption of a longitudinal photon vanishes.
(b) Compute the square of the matrix element for absorption of a photon with helicity $= +1$, that is, a transverse photon.
(c) How would your result for a longitudinal photon differ if the incident quark and photon were not precisely (anti)collinear?

1.4. Using the Feynman rules given in appendix B.5, compute the differential cross section $d\sigma/d\Omega$ and the total (integrated) cross section $\sigma \equiv \int d\Omega(d\sigma/d\Omega)$ for the reaction $e^+e^- \to \sigma^+\sigma^-$, where $\sigma^\pm$ is a charged scalar particle. Work in the center-of-momentum frame and in the high-energy limit (where all the masses may be neglected). Assume that the colliding beams are unpolarized.

1.5. (a) Again referring to Appendix B.5 for the Feynman rules, compute the differential cross section $d\sigma/d\Omega$ and the total (integrated) cross section $\sigma \equiv \int d\Omega(d\sigma/d\Omega)$ for the reaction $e^+e^- \to \mu^+\mu^-$. Work in the center-of-momentum frame and in the high-energy limit (where all the masses may be neglected). Assume that the colliding beams are unpolarized, and sum over the spins of the produced muons.
(b) Look up the original evidence for quark–antiquark jets in the inclusive reaction $e^+e^- \to$ hadrons [G. J. Hanson et al., *Phys. Rev. Lett.* **35**, 1609 (1975)]. Now recompute the differential cross section for the reaction $e^+e^- \to \mu^+\mu^-$, assuming the initial beams to be transversely polarized. [See also R. F. Schwitters et al., *Phys. Rev. Lett.* **35**, 1320 (1975).]

1.6. Define the requirements for an experiment to measure the gyromagnetic ratio of the tau lepton, taking into account the τ lifetime and the anticipated result

$g_\tau \approx 2$. For background, become acquainted with the methods used to measure the magnetic anomalies of the electron [A. Rich and J. Wesley, *Rev. Mod. Phys.* **44**, 250 (1972); R. S. Van Dyck, Jr., P. B. Schwinberg, and H. G. Dehmelt, *Phys. Rev. Lett.* **38**, 93 (1977); D. Hanneke, S. Fogwell, and G. Gabrielse, *Phys. Rev. Lett.* **100**, 120801 (2008)] and muon [F. Combley, F. J. M. Farley, and E. Picasso, *Phys. Rep.* **68**, 93 (1981); G. W. Bennett et al. [Muon $g-2$ Collaboration], *Phys. Rev. D* **73**, 072003 (2006)], and the magnetic dipole moments of the nucleons [N. F. Ramsey, *Molecular Beams,* Oxford University Press, Oxford, 1956] and unstable hyperons [L. Schachinger et al., *Phys. Rev. Lett.* **41**, 1348 (1978); L. G. Pondrom, *Phys. Rept.* **122**, 57 (1985). A novel technique is described in D. Chen et al. [E761 Collaboration], *Phys. Rev. Lett.* **69**, 3286 (1992).] For the current state of the art, see J. Abdallah et al. [DELPHI Collaboration], *Eur. Phys. J.* **C35**, 159 (2004).

1.7. Assume that the charged weak current has the left-handed form of equation (1.2.2) and that the interaction Hamiltonian is of the "current–current" form, $\mathcal{H}_{\mathrm{W}} \sim JJ^\dagger + J^\dagger J$.

(a) Enumerate the kinds of interactions (i.e., terms) in the Hamiltonian that may occur in a world composed of the electron and muon doublets.
(b) List the leptonic processes such as $\nu_\mu e \to \nu_\mu e$ that are consistent with the known selection rules but do not appear in the charged-current Hamiltonian.

