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**Michael Wickens: Macroeconomic Theory**

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# 9

## Imperfectly Flexible Prices

### 9.1 Introduction

A key feature distinguishing neoclassical from Keynesian macroeconomics is the assumed speed of adjustment of prices. Neoclassical macroeconomic models commonly assume that prices are “perfectly” flexible, i.e., they adjust instantaneously to clear goods, labor, and money markets. Keynesian macroeconomic models assume that prices are sticky, or even fixed, and as a result, at best, they adjust to clear markets only slowly; at worst, they fail to clear markets at all, leaving either permanent excess demand (shortages) or excess supply (unemployment). Such market failures provided the main justification for the adoption of active fiscal and monetary policies. The aim was to return the economy to equilibrium (usually interpreted as full employment) faster than would happen without intervention.

Disillusion with the lack of success of stabilization policy and with the weak microeconomic foundations of Keynesian models, particularly the assumption of ad hoc rigidities in nominal prices and wages, which were usually attributed to institutional factors, led to the development of DGE macroeconomic models, with their emphasis on strong microfoundations and flexible prices. Instead of treating the macroeconomy as if it were in a permanent state of disequilibrium with its behavior being explained by ad hoc assumptions, DGE models returned to examining how the economy would behave if it were able to attain equilibrium and how the equilibrium characteristics of the economy would be affected by shocks and by policy changes.

An extensive program of research followed with the aim of investigating whether the dynamic behavior of the economy could be explained by the propagation of shocks in a flexible-price DGE model, or whether it was necessary to restore elements of market failure, including price inflexibility, in order to adequately capture fluctuations in macroeconomic variables over the business cycle. Early work by Kydland and Prescott (1982) focused on whether the business cycle could be explained solely by productivity shocks that were propagated by the internal dynamics of the DGE model to produce serially correlated movements in output. A discussion of the methodology and findings of this research program is provided in chapter 14. Although this research caused a dramatic and far-reaching change in the methodology of macroeconomic analysis, and in the process generated much controversy, the evidence seems to point

to the need for more price inflexibility in macroeconomic models than is provided by a perfectly flexible DGE model. As a result, current research has sought a way to combine the insights obtained from DGE models with a rigorous treatment of price adjustment based on microfounded price theory. The resulting models are often called New Keynesian models, though they might be better described as sticky-price DGE models.

These models usually have three key features. First, they retain the assumption of an optimizing framework. Second, they assume that there is imperfect competition in either goods or labor markets (or both), which gives monopoly power to producers. This causes higher prices, and lower output and employment, than under perfect competition. Third, once firms have some control over their prices, they can choose the rate of adjustment of prices. This allows the optimal degree of price flexibility for firms to become a strategic, or endogenous, issue and not an ad hoc additional assumption.

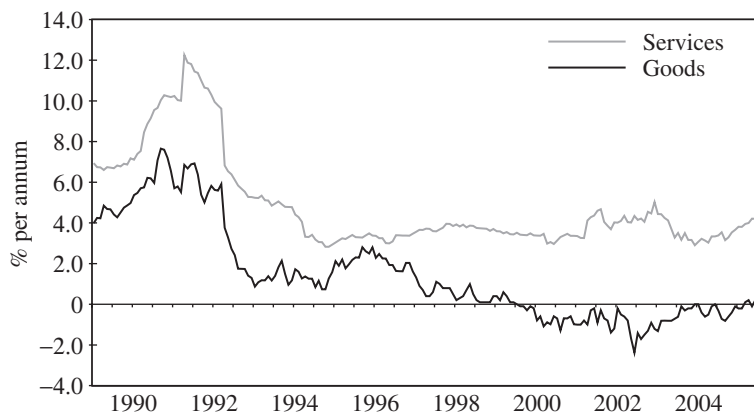
Previously, in our discussion of prices, we focused on the general price level, not on the prices of individual goods and services or on their relative prices, and we assumed that the general price level and inflation adjust instantaneously. In examining imperfect price flexibility, we note that the behaviors of individual prices differ, with some changing more frequently than others. As a result, the relative importance of components of the general price level also changes. This affects the speed of adjustment of both the general price level and inflation, which are said to be sticky, i.e., to show slow or sluggish adjustment.

A closely related argument is that intermediate outputs are required to produce the final output, but that the price of the final good is not just a weighted average of the prices of intermediate goods. The difference between final goods prices and the average price of intermediate goods represents a resource cost; the greater the dispersion of prices across intermediate goods, possibly initiated by inflation and prolonged by sticky prices, the greater is the resource cost. The main interest in this argument is that it suggests that inflation may be costly, and hence provides a reason for controlling inflation.

We now examine optimal price setting when goods and labor markets are imperfect but prices are flexible. We then analyze the intermediate-goods model. Next we consider different models that seek to explain why prices may not adjust instantaneously but may be sticky. The chapter ends by examining the implications of these theories for the dynamic behavior of prices and inflation. Before developing our theoretical models, we consider some evidence on the speed of adjustment of prices and wages. Useful surveys on these issues are Taylor (1999), Rotemberg and Woodford (1999), and Gali (2008).

## 9.2 Some Stylized “Facts” about Prices and Wages

Information comparing the behavior of different U.S. price series has been obtained by Bils et al. (2003), Bils and Klenow (2004), and Klenow and Kryvtov (2005). They find that the average time between price changes is around



**Figure 9.1.** U.K. goods and services price inflation 1989.1–2005.8.

six months, whereas Blinder et al. (1998), using data for a much broader range of U.S. industries than Klenow and Kryvtsov, found the average to be twelve months and Rumler and Vilmunen (2005) found an average of thirteen months for countries in the euro area.

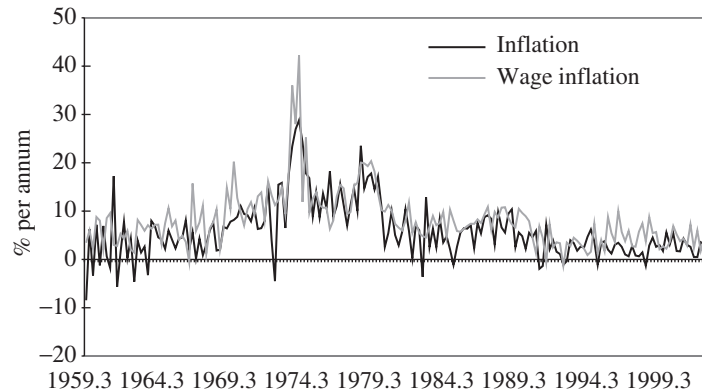
The frequency of price changes varies across sectors. Bills, Klenow, and Kryvtsov, using unpublished data on 350 categories of goods and services collected by the Bureau of Labor Statistics of the U.S. Department of Labor, report that the median duration between price changes for all items is 4.3 months, that for goods alone (which comprise 30.4% of the CPI) is 3.2 months, and that for services (40.8% of the CPI) is 7.8 months. Individual items differ even more. The median durations between price changes for apparel, food, and home furnishings (37.3%) range between 2.8 and 3.5 months, while for transportation (15.4%) the figure is 1.9 months, for entertainment (3.6%) it is 10.2 months, and for medical services (6.2%) it is 14.9 months. A similar distribution is found by Rumler and Vilmunen for the euro area; there are very frequent changes for energy products and unprocessed food, and relatively frequent changes for processed food, nonenergy industrial goods, and, particularly, services.

In the United Kingdom, the time-series evidence on goods and services price inflation shows very different behavior (see figure 9.1). Services price inflation has been larger and has fluctuated less in the short term. Goods price inflation has been very small—recently even negative—and shows greater short-term variability than services prices. General price inflation is roughly the average of the two.

The rates of change of nominal-wage rates and the general price level in the United Kingdom tend to be similar to each other both in level and volatility, but both the level and the volatility vary considerably over time (see figure 9.2).

More generally, the key stylized “facts” about price and wage changes are the following.

1. Price and wage rigidities are temporary. Hence we expect the DGE model to work in the longer term.



**Figure 9.2.** U.K. price and wage inflation 1959.3–2005.1.

2. Prices and wages change on average about two or three times a year.
3. The higher inflation is, the more frequently price and wage changes occur.
4. Price and wage changes are not synchronized.
5. Price changes (and to some extent wage changes) occur with different frequencies in different industries (e.g., food/groceries price changes are more frequent than those of manufactures, magazines, or services). Roughly speaking, it seems that changes in the prices of tradeables are more frequent than those of nontradeables.
6. Prices and costs change at different rates at different stages of the business cycle. For example, in the late expansion phase, costs rise more than prices, implying that profit margins fall.

