4
The Decentralized Economy

4.1 Introduction

So far we have considered a stylized model of the economy in which a single economic agent makes every decision: consumption, saving, leisure, work, investment, and capital accumulation. An alternative interpretation we gave to this model was that a central planner was making all of these decisions for each person in the economy and was taking the same decision for everyone so that there was, in effect, a single household (or person) or, more generally, representative economic agent. In this interpretation there is no need for a market structure as all decisions are automatically coordinated.

We now generalize this model by introducing a distinction between households and firms. Households will take consumption decisions, they will own firms (and will therefore receive dividend income from firms), they will supply labor to firms, and they will save in the form of financial assets. Firms act as the agents of households. They make output, investment, and employment decisions, determine the size of the capital stock, borrow from households to finance investment, pay wages to households, and distribute their profits to households in the form of dividends. In separating the decisions of households and firms we introduce a number of additional economic variables. In order to coordinate the separate decisions of households and firms, we also need to introduce product, labor, and capital markets.

As a result of making these changes, the model is becoming more recognizable as a macroeconomic system. The model is also becoming considerably more complex. To simplify the analysis, we delay considering labor issues. First we consider household decisions on consumption and savings, taking the supply of labor as fixed. We then make the work/leisure decision endogenous. Next we derive the firm's decisions on investment, capital accumulation, debt finance, and, after these, employment. We then show how markets coordinate the separate decisions of households and firms to bring about general equilibrium in the economy. In the process we require markets for goods, labor, equity, and bonds. We find that the behavior of the decentralized economy when in general equilibrium is remarkably similar to that of the basic representative-agent model discussed previously.

4.2 Consumption

4.2.1 The Consumption Decision

It is assumed that the representative household seeks to maximize the present value of utility,

$$\max_{\{c_{t+s}, a_{t+s}\}} V_t = \sum_{s=0}^{\infty} \beta^s U(c_{t+s}), \tag{4.1}$$

subject to its budget constraint

$$\Delta a_{t+1} + c_t = x_t + r_t a_t, \tag{4.2}$$

where, as before, c_t is consumption, $U(c_t)$ is instantaneous utility ($U_t' > 0$ and $U_t'' \leqslant 0$), the discount factor is $0 < \beta = 1/(1+\theta) < 1$. a_t is the (net) stock of financial assets at the beginning of period t; if $a_t > 0$ then households are net lenders, and if $a_t < 0$ they are net borrowers. r_t is the interest rate on financial assets during period t and is paid at the beginning of the period, and x_t is household income, which is assumed for the present to be exogenous. At this point we do not need to specify what a_t and x_t are. Later, in the absence of government, we show that a_t is solely corporate debt and x_t is income from labor plus dividend income from the ownership of firms. All of these variables continue to be specified in real terms.

At the beginning of period t the stock of financial assets (and firm capital, which is not a variable chosen by households) is given. Thus households must choose $\{c_t, a_{t+1}\}$ in period t, $\{c_{t+1}, a_{t+2}\}$ in period $t+1$, and so on. This is equivalent to choosing the complete path of consumption, i.e., current and all future consumption, $\{c_t, c_{t+1}, c_{t+2}, \dots\}$. The main changes compared with the basic model, therefore, are the replacement of the capital stock with the stock of financial assets, the introduction of the interest rate explicitly, and the replacement of the national resource constraint with the household budget constraint.

The solution to this problem can be obtained, as before, using the method of Lagrange multipliers. The Lagrangian is defined as

$$\mathcal{L} = \sum_{s=0}^{\infty} \{\beta^s U(c_{t+s}) + \lambda_{t+s}[x_{t+s} + (1+r_{t+s})a_{t+s} - c_{t+s} - a_{t+s+1}]\}. \tag{4.3}$$

The first-order conditions are

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial c_{t+s}} &= \beta^s U'(c_{t+s}) - \lambda_{t+s}, & s \geqslant 0, \\ \frac{\partial \mathcal{L}}{\partial a_{t+s}} &= \lambda_{t+s}(1+r_{t+s}) - \lambda_{t+s-1} = 0, & s > 0, \end{aligned}$$

together with the budget constraint.