1.8. Derive the connection between $|\Psi(0)|^2$ and the leptonic decay rate of a $(q\bar{q})$ vector meson. It is convenient to proceed by the following steps:
(a) Compute the spin-averaged cross section for the reaction $q\bar{q} \to e^+e^-$. Show that it is

$$\sigma = \frac{\pi\alpha^2 e_q^2}{12E^2} \cdot \frac{\beta_e}{\beta_q}(3-\beta_e^2)(3-\beta_q^2),$$

where E is the c.m. energy of a quark and β_i is the speed of particle i.
(b) The annihilation rate in a ${}^3\mathrm{S}_1$ vector meson is the density × relative velocity × 4/3 (to undo the spin average) × the cross section, or

$$\Gamma = |\Psi(0)|^2 \times 2\beta_q \times \tfrac{4}{3} \times \sigma.$$

(c) How is the result modified if the vector-meson wave function is

$$|V^0\rangle = \sum_i c_i |q_i \bar{q}_i\rangle?$$

(d) Now neglect the lepton mass and the quark binding energy and assume that the quarks move nonrelativistically. Show that

$$\Gamma(V^0 \to e^+e^-) = \frac{16\pi\alpha^2}{3M_V^2} |\Psi(0)|^2 \left(\textstyle\sum_i c_i e_i\right)^2.$$

(e) How is this result modified if quarks come in N_c colors and hadrons are color singlets? [This result is due to R. Van Royen and V. F. Weisskopf, *Nuovo Cim.* **50**, 617 (1967); *Nuovo Cim.* **51**, 583 (1967); and to H. Pietschmann and W. Thirring, *Phys. Lett.* **21**, 713 (1966).]

1.9. Outline a "three-neutrino experiment" to establish that a neutral, penetrating beam of ν_τ materializes into τ upon interacting in matter. [For background, look at the first two-neutrino experiment, J. Danby et al., *Phys. Rev. Lett.* **9**, 36 (1962). See also the Nobel lectures of M. Schwartz, J. Steinberger, and L. M. Lederman, reprinted in *Rev. Mod. Phys.* **61**, 527, 533, 547 (1989).] What would provide a copious source of ν_τ? What energy would be advantageous for the detection of the produced τ? What characteristics would be required of the detector? What are the important backgrounds, and how would you handle them? [K. Kodama et al. [DONuT Collaboration], *Phys. Rev. D***78**, 052002 (2008)].

FOR FURTHER READING

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These articles in the Spring 1997 issue of the SLAC *Beam Line* (http://j.mp/ACcaWv) celebrate the 100th anniversary of the discovery of the electron and the beginning of particle physics:

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S. Weinberg, "What Is an Elementary Particle?";
C. Quigg, "Elementary Particles: Yesterday, Today, Tomorrow."

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M. Gell-Mann and Y. Ne'eman, *The Eightfold Way,* Benjamin, New York, 1964,
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S. Weinberg, *Rev. Mod. Phys.* **52**, 515 (1980).

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G. 't Hooft, *In Search of the Ultimate Building Blocks,* Cambridge University Press, Cambridge and New York, 1997;

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3. See K. Hagiwara, K. Hikasa, and M. Tanabashi "Searches for Quark and Lepton Compositeness" in Ref. [2] for a more detailed discussion.
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7. See the listing of $b\bar{b}$ mesons in Ref. [2], p. 1100, for the $b\bar{b}$ spectrum [http://j.mp/wGW7Kn]. The top-quark lifetime, $\tau_t \lesssim 10^{-24}$ s, is too short to allow the formation of top hadrons and, specifically, toponium states. See, for example, V. M. Abazov et al. [D0 Collaboration], *Phys. Rev. D* **85**, 091104 (2012).