It is clear from this evidence that prices of individual items behave differently: their relative prices change over time, their short-term fluctuations are different, and the frequency with which they change differs. Only for models of the economy with an implicit time period of about one year is it reasonable to assume no lag in the adjustment of prices. Even then, lags elsewhere in the system can delay the completion of price adjustment from one equilibrium to another.

### 9.3 Price Setting under Imperfect Competition

Under perfect competition in goods markets, firms (or, more generally, suppliers) have no individual power to set prices as consumers, possessing full information, search for the lowest price. Consequently, prices change only when all firms face the same increase in costs. To be able to set prices, firms require a degree of monopoly power. This arises under imperfect competition. Prices are then a markup over costs. The markup depends on the response of demand to prices. As a result, prices may also respond to demand factors.

Similar arguments apply to labor markets. When either the employer or the supplier of labor, whether it be unionized or nonunionized labor, has monopoly power, labor is not paid its marginal product. Depending on who exercises the most monopoly power, real wages will be either below (when employers dominate) or above (when employees dominate) the marginal product of labor. We refer to such discrepancies as wedges. Next we consider some basic results of pricing in imperfectly competitive goods and factor markets.

### 9.3.1 Theory of Pricing in Imperfect Competition

In the standard theory of pricing in imperfect competition there is a single firm which faces a downward-sloping demand for its product:

$$P = P(Q), \quad P'(Q) < 0,$$

where  $P$  is the price and  $Q$  now represents the quantity produced. The firm's production function is

$$Q = F(X_1, \dots, X_n), \quad F' > 0, \quad F'' \leq 0,$$

where  $X_i$  is the  $i$ th factor input, including raw materials. The cost of production is

$$C = \sum_{i=1}^n W_i X_i,$$

where, in order to capture monopoly supply features, the factor prices  $W$  are a nondecreasing function of the quantity of the factor used, so that

$$W_i = W(X_i), \quad W'(X_i) > 0.$$

The firm chooses  $Q$  and  $X_i$  ( $i = 1, \dots, n$ ) to maximize profits  $\Pi = R - C$  subject to the production technology, where  $R = PQ$  is total revenue. The Lagrangian for this problem is

$$\mathcal{L} = P(Q)Q - \sum_{i=1}^n W(X_i)X_i + \lambda[F(X_1, \dots, X_n) - Q].$$

The first-order conditions are

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial Q} &= P + P'(Q)Q - \lambda = 0, \\ \frac{\partial \mathcal{L}}{\partial X_i} &= -W_i - W'(X_i)X_i + \lambda F'_i = 0. \end{aligned}$$

Hence,

$$\begin{aligned} \lambda &= P + P'(Q)Q = MR \\ &= \frac{W_i + W'(X_i)X_i}{F'_i} = \frac{MC_i}{MP_i} = MC, \end{aligned}$$

where  $MR$  is marginal revenue,  $MC_i$  is the marginal cost, and  $MP_i$  the marginal product of the  $i$ th factor, and where

$$MC = \frac{\partial C}{\partial Q} = \frac{\partial C}{\partial X_i} \frac{\partial X_i}{\partial Q}$$

is total marginal cost. It follows that

$$p = \frac{1}{1 - (1/\epsilon_D)} MC \quad (9.1)$$

$$= \frac{1}{1 - (1/\epsilon_D)} \frac{MC_i}{MP_i} \quad (9.2)$$

$$= \frac{1 + (1/\epsilon_{X_i})}{1 - (1/\epsilon_D)} \frac{W_i}{MP_i}, \quad (9.3)$$

where

$$\epsilon_D = -\frac{\partial Q}{\partial P} \frac{P}{Q} > 0$$

is the price elasticity of demand and

$$\epsilon_{X_i} = \frac{\partial X_i}{\partial W} \frac{W}{X_i} > 0$$

is the factor supply elasticity. The labor-supply elasticity may reflect the monopsony power of unionized labor as well as the labor supply of an individual. Hence the price of goods and services is dependent on the unit costs of the factors, their marginal product, the elasticity of their supply, and on the elasticity of demand for the good.

For the Cobb–Douglas production function

$$Q = \prod_{i=1}^n X_i^{\alpha_i}, \quad \sum_{i=1}^n \alpha_i = 1,$$

we can obtain the share going to the  $i$ th factor, which is

$$\frac{w_i X_i}{pq} = \alpha_i \frac{1 - (1/\epsilon_D)}{1 + (1/\epsilon_{X_i})}.$$

The first-order conditions imply that  $MC_i/MP_i$  is equal for each factor. It follows that an increase in the unit cost of a single factor would result in a decrease in its use and hence an increase in its marginal product. If  $\epsilon_{X_i}$  is constant, then  $W_i/MP_i$ , and hence  $MC_i/MP_i$ , will remain unchanged. As a result, the price of goods would be unaffected; because this is a relative price change, only the factor proportions have altered. In contrast, if a factor is required in fixed proportion to output, then substitutability between factors is not possible. In this case its marginal product is fixed and so marginal cost, and hence the price of the good, will increase. Output will fall, which will reduce the demand for all factors. This analysis applies to the medium and to the long run; in the short run all factors will tend to be less flexible. Consequently, the case of fixed proportions may also be a good approximation to the short-run response of an increase in the price of a factor.

If all factor prices increase in the same proportion, and their supply elasticities and the price elasticity of demand are constant, then the prices of goods would increase by the same proportion. Accordingly, with constant elasticities, inflation must be due to a general increase in factor prices. We have argued that relative factor price changes would have no effect on prices in the long run; they would only affect relative factor usage. However, they may affect the general price level in the short run.

These elementary principles of pricing are the basis of New Keynesian models of inflation. They underpin the supply side of the economy through new theories of the Phillips curve—a relation between price or wage inflation and a measure of excess supply in the goods or labor market, such as the deviation of output from full capacity (or from trend output) or unemployment. For further discussion of the role of marginal cost pricing and the output gap in the New Keynesian inflation equation see Neiss and Nelson (2005), Batini et al. (2005), and Gali (2008).

### 9.3.2 Price Determination in the Macroeconomy with Imperfect Competition

Modern macroeconomic theories of price determination emphasize the fact that in the economy a large number of different goods and services are produced. A widely used model of price setting when these goods are imperfect substitutes is that of Dixit and Stiglitz (1977). We consider a variant of this that is closely related to work by Blanchard and Kiyotaki (1987), Ball and Romer (1991), and Dixon and Rankin (1995) (see also Mankiw and Romer (1991) and the articles cited therein). For simplicity, the model is highly stylized.

We assume that the economy is composed of  $N$  firms each producing a different good that is an imperfect substitute for the other goods, and that a single factor of production is used, namely, labor that is supplied by  $N$  households. The production function for the  $i$ th firm is assumed to be

$$y_t(i) = F_i[n_t(i)],$$

where  $n_t(i)$  is the labor input of the  $i$ th firm. The production function is indexed by  $i$  to denote that each good may be produced with a different production function. The profits of the  $i$ th firm are

$$\Pi_t(i) = P_t(i)F_i[n_t(i)] - W_t(i)n_t(i), \quad (9.4)$$

where  $P_t(i)$  is the output price and  $W_t(i)$  is the wage rate paid by firm  $i$ .

#### 9.3.2.1 Households

We assume that there are also  $N$  households and these are classified by their type of employment, with each household working for one type of firm  $i$ . Households are assumed to have an identical instantaneous utility function:

$$U[c_t, l_t(i)] = u[c_t] + \eta l_t(i)^\varepsilon, \quad \varepsilon \geq 1,$$



where  $c_t$  is their total consumption,  $l_t(i)$  is leisure, and  $n_t(i) + l_t(i) = 1$ . We assume that  $u_c > 0$  and that  $u_{cc} < 0$ .

We also assume that total consumption  $c_t$  is obtained by aggregating over the  $N$  different types of goods and services  $c_t(i)$  using the constant elasticity of substitution function

$$c_t = \left[ \sum_{i=1}^N c_t(i)^{(\phi-1)/\phi} \right]^{\phi/(\phi-1)}, \quad (9.5)$$

where  $\phi > 1$  is the elasticity of substitution; we recall that a higher value of  $\phi$  implies greater substitutability. Thus goods and services are imperfect substitutes if  $\phi$  is finite.

Total household expenditure on goods and services is

$$P_t c_t = \sum_{i=1}^N P_t(i) c_t(i);$$

hence the general price index  $P_t$  satisfies

$$P_t = \sum_{i=1}^N P_t(i) \frac{c_t(i)}{c_t}. \quad (9.6)$$

The household budget constraint is

$$P_t c_t = \sum_{i=1}^N P_t(i) c_t(i) = W_t(i) n_t(i) + \sum_{i=1}^N \Pi_t(i),$$

where each household is assumed to hold an equal share in each firm.