Solving the first-order conditions for $s = 1$ to eliminate λ_{t+s} gives the Euler equation:

$$\frac{\beta U'(c_{t+1})}{U'(c_t)}(1 + r_{t+1}) = 1. \tag{4.4}$$

Equation (4.4) is identical to the Euler equation derived for the basic model if $r_{t+1} = F'(k_{t+1}) - \delta$. This is why previously we interpreted the net marginal product of capital, $F'(k_{t+1}) - \delta$, as the real interest rate.

4.2.2 The Intertemporal Budget Constraint

The household's problem can be expressed in another way. This uses the intertemporal budget constraint, which is derived from the one-period budget constraints by successively eliminating $a_{t+1}, a_{t+2}, a_{t+3}, \dots$. The budget constraints in periods t and $t+1$ are

$$a_{t+1} + c_t = x_t + (1 + r_t)a_t,$$
$$a_{t+2} + c_{t+1} = x_{t+1} + (1 + r_{t+1})a_{t+1}.$$

Combining these to eliminate a_{t+1} gives the two-period intertemporal budget constraint

$$a_{t+2} + c_{t+1} + (1 + r_{t+1})c_t = x_{t+1} + (1 + r_{t+1})x_t + (1 + r_{t+1})(1 + r_t)a_t. \quad (4.5)$$

This can be rewritten as

$$\frac{a_{t+2}}{1 + r_{t+1}} + \frac{c_{t+1}}{1 + r_{t+1}} + c_t = \frac{x_{t+1}}{1 + r_{t+1}} + x_t + (1 + r_t)a_t. \quad (4.6)$$

Further substitutions of $a_{t+2}, a_{t+3}, \dots$ give the wealth of the household as

$$W_t = \frac{a_{t+n}}{\prod_{s=1}^{n-1}(1 + r_{t+s})} + \sum_{s=0}^{n-1} \frac{c_{t+s}}{\prod_{s=1}^{n-1}(1 + r_{t+s})} \quad (4.7)$$
$$= \sum_{s=0}^{n-1} \frac{x_{t+s}}{\prod_{s=1}^{n-1}(1 + r_{t+s})} + (1 + r_t)a_t. \quad (4.8)$$

Thus wealth can be measured either in terms of its source as the present value of current and future income plus initial financial assets (equation (4.8)), or in terms of its use as the present value of current and future consumption plus the discounted value of terminal financial assets (equation (4.7)).

Taking the limit of wealth as $n \to \infty$ gives the infinite intertemporal budget constraint. When the interest rate is constant (equal to r), wealth can be written as

$$W_t = \sum_0^{\infty} \frac{c_{t+s}}{(1 + r)^s} = \sum_0^{\infty} \frac{x_{t+s}}{(1 + r)^s} + (1 + r)a_t. \quad (4.9)$$

An alternative way to express the household's problem would be to maximize V_t (equation (4.1)) subject to the constraint on wealth (equation (4.9)). This would then involve a single Lagrange multiplier.

We note that we also need an extra optimality condition, namely, the transversality condition, which is

$$\lim_{n \to \infty} \beta^n a_{t+n} U'(c_{t+n}) = 0. \quad (4.10)$$

Financial assets in period $t + n$ if consumed would give a discounted utility of $\beta^n a_{t+n} U(c_{t+n})$. Equation (4.10) implies that as $n \to \infty$ their discounted value goes to zero. Since $U'(c_{t+n})$ is positive for finite c_{t+n}, this implies that $\lim_{n\to\infty} \beta^n a_{t+n} = 0$. And as

$$\beta^n = \frac{1}{\prod_{s=1}^{n-1}(1 + r_{t+s})}$$

in the steady state, we obtain

$$\lim_{n\to\infty} \frac{a_{t+n}}{\prod_{s=1}^{n-1}(1 + r_{t+s})} \geqslant 0. \tag{4.11}$$

This is known as the no-Ponzi-game (NPG) condition. It implies that households are unable to finance consumption indefinitely by borrowing, i.e., by having negative financial assets.