8. Many predictions are in fact shared by a model of integrally charged colored quarks due originally to M. Y. Han and Y. Nambu, *Phys. Rev.* **139**, B1005 (1965), and it is notoriously difficult to draw distinctions. A straightforward experimental test will be developed in section 8.7 and problem 8.27. Properties of the $\eta(549)$ and $\eta'(958)$ strongly favor the fractional charge assignment, as explained in the articles cited in problem 8.27.
9. M. Gell-Mann, *Phys. Rev.* **92**, 833 (1953); T. Nakano and K. Nishijima, *Prog. Theor. Phys. (Kyoto)* **10**, 581 (1955).
10. O. W. Greenberg, *Phys. Rev. Lett.* **13**, 598 (1964).
11. Refined theoretical estimates for the leptonic branching ratio (17.75%) have been given by F. J. Gilman and D. H. Miller, *Phys. Rev. D* **17**, 1846 (1978) and by N. Kawamoto and A. I. Sanda, *Phys. Lett.* **76B**, 446 (1978).
12. A clear and thorough discussion of the calculation of the π^0 lifetime is given in J. F. Donoghue, E. Golowich, and B. R. Holstein, *Dynamics of the Standard Model,* Cambridge University Press, Cambridge, 1992.
13. The first thorough discussions of this phenomenon are to be found in the reports by H. Newman (Mark-J Collaboration), Ch. Berger (PLUTO Collaboration), G. Wolf (TASSO Collaboration), and S. Orito (JADE Collaboration), in *Proceedings of the 1979 International Symposium on Lepton and Photon Interactions at High Energies,* ed. T.B.W. Kirk and H.D.I. Abarbanel, Fermilab, Batavia, Illinois, 1979, pp. 3, 19, 34, 52, http://j.mp/AEG8vn.
14. Today's electroweak theory developed from a proposal by S. Weinberg, *Phys. Rev. Lett.* **19**, 1264 (1967) and by A. Salam, in *Elementary Particle Theory: Relativistic Groups and Analyticity* (8th Nobel Symposium), ed. N. Svartholm, Almqvist and Wiksell International, Stockholm, 1968, p. 367. The theory is built on the $SU(2)_L \otimes U(1)_Y$ gauge symmetry investigated by S. L. Glashow, *Nucl. Phys.* **22**, 579 (1961). For a meticulous intellectual history see M. Veltman, in *Proceedings of the 6th International Symposium on Electron and Photon Interactions at High Energies,* ed. H. Rollnik and W. Pfeil, North-Holland, Amsterdam, 1974, p. 429. A later historical perspective is that of S. Coleman, *Science* **206**, 1290 (1979).
15. N. Cabibbo, *Phys. Rev. Lett.* **10**, 531 (1963).
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18. J. J. Aubert et al., *Phys. Rev. Lett.* **33**, 1404 (1974); J.-E. Augustin et al., *Phys. Rev. Lett.* **33**, 1406 (1974).
19. Definitive evidence for charmed mesons was presented by G. Goldhaber et al., *Phys. Rev. Lett.* **37**, 255 (1976), and by I. Peruzzi, et al., *Phys. Rev. Lett.* **37**, 569 (1976). The first example of a charmed baryon was reported by E. G. Cazzoli et al., *Phys. Rev. Lett.* **34**, 1125 (1975).

Two

Lagrangian Formalism and Conservation Laws

There are many ways to formulate the relativistic quantum field theory of interacting particles, each with its own set of advantages and shortcomings or inconveniences. The Lagrangian formalism has a number of attributes that make it particularly felicitous for our rather utilitarian purposes. Not to be neglected among its assets are that it is a familiar construct in classical mechanics and that many of its practical advantages can be understood already at the classical level. The Lagrangian approach is characterized by a simplicity in that field theory may be regarded as the limit of a system with n degrees of freedom as n tends toward infinity. Perhaps more to the point, it provides a formalism in which relativistic covariance is manifest, because the four coordinates of spacetime enter symmetrically. This is a decided, though by no means indispensable, advantage for the construction of a relativistic theory.

Lagrangian field theory is also particularly suited to the systematic discussion of invariance principles and the conservation laws to which they are related. In addition, a variational principle provides a direct link between the Lagrangian and the equations of motion. The foregoing properties are of particular value for the development of gauge theories, in which the interactions arise as consequences of local gauge symmetries. Finally, the path that leads from a Lagrangian to a quantum field theory by the method of canonical quantization is extremely well traveled. This makes it possible to conduct much of the discussion of the formulation of gauge theories at what is essentially the classical level and to make the leap to quantum field theory simply by using standard results, without repeating developments that are to be found in every field theory textbook. Thus equipped with the Feynman rules for a theory, we may proceed to calculate the consequences [1].

We do three things in this brief chapter. First, we summarize the basic elements of the Lagrangian formulation of classical mechanics and of field theory and the