In the absence of capital (and trading in shares) the budget constraint is static. Consequently, optimization can be carried out each period without regard to future periods. Thus, in the absence of assets, the intertemporal aspect of the DGE model of the model is eliminated. We assume, therefore, that households maximize utility with respect to  $\{c_t(1), \dots, c_t(N), n_t(i)\}$  subject to their budget constraint and to  $n_t(i) + l_t(i) = 1$ . The Lagrangian is defined as

$$\begin{aligned} \mathcal{L} = u \left( \left[ \sum_{i=1}^N c_t(i)^{(\phi-1)/\phi} \right]^{\phi/(\phi-1)} \right) &+ \eta l_t(i)^\varepsilon \\ &+ \lambda_t \left[ W_t(i) n_t(i) + \sum_{i=1}^N \Pi_t(i) - \sum_{i=1}^N P_t(i) c_t(i) \right]. \end{aligned}$$

The first-order conditions are

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial c_t(i)} &= u_{c,t} \left[ \frac{c_t}{c_t(i)} \right]^{1/\phi} - \lambda_t P_t(i) = 0, \quad i = 1, \dots, N, \\ \frac{\partial \mathcal{L}}{\partial l_t(i)} &= \eta \varepsilon l_t(i)^{\varepsilon-1} - \lambda_t W_t(i) = 0, \end{aligned}$$

giving

$$\frac{c_t(i)}{c_t} = \left[ \frac{\lambda_t P_t(i)}{u_{c,t}} \right]^{-\phi}. \quad (9.7)$$

The household's problem can also be expressed in terms of maximizing utility with respect to aggregate consumption, as the Lagrangian can be rewritten as

$$\mathcal{L} = u(c_t) + \eta l_t(i)^\varepsilon + \lambda_t \left[ W_t(i) n_t(i) + \sum_{i=1}^N \Pi_t(i) - P_t c_t \right].$$

The first-order condition with respect to  $c_t$  is

$$\frac{\partial \mathcal{L}}{\partial c_t} = u_{c,t} - \lambda_t P_t = 0;$$

hence  $u_{c,t}/P_t = \lambda_t$  and so equation (9.7) can also be written as

$$\frac{c_t(i)}{c_t} = \left[ \frac{P_t(i)}{P_t} \right]^{-\phi}. \quad (9.8)$$

This is the demand function for the  $i$ th good.

Substituting (9.8) into (9.6) gives the general price index expressed solely in terms of individual prices:

$$\begin{aligned} P_t &= \sum_{i=1}^n P_t(i) \left[ \frac{P_t(i)}{P_t} \right]^{-\phi} \\ &= \left[ \sum_{i=1}^n P_t(i)^{1-\phi} \right]^{1/(1-\phi)}. \end{aligned} \quad (9.9)$$

From the first-order condition with respect to labor, the total supply of labor by the household is

$$l_t(i) = \left( \frac{u_{c,t} W_t(i)}{\eta \varepsilon P_t} \right)^{1/(\varepsilon-1)}. \quad (9.10)$$

As  $\varepsilon \geq 1$ , an increase in  $W_t(i)$  will raise labor supply  $l_t(i)$ . If labor markets are competitive, households have the same utility function (implying complete markets) and work equally hard (implying firms are indifferent about who they hire), in which case  $W_t(i)$  will be equal across firms. We denote the common wage by  $W_t$ . If households have different utility functions (or do not work equally hard), then the marginal utilities will differ and so will wages.

### 9.3.2.2 Firms

The problem for the  $i$ th firm is to maximize profits subject to its demand function, equation (9.8). In the absence of investment and government expenditures, we have  $c_t(i) = y_t(i) = F_i[n_t(i)]$ . The first-order condition of  $\Pi_t(i)$ , equation (9.4), with respect to  $c_t(i)$  is

$$\frac{d\Pi_t(i)}{dc_t(i)} = P_t(i) + \frac{\partial P_t(i)}{\partial c_t(i)} c_t(i) - W_t \frac{dn_t(i)}{dc_t(i)} = 0,$$

where

$$\frac{dc_t(i)}{dn_t(i)} = \frac{dy_t(i)}{dn_t(i)} = F'_i[n_t(i)].$$

It follows that

$$P_t(i) = \frac{\phi}{\phi - 1} \frac{W_t}{F'_i[n_t(i)]}. \quad (9.11)$$

This is a key result. It indicates that price is a markup over  $W_t/F'_i$ , which is the marginal cost of an extra unit of output; the markup or wedge is  $\phi/(\phi - 1) > 1$ . As  $\phi \rightarrow \infty$ , i.e., as the consumption goods become perfect substitutes, the markup tends to unity and price falls to equal marginal cost. This solution is the standard outcome for monopoly pricing. Prices vary across goods due to differences in the marginal product of labor,  $F'_i[n_t(i)]$ . Equation (9.11) implies that firms have some control over their prices. This entails a source of inefficiency because output, and hence consumption, are lower than in perfect competition. An increase in the economy-wide wage would therefore cause an increase in the price of each good and in the general price level.

The demand for labor can be obtained from equation (9.11). Suppose that the production function is Cobb–Douglas so that

$$y_t(i) = A_{it}n_t(i)^{\alpha_i}, \quad \alpha_i \leq 1,$$

where  $A_{it}$  can be interpreted as an efficiency term for the  $i$ th firm at time  $t$ . Labor demand is then given by

$$n_t(i) = \left( \frac{\phi}{\alpha_i A_{it}(\phi - 1)} \frac{W_t}{P_t(i)} \right)^{-1/(1-\alpha_i)}. \quad (9.12)$$

The greater  $\phi$  is, and hence the lower the markup, the greater labor demand and output are, reflecting once more the inefficiency of monopolies in terms of lost output and employment.

Equating labor demand and supply (equations (9.10) and (9.12)) for firm  $i$  gives

$$n_t(i) = \left( \frac{\phi}{\alpha_i A_{it}(\phi - 1)} \frac{W_t}{P_t(i)} \right)^{-1/(1-\alpha_i)} = \left( \frac{u_{c,t} W_t}{\eta \varepsilon P_t} \right)^{1/(\varepsilon-1)}.$$

Hence

$$\frac{P_t(i)}{P_t} = \frac{\phi}{\alpha_i A_{it}(\phi - 1)} \left( \frac{u_{c,t}}{\eta \varepsilon} \right)^{(1-\alpha_i)/(\varepsilon-1)} \left( \frac{W_t}{P_t} \right)^{(\varepsilon-\alpha_i)/(\varepsilon-1)}. \quad (9.13)$$

Thus differences between firm prices are due to  $A_{it}$  and  $\alpha_i$ . Equation (9.13) implies that, as  $\varepsilon \geq 1$ , an increase in the economy-wide real-wage rate would raise the relative price of firm  $i$ .

In the special case where the efficiency term  $A_{it}$  and the production elasticities are the same, so that  $A_{it} = A_t$  and  $\alpha_i = \alpha$ , firm prices will be identical. In this case we can solve equation (9.13) for the real wage as

$$\frac{W_t}{P_t} = \left( \frac{\phi}{\alpha A_t(\phi - 1)} \right)^{-(\varepsilon-1)/(\varepsilon-\alpha)} \left( \frac{u_{c,t}}{\eta \varepsilon} \right)^{-(1-\alpha)/(\varepsilon-\alpha)}. \quad (9.14)$$

As  $u_{c,t}$  is negatively related to  $c_t$  ( $= y_t$ ) and  $\varepsilon > \alpha$ , an increase in the real wage will raise output. Moreover, the lower the markup  $\phi/(\phi - 1)$  is, the greater the response of output to the real wage will be. Equation (9.14) also shows that the

economy is then neutral with respect to nominal values. An example of this is when each production function is linear in labor when  $\alpha_i = 1$ . In this case employment is determined by the supply side (equation (9.10)).

More generally, when the  $\alpha_i$  are different, we do not obtain a closed-form solution for the real wage. To see this, substitute equation (9.13) into (9.9). An analytic solution for the general price level cannot be derived and hence total output is not a function of the real wage. Nonetheless, the economy remains neutral with respect to a nominal shock. This can be seen by noting that equation (9.13) is still homogeneous of degree zero in wages and prices.

### 9.3.3 Pricing with Intermediate Goods

Once more our model of the economy is highly stylized for simplicity. The key assumption is that a final good is produced by a profit-maximizing firm using  $N$  inputs that are all intermediate goods. It is assumed that the intermediate goods are produced by  $N$  monopolistically competitive firms using only one factor of production: labor. Households consume only the final good and supply labor.