4.2.3 Interpreting the Euler Equation

An interpretation similar to that proposed in chapter 2 can be given to the Euler equation. Again we reduce the problem to two periods, and then consider reducing c_t by a small amount $\mathrm{d}c_t$ and asking how much larger c_{t+1} must be to fully compensate for this, i.e., in order to leave V_t unchanged. Thus we let

$$V_t = U(c_t) + \beta U(c_{t+1}).$$

Differentiating V_t, and recalling that V_t remains constant, implies that

$$0 = \mathrm{d}V_t = \mathrm{d}U_t + \beta\,\mathrm{d}U_{t+1} = U'(c_t)\,\mathrm{d}c_t + \beta U'(c_{t+1})\,\mathrm{d}c_{t+1},$$

where $\mathrm{d}c_{t+1}$ is the small change in c_{t+1} brought about by reducing c_t. The loss of utility in period t is therefore $U'(c_t)\,\mathrm{d}c_t$. In order for V_t to be constant, this must be compensated by the discounted gain in utility $\beta U'(c_{t+1})\,\mathrm{d}c_{t+1}$. Hence we need to increase c_{t+1} by

$$\mathrm{d}c_{t+1} = -\frac{U'(c_t)}{\beta U'(c_{t+1})}\,\mathrm{d}c_t. \tag{4.12}$$

All of this is the same as for the centralized model.

We now use the two-period intertemporal budget constraint, equation (4.5). Assuming that the interest rate, exogenous income, and the asset holdings a_t and a_{t+2} are unchanged, the intertemporal budget constraint implies that

$$\mathrm{d}c_{t+1} = -(1 + r_{t+1})\,\mathrm{d}c_t,$$

and hence that

$$-\frac{\mathrm{d}c_{t+1}}{\mathrm{d}c_t} = 1 + r_{t+1}. \tag{4.13}$$

Combining (4.12) and (4.13) gives

$$-\frac{\mathrm{d}c_{t+1}}{\mathrm{d}c_t} = \frac{U'(c_t)}{\beta U'(c_{t+1})} = 1 + r_{t+1}, \tag{4.14}$$

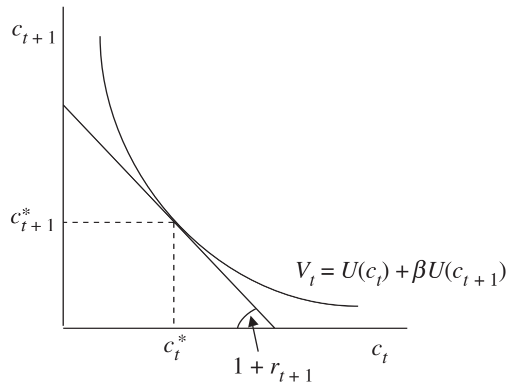

Figure 4.1. Two-period solution.

implying that

$$U'(c_t)(-\mathrm{d}c_t) = \beta U'(c_{t+1})[1 + r_{t+1}]\,\mathrm{d}c_t.$$

Thus the reduction in utility in period t due to cutting consumption and increasing saving, $U'(c_t)\,\mathrm{d}c_t$, is compensated by the discounted increase in utility in period $t+1$ created by the interest income generated from the additional saving, $\beta U'(c_{t+1})[1 + r_{t+1}]\,\mathrm{d}c_{t+1}$.

Solely for the sake of convenience, we consider the case where the interest rate is a constant equal to r in depicting the solution in figure 4.1. The maximum value of c_t occurs when $c_{t+1} = 0$ and we consume the whole of next period's income by borrowing today and repaying the loan with next period's income. The maximum value of c_{t+1} occurs when $c_t = 0$ and we save all of the current period's income. Thus, from equations (4.5) and (4.6),

$$\max c_t = x_t + \frac{x_{t+1}}{1+r} + (1+r)a_t,$$
$$\max c_{t+1} = (1+r)x_t + x_{t+1} + (1+r)^2 a_t.$$

These determine the points at which the budget constraint touches the two axes. The slope of the budget constraint is $-(1+r)$. The optimal solution occurs where the budget constraint is tangent to the highest attainable indifference curve.

An increase in income in either period t or $t+1$ shifts the budget constraint to the right and results in higher c_t, c_{t+1}, and V_t.