#### 9.3.3.1 Final-Goods Production

The final good is  $y_t$  and the intermediate goods are  $y_t(i)$ ,  $i = 1, \dots, N$ . It is assumed that the final output satisfies the CES production function

$$y_t = \left[ \sum_{i=1}^N y_t(i)^{(\phi-1)/\phi} \right]^{\phi/(\phi-1)}, \quad \phi > 1,$$

and that there are no other factors of production for final output.

The final-output producer is assumed to choose the inputs  $y_t(i)$  to maximize profits, which are given by

$$\begin{aligned} \Pi_t &= P_t y_t - \sum_{i=1}^N P_t(i) y_t(i) \\ &= P_t \left[ \sum_{i=1}^N y_t(i)^{(\phi-1)/\phi} \right]^{\phi/(\phi-1)} - \sum_{i=1}^N P_t(i) y_t(i), \end{aligned}$$

where  $P_t$  is the price of the final output and  $P_t(i)$  are the prices of the intermediate inputs. The first-order condition is

$$\frac{\partial \Pi_t}{\partial y_t(i)} = P_t \left( \frac{y_t}{y_t(i)} \right)^{1/\phi} - P_t(i) = 0;$$

hence the demand for the  $i$ th input is

$$y_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\phi} y_t. \quad (9.15)$$

Since equilibrium profits are zero, the price of the final good is

$$\begin{aligned} P_t &= \sum_{i=1}^N P_t(i) \frac{y_t(i)}{y_t} \\ &= \left[ \sum_{i=1}^N P_t(i)^{1-\phi} \right]^{1/(1-\phi)}. \end{aligned}$$

### 9.3.3.2 Intermediate-Goods Production

The intermediate goods are assumed to be produced with the constant returns to scale production function

$$y_t(i) = A_i n_t(i),$$

where  $n_t(i)$  is labor input. Intermediate-goods firms maximize the profit function

$$\Pi_t(i) = P_t(i) y_t(i) - W_t n_t(i)$$

subject to the demand function, equation (9.15), where  $W_t$  is the economy-wide wage rate. The profit function can therefore be written as

$$\begin{aligned} \Pi_t(i) &= P_t \left( \frac{y_t}{y_t(i)} \right)^{1/\phi} y_t(i) - W_t n_t(i) \\ &= P_t y_t^{1/\phi} y_t(i)^{1-(1/\phi)} - W_t n_t(i) \\ &= P_t y_t^{1/\phi} [A_i n_t(i)]^{1-(1/\phi)} - W_t n_t(i). \end{aligned}$$

Maximizing  $\Pi_t(i)$  with respect to  $n_t(i)$ , taking  $P_t$  and  $y_t$  as given, yields

$$\begin{aligned} n_t(i) &= A_i^{\phi-1} \left[ \frac{\phi-1}{\phi} \frac{P_t}{W_t} \right]^{1/\phi} y_t, \\ y_t(i) &= A_i^\phi \left[ \frac{\phi-1}{\phi} \frac{P_t}{W_t} \right]^{1/\phi} y_t, \\ P_t(i) &= \frac{\phi}{A_i(\phi-1)} W_t. \end{aligned}$$

### 9.3.3.3 The Inefficiency Loss

Total output is derived from the outputs of the intermediate goods as

$$y_t(i) = A_i n_t(i) = \left[ \frac{P_t(i)}{P_t} \right]^{-\phi} y_t.$$

Total labor is given by

$$\begin{aligned} n_t &= \sum_{i=1}^N n_t(i) \\ &= \sum_{i=1}^N \frac{1}{A_i} \left[ \frac{P_t(i)}{P_t} \right]^{-\phi} y_t. \end{aligned}$$

This gives a relation between the output of the final good and aggregate labor, which can be written as

$$\mathcal{Y}_t = v_t n_t,$$

$$v_t = \frac{1}{\sum_{i=1}^N (1/A_i) [P_t(i)/P_t]^{-\phi}}.$$

If  $v_t < 1$ , then there is an inefficiency loss in the use of labor in producing the final output but not in producing intermediate outputs. Or, put another way, since it is necessary to use intermediate inputs to produce final output, an inefficiency loss occurs in the use of labor in the economy. Moreover, if  $A_i = 1$ , then the inefficiency loss is due solely to price dispersion as we can then express  $v_t$  as

$$v_t = \left[ \frac{\tilde{P}_t}{P_t} \right]^\phi,$$

$$\tilde{P}_t = \left[ \sum_{i=1}^N P_t(i)^{-\phi} \right]^{-1/\phi},$$

$$P_t = \left[ \sum_{i=1}^N P_t(i)^{1-\phi} \right]^{1/(1-\phi)}.$$

This implies that  $v_t < 1$ .

To see the effect of inflation, totally differentiate  $P_t$  to obtain

$$dP_t^{-\phi} = \sum_{i=1}^N dP_t(i)^{-\phi}.$$

Hence, the general level of inflation is related to individual intermediate-goods inflation rates through

$$\frac{dP_t}{P_t} = \left\{ \sum_{i=1}^N \left[ \frac{dP_t(i)}{P_t(i)} \frac{P_t(i)}{P_t} \right]^{-\phi} \right\}^{-1/\phi}.$$

Suppose that  $dP_t(i)$  is the same for all  $i$ , then  $dP_t < dP_t(i)$ . This implies that if  $P_t(i)/P_t$  is the same for  $i$ , then

$$\frac{dP_t}{P_t} < \frac{dP_t(i)}{P_t(i)},$$

and so the overall level of inflation for the economy is less than the common individual inflation rate. The presence of intermediate goods therefore ameliorates general inflation.

Putting the two results together, we conclude that the presence of intermediate goods leads to an output loss but also a lower level of inflation for the final good.

### 9.3.4 Pricing in the Open Economy: Local and Producer-Currency Pricing

Previously, in our discussion of the open economy in chapter 7, we discussed imperfect substitutability between domestic and foreign tradeables. We argued that when they are perfect substitutes, after taking account of transportation costs, the law of one price holds. In other words, there is a single world price for tradeables, which may be expressed in foreign currency, typically the U.S. dollar, or in terms of domestic currency. This relation implies that  $P_t^H = P_t^F = S_t P_t^{F*} = S_t P_t^{H*}$ , where we are using the notation of chapter 7, i.e., that  $P_t^H$  is the domestic currency price of home tradeables,  $P_t^F$  is the domestic currency price of imports,  $S_t$  is the domestic price of foreign exchange, and an asterisk denotes the foreign equivalent. Not only does this prevent domestic producers from being able to pass on increases in costs that are purely domestic, it also affects the response of prices to changes in the nominal exchange rate, and hence the effectiveness of monetary policy.

Since for a small economy the prices of domestic tradeables are set at the world price, and the world price is given in foreign currency terms, an exchange-rate depreciation—which raises the number of units of domestic currency per unit of foreign currency—would raise the domestic currency price of imports, and hence cause an increase in the price of domestic tradeables sold at home. The foreign currency price of home-country exports would be unaffected as this is set in terms of the world currency price, which is taken as given. Exports therefore become more profitable in terms of domestic currency. An exchange-rate appreciation would reduce the domestic currency price of imports and hence domestic tradeables prices. As the foreign currency price of exports is unchanged, the domestic currency price will decrease; therefore exporting becomes less profitable.

If, instead, domestic and foreign tradeables are imperfect substitutes, then domestic and foreign producers have a measure of monopoly power in setting prices. As previously noted in chapter 7, a situation where producers have monopoly power both at home and abroad has been named producer-currency pricing (PCP) by Betts and Devereux (1996) and Devereux (1997). A situation where producers have monopoly power at home, but not abroad, has been named local-currency pricing (LCP). In this case imports are priced at the domestic producer price; this is known as pricing-to-market.

Consider PCP. In the extreme case of a pure monopoly, the export price is just the domestic currency price expressed in foreign currency, but domestic and foreign tradeables will differ in price. In this case,  $P_t^F = S_t P_t^{H*}$  and  $P_t^{F*} = S_t^{-1} P_t^H$  but  $P_t^H \neq P_t^F$  and  $P_t^{H*} \neq P_t^{F*}$ . Domestic producers can now pass on cost increases both at home and abroad, and an exchange-rate appreciation would result in an increase in the foreign currency price of exports. If the foreign producer has monopoly power in the domestic market, then a depreciation would raise the domestic currency price of imports.

In contrast, under pure LCP,  $P_t^H = P_t^F$  and  $P_t^{H*} = P_t^{F*}$  but  $P_t^F \neq S_t P_t^{H*}$  and  $P_t^{F*} \neq S_t^{-1} P_t^H$ . Here the domestic producer would be able to pass on cost increases only

in the domestic market and not in the foreign market. An appreciation would have no effect on the foreign currency price of exports, and a depreciation would not affect domestic tradeables prices.

The evidence shows that import prices are relatively sticky and do not fluctuate one-for-one with changes in the nominal exchange rate, or with changes in foreign prices. It is the terms of trade and the real exchange rate that seem to absorb shocks, especially those to the nominal exchange rate. Engel (2000), among others, has found that traded goods prices in Europe are not much influenced by exchange-rate movements. This seems to suggest that either foreign goods are highly substitutable with home goods or they may not be perfect substitutes, but because producers lack monopoly power in foreign markets, the prices of imported goods are priced-to-market, i.e., LCP prevails. This finding has important policy consequences. It implies that a depreciation of the exchange rate tends not to be passed on in the form of higher prices for imported goods. This considerably reduces the effectiveness of an exchange-rate depreciation, a traditional way to stimulate the domestic economy and improve the trade balance.

These arguments apply primarily to a small open economy and are less applicable to the United States, which is a large and relatively closed economy compared with nearly all other countries. By virtue of its size, the U.S. price will often be the principal determinant of the world price. And because world prices are often set in terms of the U.S. dollar, U.S. domestic prices are well-insulated against changes in the value of the dollar. As a result, to a first approximation, it is common to treat the United States as a closed economy. However, this would miss a crucial aspect of the U.S. economy. To the extent that the world prices of commodities are set in dollars, it would be more difficult for the United States to improve its competitiveness through a depreciation. Instead, it would have to rely more on improvements to productive efficiency through technological growth and innovation in new products, which would create, at least for a time, monopoly power in world markets. Thus, in this case, the arguments above concerning pricing under imperfect competition apply to the United States, but those relating to the effect of exchange-rate changes on prices may be less relevant. Like other countries, of course, the United States also exports commodities whose foreign currency price would fall following a dollar depreciation.

## 9.4 Price Stickiness

We have discussed how optimal prices are determined in the long run. We now consider the dynamic behavior of prices in the short run. This will give us a complete picture of pricing behavior. There are several competing theories of price dynamics adjustment, but they have similar implications for price dynamics. These theories have in common the notion that the general price level is made up of the prices of many individual items, and that the prices of these components adjust at different speeds. Over time the prices of individual items



are revised. As a result, the general price level displays inertia. The key distinguishing feature of these theories lies in whether they attribute price changes to chance or to choice: i.e., whether the changes are exogenous or endogenous, optimal or constrained, and hence suboptimal.

We focus on three theories commonly used in modern macroeconomics:

1. The overlapping contracts model of Taylor (1979), where wages are the main cause of price change.
2. The staggered pricing model of Calvo (1983), where price changes occur randomly.
3. The optimal dynamic adjustment model used, for example, by Rotemberg (1982), where the speed of price adjustment is chosen optimally.

We then consider the implications for price dynamics.

#### 9.4.1 Taylor Model of Overlapping Contracts

This model is based on the following assumptions.

1. Price is a markup over marginal cost and the markup may be time-varying and affected in the short-run mainly by the wage rate.
2. The wage rate at any point in time is an average of wage contracts that were set in the past but are still in force, and of those set in the current period.
3. When they were first set, wage contracts were profit maximizing and reflected the prevailing marginal product of labor and the expected future price level.

We define the following variables:  $P_t$  is the general price level,  $p_t = \ln P_t$ ,  $\pi_t = \Delta p_t$  is the inflation rate,  $W_t$  is the economy-wide wage rate,  $w_t = \ln W_t$ ,  $W_t^N$  is the new wage contract made in period  $t$ ,  $w_t^N = \ln W_t^N$ ,  $v_t$  is the price markup over costs, and  $z_t$  is the logarithm of the marginal product of labor.

Price is assumed to be a markup over wage costs:

$$p_t = w_t + v_t. \quad (9.16)$$

This implies a degree of monopoly power and a single factor, labor. Taylor assumed that wage contracts last for four quarters. For simplicity, we assume that they last for only two periods. The average wage  $w_t$  is the geometric mean of the wage contracts  $w_t^N$  and  $w_{t-1}^N$  made in periods  $t$  and  $t - 1$ :

$$w_t = \frac{1}{2}(w_t^N + w_{t-1}^N). \quad (9.17)$$

New wage contracts are assumed to be set to take account of the possibility that the future price level  $p_{t+1}$  might differ from the current level  $p_t$ ; hence the real wage, defined by taking the average of the current and future expected

price levels over two periods, equals the current marginal product of labor  $z_t$ . The new nominal wage is therefore

$$w_t^N - \frac{1}{2}(p_t + E_t p_{t+1}) = z_t. \quad (9.18)$$

Combining equations (9.16), (9.17), and (9.18) gives

$$p_t = \frac{1}{2} \{ [\frac{1}{2}(p_t + E_t p_{t+1}) + z_t] + [\frac{1}{2}(p_{t-1} + E_{t-1} p_t) + z_{t-1}] \} + v_t.$$

Consequently, the price level depends on past prices as well as future expected prices. The rate of inflation implied by this equation is

$$\Delta p_t = E_t \Delta p_{t+1} + 2(z_t + z_{t-1}) + 4v_t + \eta_t.$$

And if expectations are rational so that

$$\eta_t = -(p_t - E_{t-1} p_t)$$

with  $E_{t-1} \eta_t = 0$ , then the inflation rate is given by

$$\pi_t = E_t \pi_{t+1} + 2(z_t + z_{t-1}) + 4v_t + \eta_t. \quad (9.19)$$

This has the forward-looking solution

$$\pi_t = E_t \sum_{s=0}^{\infty} 4(z_{t+s} + v_{t+s}) + 2z_{t-1} + \eta_t. \quad (9.20)$$

Hence, following a temporary unit increase in log marginal productivity  $z_t$ , inflation increases in period  $t$  by 4 units and in period  $t+1$  by 2 units before returning to its initial level in period  $t+2$ . Equation (9.20) also implies that, following a permanent increase to  $z_t$ , inflation instantly increases without bound, which is implausible. The model only makes sense, therefore, if the long-run level of  $z_t$  is constrained to be zero but may temporarily depart from this. In this case, the steady-state level of inflation is equal to the logarithm of the markup.

Assuming that wage contracts last longer than  $n$  periods results in a price equation of the form

$$p_t = \sum_{s=1}^{n-1} \alpha_s E_t p_{t+s} + \sum_{s=1}^n \beta_s p_{t-s} + \frac{1}{n} \sum_{s=0}^{n-1} z_{t-s} + v_t + \xi_t,$$

where  $\xi_t$  is a linear combination of innovations in price; hence  $\xi_t$  is serially correlated. For each additional period there is an extra forward-looking and lagged price term and an additional lag in productivity.

#### 9.4.2 The Calvo Model of Staggered Price Adjustment

This is perhaps the most popular pricing model as it offers a simple way to derive a theory of dynamic behavior of the general price level while starting from a disaggregated theory of prices. The general price level is the average price of all firms. It is assumed that firms are forward looking and they forecast what the optimal price  $p_{t+s}^*$  ( $s \geq 0$ ), which is the same for all firms, should

be both in the future and in the current period. The crucial distinguishing features of Calvo pricing are that not all firms are able to adjust to the optimal price immediately and that adjustment, when it does occur, is exogenous to the firm and happens randomly. It is assumed that in any period there is a given probability  $\rho$  of a firm being able to make an adjustment to its price. Consequently,  $(1 - \rho)^s$  is the probability that in period  $t + s$  the price is still  $p_t$ . The drawback of this theory, therefore, is the restriction that firms have no control over when they can adjust their price.

When firms do adjust their price they set it to minimize the present value of the cost of deviations of the newly adjusted price  $p_t^\#$  from the optimal price. As soon as the adjustment takes place, given current information, this cost is expected to be zero both for the period of adjustment and for future periods. Thus the aim is to choose  $p_t^\#$  to minimize

$$\frac{1}{2} \sum_{s=0}^{\infty} \gamma^s E_t [p_t^\# - p_{t+s}^*]^2,$$

where  $\gamma = \beta(1 - \rho)$ .

Differentiating with respect to  $p_t^\#$  gives the first-order condition

$$\sum_{s=0}^{\infty} \gamma^s E_t [p_t^\# - p_{t+s}^*] = 0.$$

Hence, after adjustment, the new price is

$$p_t^\# = (1 - \gamma) \sum_{s=0}^{\infty} \gamma^s E_t p_{t+s}^*. \quad (9.21)$$

This can also be written as the recursion

$$p_t^\# = (1 - \gamma)p_t^* + \gamma E_t p_{t+1}^\#.$$

Consequently, like the Taylor model, the solution is forward looking.

Since the general price level is the average of all prices, and a proportion  $\rho$  of firms adjust their price in period  $t$ , the actual price level  $p_t$  is a weighted average of firms that are able to adjust and those that are not. Thus

$$p_t = \rho p_t^\# + (1 - \rho)p_{t-1}. \quad (9.22)$$

Eliminating  $p_t^\#$  using equation (9.21) gives

$$p_t = \rho(1 - \gamma) \sum_0^{\infty} \gamma^s E_t p_{t+s}^* + (1 - \rho)p_{t-1}.$$

Hence inflation is given by

$$\pi_t = \rho(1 - \gamma) \sum_0^{\infty} \gamma^s E_t [p_{t+s}^* - p_{t+s-1}],$$

or, expressed as a recursion, it is given by

$$\pi_t = \rho(1 - \gamma)(p_t^* - p_{t-1}) + \gamma E_t \pi_{t+1}. \quad (9.23)$$

Once again, therefore, we obtain a forward-looking solution for inflation. At time  $t$ , equation (9.23) shows that the actual change in price is related to the “desired” change in price  $p_t^* - p_{t-1}$  and to the expected future change in price. In steady state the actual price level equals the desired level and inflation is zero.

A modification of the basic Calvo model assumes that, if firms cannot reset their prices optimally, then they index their current price change to the past inflation rate. As a result, equation (9.22) is replaced by

$$\begin{aligned} p_t &= \rho p_t^\# + (1 - \rho)(\pi_{t-1} + p_{t-1}) \\ &= \rho p_t^\# + (1 - \rho)(2p_{t-1} - p_{t-1}). \end{aligned}$$

The auxiliary equation is

$$A(L) = 1 - 2(1 - \rho)L + (1 - \rho)L^2 = 0,$$

where  $L$  is the lag operator. As  $A(1) = \rho > 0$  and the coefficient of  $L^2$  is less than unity, both roots lie outside the unit circle and so the equation is stable. The solution may therefore be written as

$$\pi_t = \rho(p_t^\# - p_{t-1}) + (1 - \rho)\pi_{t-1}.$$

The inflation equation is then

$$\pi_t = \frac{\rho(1 - \gamma)}{1 + \rho(1 - \gamma)}(p_t^* - p_{t-1}) + \frac{\gamma}{\rho(1 - \gamma)}E_t\pi_{t+1} + \frac{1 - \rho}{\rho(1 - \gamma)}\pi_{t-1}.$$

Hence, there is an additional term in  $\pi_{t-1}$  on the right-hand side of the inflation equation, which implies that inflation takes time to adjust, and the coefficients of the other terms are different.

### 9.4.3 Optimal Dynamic Adjustment

Here we assume that firms trade off two types of distortion. One arises because changing prices is costly. The other is the cost of being out of equilibrium. The trade-off is expressed in terms of an intertemporal cost function involving the change in the logarithm of the price level,  $\Delta p_t$ , and deviations of the price level from its optimal long-run price  $p_t^*$ . The resulting intertemporal cost function is

$$C_t = \sum_{s=0}^{\infty} \beta^s E_t [\alpha(p_{t+s}^* - p_{t+s})^2 + (\Delta p_{t+s})^2].$$

The first term is the cost of being out of long-run equilibrium and the second is the cost of changing the price level. Firms seek to minimize the present value of these costs by a suitable choice of the current price  $p_t$ . The solution is the optimal short-run price level, as opposed to the optimal, or equilibrium, long-run price level.

The first-order condition is

$$\frac{\partial C_t}{\partial p_{t+s}} = 2E_t[\beta^s \{-\alpha(p_{t+s}^* - p_{t+s}) + \Delta p_{t+s}\} - \beta^{s+1}\Delta p_{t+s+1}] = 0.$$

For  $s = 0$ , this implies that

$$\Delta p_t = \alpha(p_t^* - p_t) + \beta E_t \Delta p_{t+1} \quad (9.24)$$

or

$$\pi_t = \frac{\alpha}{1 + \alpha}(p_t^* - p_{t-1}) + \frac{\beta}{1 + \alpha} E_t \pi_{t+1}. \quad (9.25)$$

Once more we have a forward-looking equation for inflation with  $\pi_t$  depending on the desired change in the price level and on  $E_t \pi_{t+1}$ . The greater is  $\alpha$ , the relative cost of being out of equilibrium, the larger is the coefficient on “desired” inflation  $\pi_t^* = p_t^* - p_{t-1}$ ; the greater is the discount factor  $\beta$ , the larger is the coefficient of future expected inflation. In steady state,  $p_t^* = p_t$  and  $\pi_t = E_t \pi_{t+1}$ .

A variant of the optimal dynamic adjustment model that results in  $\pi_{t-1}$  being an additional variable is obtained by assuming that only a fraction  $\lambda$  of firms set their prices in this way and that the rest,  $1 - \lambda$ , set prices using a rule of thumb based on the previous period’s inflation. The resulting inflation equation is

$$\pi_t = \lambda \frac{\beta}{1 + \alpha} E_t \pi_{t+1} + \lambda \frac{\alpha}{1 + \alpha} (p_t^* - p_{t-1}) + (1 - \lambda) \pi_{t-1}.$$

In this way inflation takes time to adjust. An alternative way of adding lagged inflation terms is to include additional terms in the cost function. For example, if it is costly to change the inflation rate, then a term in  $(\Delta p_t)^s$ ,  $s \geq 3$ , can be added. For  $s = 3$  the lag in inflation becomes an extra variable in the price equation.

#### 9.4.4 Price Level Dynamics

The Calvo model and the optimal dynamic adjustment model have the same form—only the interpretation of the coefficients differs. The Taylor model has a similar dynamic structure but a different coefficient on expected future inflation. The evidence suggests that additional dynamics may be required in the inflation equations (see, for example, Smith and Wickens 2007). These can be added to the Taylor model by extending the contract period; extra lags can be added to the Calvo model by assuming that firms who are unable to adjust prices optimally index on past inflation; extra lags to the optimal dynamic adjustment model may also be generated by assuming that firms set prices using a rule of thumb or by adding terms to the cost function.

A general formulation of the price equation that captures all three theories is

$$\pi_t = \alpha \pi_t^* + \beta E_t \pi_{t+1}, \quad |\beta| \leq 1, \quad (9.26)$$

where  $\pi_t = \Delta p_t$  and  $\pi_t^* = p_t^* - p_{t-1}$ . At first sight, this equation may seem to imply that there is no price stickiness, because it has the forward-looking solution

$$\pi_t = \alpha \sum_{s=0}^{\infty} \beta^s E_t \pi_{t+s}^*.$$

If we rewrite the model in terms of the price level, however, then we obtain

$$\Delta p_t = \alpha(p_t^* - p_{t-1}) + \beta E_t \Delta p_{t+1}, \quad (9.27)$$

or, in terms of the price level,

$$-\beta E_t p_{t+1} + (1 + \beta)p_t - (1 - \alpha)p_{t-1} = \alpha p_t^*, \quad (9.28)$$

which is a second-order difference equation.

Using the lag operator, this can be written as

$$-A(L)L^{-1}p_t = \alpha p_t^*.$$

The auxiliary equation associated with equation (9.28) is

$$A(L) = \beta - (1 + \beta)L + (1 - \alpha)L^2 = 0.$$

As  $A(1) = -\alpha < 0$ , the solution is a saddlepath. If the roots are denominated  $|\lambda_1| \geq 1$ ,  $|\lambda_2| < 1$ , then the solution can be written as (see the mathematical appendix)

$$(1 - \alpha)\lambda_1 \left(1 - \frac{1}{\lambda_1}L\right) (1 - \lambda_2 L^{-1}) p_t = \alpha p_t^*$$

or as the partial adjustment model

$$\Delta p_t = \left(1 - \frac{1}{\lambda_1}\right) (p_t^\# - p_{t-1}), \quad (9.29)$$

$$p_t^\# = \frac{\alpha}{(1 - \alpha)(\lambda_1 - 1)} \sum_{s=0}^{\infty} \lambda_2^s E_t p_{t+s}^*. \quad (9.30)$$

If expectations are static, so that  $E_t p_{t+s}^* = p_t^*$ , then

$$p_t^\# = \frac{\alpha}{(1 - \alpha)(\lambda_1 - 1)(1 - \lambda_2)} p_t^* = p_t^*.$$

Equations (9.29) and (9.30) show that, following a temporary or permanent disturbance to equilibrium, the adjustment of the price level takes time. In other words, prices are sticky.

For prices to be perfectly flexible—and price adjustment instantaneous—we require that  $\lambda_1 = 1$ , implying that  $\lambda_2 = \beta/(1 - \alpha)$ . If we rewrite equation (9.27) as

$$\Delta p_t = \frac{\alpha}{1 - \alpha} (p_t^* - p_t) + \frac{\beta}{1 - \alpha} E_t \Delta p_{t+1}, \quad (9.31)$$

then it is clear that this requires that  $\alpha = 1$ . Translated in terms of the parameters of the Calvo model, we require that  $\rho = 0$ , and in the terms of the optimal dynamic adjustment model we require that  $\alpha = \infty$ .

#### 9.4.4.1 Long-Run Equilibrium

In long-run equilibrium we expect that the solution for the price level is  $p_t = p_t^*$ . But equation (9.31) does not have this solution unless either the long-run rate of inflation is zero or  $\beta/(1 - \alpha) = 1$ . If  $\beta/(1 - \alpha) \neq 1$ , and long-run inflation is  $\pi$ , then, in order for the long-run solution to be  $p_t = p_t^*$ , equation (9.31) must include an intercept term so that the equation becomes

$$\Delta p_t = -\left(1 - \frac{\beta}{1 - \alpha}\right)\pi + \frac{\alpha}{1 - \alpha}(p_t^* - p_t) + \frac{\beta}{1 - \alpha}E_t\Delta p_{t+1}. \quad (9.32)$$

In the Calvo model,  $p_t = p_t^*$  in the long run only if  $\pi = 0$  or, when  $\pi > 0$ , if the probability of being able to adjust to equilibrium is unity, i.e., if  $\rho = 1$ . Similarly, in the optimal adjustment model, we require either that  $\pi = 0$  or, for  $\pi > 0$ , that the discount rate  $\beta = 1$ .

### 9.5 The New Keynesian Phillips Curve

The inflation equation is a key relation in models of inflation and monetary-policy analysis. In Keynesian models inflation is determined from the Phillips curve, an ad hoc relation between inflation and unemployment, which we may write in terms of price inflation and unemployment as

$$\pi_t = \alpha - \beta u_t, \quad (9.33)$$

where  $u_t$  is the unemployment rate. Equation (9.33) implies a permanent trade-off between inflation and unemployment. We note that the original Phillips curve used wage inflation and not price inflation.

Observing that the evidence increasingly failed to support a stable negative relation between inflation and unemployment, the Phillips curve came to be replaced by the expectations-augmented Phillips curve (see Friedman 1968; Phelps 1966), which takes the form

$$\pi_t = E_t\pi_{t+1} - \beta(u_t - u_t^n), \quad (9.34)$$

where  $u_t^n$  is the natural (or long-run equilibrium) rate of unemployment (i.e., the “nonaccelerating inflation rate of unemployment” (NAIRU)). In equation (9.34) there is only a short-run trade-off between inflation and unemployment, not a long-run trade-off. This is because unemployment will eventually return to its natural rate. When this happens inflation will equal expected future inflation, and so is not determined within the equation; i.e., equation (9.34) allows inflation to take any value in the long run. (We note that this is also a property of equation (9.31) when  $\beta/(1 - \alpha) = 1$ .) The absence of a long-run trade-off between inflation and unemployment seems to accord better with the evidence during the high-inflation years of the 1970s and 1980s. Figures 13.1–13.3 in chapter 13 plot Phillips curves for the United States and United Kingdom.

Later, in the 1990s, the evidence seemed to show that the natural rate of unemployment varied as much as the actual rate of unemployment, thereby

largely destroying any link between inflation and unemployment. This led to the development of the New Keynesian Phillips curve. This is closely related to the NAIRU model, but it has more explicit microfoundations and does not depend on unemployment to provide the link relating the real economy to inflation.

The New Keynesian Phillips curve is based on a model of optimal pricing in imperfect competition and a theory of price stickiness (see Roberts 1995, 1997; Clarida et al. 1999; McCallum and Nelson 1999; Svensson and Woodford 2003, 2004; Woodford 2003; Giannoni and Woodford 2005). From equation (9.32) we may express inflation as

$$\pi_t = -\left(1 - \frac{\beta}{1 - \alpha}\right)\pi + \frac{\alpha}{1 - \alpha}(p_t^* - p_t) + \frac{\beta}{1 - \alpha}E_t\pi_{t+1}, \quad (9.35)$$

where in long-run equilibrium  $p_t^* = p_t$  and  $\pi_t = \pi$ . In equation (9.35) inflation is generated by current expected future deviations of the actual price from the optimal price.

Our previous discussion of pricing in imperfect competition showed that the optimal price is a markup over marginal cost (see equations (9.1)–(9.3)). In this case we may write the logarithm of the optimal price  $p_t^*$  as

$$p_t^* = \mu_t + mc_t,$$

where  $mc_t$  is the log of marginal cost and  $\mu_t$  is the markup over marginal cost. The markup depends on the price elasticity of demand. From equation (9.1),

$$\mu_t \simeq -\epsilon_{D,t}.$$

Hence the greater the price elasticity, the smaller the markup. From equation (9.3), and in the case of a single factor, namely, labor,

$$\begin{aligned} mc_t &= -v_t + w_t - mp_t, \\ v_t &= \epsilon_{X_i,t}, \end{aligned}$$

where  $v_t$  is the labor markup, which depends on the labor-supply elasticity  $\epsilon_{X_i,t}$ ,  $w_t$  is the log of the nominal-wage rate, and  $mp_t$  is the log of the marginal product of labor. The less elastic the labor-supply function is, the higher the marginal cost. For a Cobb–Douglas production function written in logs

$$y_t = a_t + \phi n_t, \quad 0 < \phi < 1,$$

where  $y_t$  is output,  $n_t$  is employment, and  $a_t$  is technological progress, we have

$$\begin{aligned} mp_t &= \ln \phi + y_t - n_t \\ &= \ln \phi + \frac{a_t}{\phi} - \frac{1 - \phi}{\phi} y_t. \end{aligned} \quad (9.36)$$

The optimal price is therefore

$$p_t^* = -\ln \phi + \mu_t - v_t + w_t - \frac{a_t}{\phi} + \frac{1 - \phi}{\phi} y_t, \quad (9.37)$$



and so the deviation of the optimal price from the actual price is

$$p_t^* - p_t = -\ln \phi + \mu_t - \nu_t - \frac{a_t}{\phi} + \frac{1-\phi}{\phi} y_t + (w_t - p_t). \quad (9.38)$$

Substituting equation (9.38) into (9.35) gives the inflation equation

$$\begin{aligned} \pi_t = - & \left[ \left(1 - \frac{\beta}{1-\alpha}\right) \pi + \frac{\alpha \ln \phi}{(1-\alpha)} \right] + \frac{\beta}{1-\alpha} E_t \pi_{t+1} + \frac{\alpha(1-\phi)}{(1-\alpha)\phi} y_t \\ & + \frac{\alpha}{(1-\alpha)} (w_t - p_t) - \frac{\alpha}{(1-\alpha)\phi} a_t + \frac{\alpha}{(1-\alpha)} (\mu_t - \nu_t). \end{aligned} \quad (9.39)$$

By exploiting the fact that in equilibrium  $p_t^* = p_t$ , we can write the inflation equation (9.39) in another way. Denoting equilibrium values by an asterisk and deviations from equilibrium by a tilde, so that  $\tilde{p}_t = p_t^* - p_t$ , equation (9.38) implies that

$$0 = -\ln \phi + \mu_t^* - \nu_t^* - \frac{a_t^*}{\phi} + \frac{1-\phi}{\phi} y_t^* + (w_t^* - p_t^*),$$

and hence

$$\tilde{p}_t = -\tilde{\mu}_t + \tilde{\nu}_t - \frac{\tilde{a}_t}{\phi} - \frac{1-\phi}{\phi} \tilde{y}_t - (\tilde{w}_t - \tilde{p}_t). \quad (9.40)$$

It then follows from equations (9.35) and (9.40) that the inflation equation can be written as

$$\begin{aligned} \pi_t = - & \left(1 - \frac{\beta}{1-\alpha}\right) \pi + \frac{\beta}{1-\alpha} E_t \pi_{t+1} - \frac{\alpha}{1-\alpha} \tilde{\mu}_t + \frac{\alpha}{1-\alpha} \tilde{\nu}_t \\ & + \frac{\alpha}{(1-\alpha)\phi} \tilde{a}_t - \frac{\alpha(1-\phi)}{(1-\alpha)\phi} \tilde{y}_t - \frac{\alpha}{1-\alpha} (\tilde{w}_t - \tilde{p}_t). \end{aligned} \quad (9.41)$$

Hence inflation will increase if  $y_t > y_t^*$ , i.e., if output is above its equilibrium level (which is sometimes measured in empirical work by its trend level), or if the real wage or the price markup exceed their equilibrium levels, or if the labor markup is below its equilibrium level, or if there is a negative technology shock.

A number of observations may be made about equation (9.39). First, it is more complex than the usual specification of the New Keynesian inflation equation, which does not include the real-wage term, the markups, or the productivity shock. Assuming that equation (9.41) is correct, it would, of course, be a specification error to omit these terms. On the other hand, the markup and productivity terms may be small relative to the output and real-wage terms. Second, the equation is based on having a single factor of production: labor. In practice, there are other factors: for example, physical capital and material inputs. There is therefore an argument for deriving a more complete model of inflation that takes these into account. Third, and related to this, we have previously argued that a general increase in the prices of all factors is required for the effect on inflation to be sizeable in the longer term. An increase in the unit cost of a single factor (for example, an oil price increase) may not have a significant effect on

inflation for long due to factor substitution in the longer term. Fourth, in equation (9.36) we expressed the marginal product of labor in terms of output. We could, however, have expressed it in terms of labor, in which case the equation would become

$$mp_t = \ln \phi + a_t - (1 - \phi)n_t.$$

The resulting inflation equation would then be

$$\begin{aligned} \pi_t = & -\left(1 - \frac{\beta}{1 - \alpha}\right)\pi + \frac{\beta}{1 - \alpha}E_t\pi_{t+1} - \frac{\alpha}{1 - \alpha}\tilde{\mu}_t + \frac{\alpha}{1 - \alpha}\tilde{v}_t \\ & + \frac{\alpha}{(1 - \alpha)}\tilde{a}_t - \frac{\alpha(1 - \phi)}{(1 - \alpha)}\tilde{n}_t - \frac{\alpha}{1 - \alpha}(\tilde{w}_t - \tilde{p}_t), \end{aligned} \quad (9.42)$$

where we may interpret  $\tilde{n}_t$ , the deviation of employment from its long-run equilibrium value, as unemployment.

### 9.5.1 The New Keynesian Phillips Curve in an Open Economy

So far we have measured inflation in terms of the GDP deflator. Monetary policy is, however, usually conducted with reference to the consumer price index (CPI). In a closed economy there is little difference between the GDP deflator and the CPI, but in an open economy there is an important difference as the CPI also reflects the price of foreign tradeables. Inflation measured by the GDP deflator is

$$\pi_t = (1 - s_t^{\text{nt}})\pi_t^{\text{t}} + s_t^{\text{nt}}\pi_t^{\text{nt}},$$

where  $\pi_t$  is the inflation rate of domestically produced goods and services and  $s_t^{\text{nt}}$  is the share of nontraded goods. This is a weighted average of  $\pi_t^{\text{nt}}$ , the inflation rate of domestic nontraded goods, and  $\pi_t^{\text{t}}$ , the inflation rate of domestic traded goods. CPI inflation is measured by a weighted average of  $\pi_t$  and the inflation rate of imported goods,  $\pi_t^{\text{m}}$ , and is given by

$$\pi_t^{\text{cpi}} = (1 - s_t^{\text{m}})\pi_t + s_t^{\text{m}}\pi_t^{\text{m}},$$

where  $s_t^{\text{m}}$  is the share of imports.

In an economy where producers have little or no monopoly power—a typical situation for a small economy—domestic traded goods prices are equal to world prices expressed in domestic currency. Thus

$$\pi_t^{\text{t}} = \pi_t^{\text{m}} = \pi_t^{\text{w}} + \Delta s_t,$$

where  $\pi_t^{\text{w}}$  is the world inflation rate and  $\Delta s_t$  is the proportionate rate of change of the exchange rate (the domestic price of foreign exchange). In an open economy in which producers have a degree of monopoly power, such as a large economy, import prices will be fully or partly priced to market. Consequently,

$$\pi_t^{\text{m}} = \varphi(1 - \eta)(\pi_t^{\text{w}} + \Delta s_t) + (1 - \varphi)\eta\pi_t^{\text{t}},$$

where  $\varphi = 1$  for full exchange rate pass through, and  $\eta = 1$  for full pricing-to-market (both lie in the interval  $[0, 1]$ ), and  $\pi_t^{\text{t}}$  is determined domestically.

Thus, measured by the GDP deflator, inflation in a small open economy is

$$\pi_t = s_t^{\text{nt}} \pi_t^{\text{nt}} + (1 - s_t^{\text{nt}})(\pi_t^{\text{w}} + \Delta s_t),$$

and in a large open economy it is

$$\pi_t = (1 - s_t^{\text{nt}})\pi_t^{\text{t}} + s_t^{\text{nt}}\pi_t^{\text{nt}}.$$

CPI inflation in a small open economy is given by

$$\pi_t^{\text{cpi}} = (1 - s_t^{\text{m}})s_t^{\text{nt}}\pi_t^{\text{nt}} + [1 - s_t^{\text{m}}(1 - s_t^{\text{nt}})](\pi_t^{\text{w}} + \Delta s_t),$$

and in a large open economy it is given by

$$\pi_t^{\text{cpi}} = (1 - s_t^{\text{m}})s_t^{\text{nt}}\pi_t^{\text{nt}} + [(1 - s_t^{\text{m}})(1 - s_t^{\text{nt}}) + s_t^{\text{m}}(1 - \varphi)\eta]\pi_t^{\text{t}} + s_t^{\text{m}}\varphi(1 - \eta)(\pi_t^{\text{w}} + \Delta s_t).$$

Consequently, the impact of changes in the exchange rate on inflation depends on how inflation is measured and on the size of the economy. It has little or no effect on the GDP deflator for a large open economy. For a small economy, it has a greater effect on CPI inflation than on GDP inflation.

To complete the model of CPI inflation we need to specify traded and non-traded goods inflation. Suppose that they are identical, and hence equal to GDP inflation, and that they are determined by the New Keynesian Phillips curve. Then, from equation (9.35),

$$\pi_t = \frac{\alpha}{1 - \alpha} \sum_{s=0}^{\infty} \left( \frac{\beta}{1 - \alpha} \right)^s E_t(p_{t+s}^* - p_{t+s}),$$

provided  $\beta/(1 - \alpha) < 1$ . Hence CPI inflation is given by

$$\begin{aligned} \pi_t^{\text{cpi}} &= [(1 - s_t^{\text{m}}) + s_t^{\text{m}}(1 - \varphi)\eta]\pi_t + s_t^{\text{m}}\varphi(1 - \eta)(\pi_t^{\text{w}} + \Delta s_t) \\ &= \frac{\delta\alpha}{1 - \alpha}(p_t^* - p_t) + \frac{\beta}{1 - \alpha}E_t\pi_{t+1}^{\text{cpi}} + s_t^{\text{m}}\delta(1 - \eta)(\pi_t^{\text{w}} + \Delta s_t) \\ &\quad - \frac{\beta}{1 - \alpha}E_t[s_{t+1}^{\text{m}}\varphi(1 - \eta)(\pi_{t+1}^{\text{w}} + \Delta s_{t+1})], \end{aligned}$$

where  $p_t$  is the GDP price level and  $\delta = [(1 - s_t^{\text{m}}) + s_t^{\text{m}}(1 - \varphi)\eta]$ . Thus, CPI inflation replaces GDP inflation and includes world inflation expressed in domestic currency.

## 9.6 Conclusions

The evidence shows that prices are not perfectly flexible and that the frequency of price changes varies between different types of goods and services. This suggests that prices are not determined in perfectly competitive markets. Modern theories of price determination adopt an optimizing framework but seek to explain price stickiness by assuming imperfect competition. We have extended this to price determination in the open economy. As a result, prices and output differ from the levels that would prevail in perfect competition.

The three leading theories of price stickiness yield very similar models of inflation. These, together with the assumption of imperfect competition, form the basis of the New Keynesian Phillips curve. In an open economy it is necessary to distinguish between GDP and CPI inflation. The New Keynesian open-economy Phillips curve includes an additional variable: the rate of inflation of world prices measured in domestic currency. As the impact of foreign inflation on domestic inflation is affected by the exchange rate, the exchange rate provides an additional channel in the transmission mechanism of monetary policy. The significance of this channel depends on the degree of imperfect competition in traded goods markets, which affects how the prices of traded goods are set in foreign markets. After including all of the features we have discussed, the resulting price equation has become quite complex. It is therefore common in monetary-policy analysis to use a simplified version of the price equation that is closely related to the basic Calvo model.