An increase in the interest rate (from r_0 to r_1) makes the budget constraint steeper, as shown in figure 4.2. It also affects the maximum values of c_t and c_{t+1}. If $a_t = 0$, then there is a decrease in the maximum value of c_t (from $c^*_{t,0}$ to $c^*_{t,1}$) because the amount that can be borrowed on future income falls; and there is an increase in the maximum value of c_{t+1}, because the interest earned by saving current income rises. The result is an intertemporal substitution of consumption in which c_t falls and c_{t+1} rises. The effect on V_t is ambiguous: the point of tangency of the budget constraint may be on the same indifference

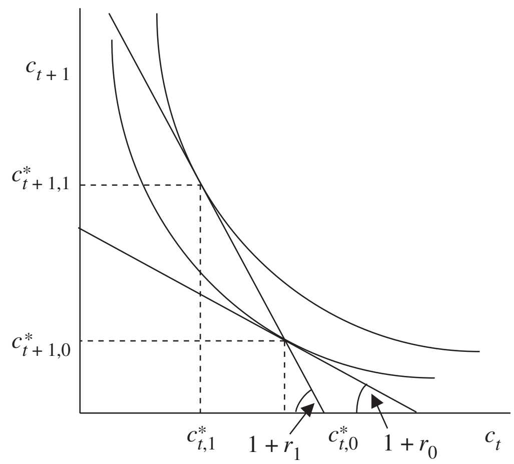

Figure 4.2. The effect of an increase in the interest rate.

curve, one to the left (implying a loss of discounted utility), or one to the right (implying a gain in discounted utility).

When $a_t < 0$, we obtain the same outcome, except that V_t is unambiguously reduced. When $a_t > 0$, $\max c_{t+1}$ still increases, and if $x_{t+1}/(1+r) < (1+r)a_t$, then $\max c_t$ now increases. This would then result in higher c_t, c_{t+1}, and V_t; this case is depicted in figure 4.2. In practice, as most of household financial wealth is in the form of pension entitlements, and this is sufficiently far in the future to be heavily discounted, households—especially those with large mortgages—probably behave in the short run as though they are net debtors (i.e., as if $a_t < 0$). We conclude, therefore, that in practice an increase in r is likely to cause c_t to fall and c_{t+1} to rise.

4.2.4 The Consumption Function

What factors affect the behavior of consumption? We have shown already that consumption in period t increases if income or net assets increase, and it is likely to decrease if the interest rate increases. We now examine the behavior of consumption in more detail. We consider the traditional consumption function—the behavior of consumption in period t—together with the future behavior of consumption along the economy's optimal path.

First we examine the behavior of consumption on the optimal path. Exactly what this means will become clear. First we take a linear approximation to the Euler equations, (2.12) and (2.12). Using a first-order Taylor series expansion of $U'(c_{t+1})$ about c_t we obtain

$$\begin{aligned}\frac{U'(c_{t+1})}{U'(c_t)} &\simeq 1 + \frac{U''}{U'}\Delta c_{t+1} \\ &= 1 - \sigma\frac{\Delta c_{t+1}}{c_t}, \end{aligned} \tag{4.15}$$

where $\sigma = -cU''/U'$ is the coefficient of relative risk aversion (CRRA). In general the CRRA will be time-varying, but, again for convenience, we consider the case where it is constant. Solving (2.12) and (4.15) we obtain the future rate of growth of consumption along the optimal path as

$$\frac{\Delta c_{t+1}}{c_t} = \frac{1}{\sigma}\left[1 - \frac{1}{\beta(1+r_{t+1})}\right] \simeq \frac{r_{t+1}-\theta}{\sigma(1+r_{t+1})}. \tag{4.16}$$

Thus, if $r_{t+1} = \theta$, then optimal consumption in the future will remain at its period t value. In this case households are willing to save until the rate of return on savings falls to equal the rate of time discount θ. This is the long-run general equilibrium solution. For interest rates below θ households prefer to consume than save. In the short run, the interest rate will typically differ from θ. If $r_{t+1} > \theta$ consumption will be growing along the optimal path, but this will not be sustained due to the rate of return to saving falling. This is a result of the diminishing marginal product of capital.

Consumption in period t—the consumption function—is obtained by combining equation (4.16) with the intertemporal budget constraint (4.7). Again for convenience we assume that the interest rate is the constant r. The generalization to a time-varying interest rate is straightforward. We also assume that $r = \theta$, its steady-state value, when optimal consumption in the future remains at its period t value. This enables us to replace c_{t+s} $(s > 0)$ in equation (4.9) by c_t to obtain

$$\begin{aligned} W_t &= \sum_0^\infty \frac{c_{t+s}}{(1+r)^s} = \frac{1+r}{r} c_t \\ &= \sum_0^\infty \frac{x_{t+s}}{(1+r)^s} + (1+r)a_t. \end{aligned}$$

Hence

$$c_t = \frac{r}{1+r} W_t = r \sum_0^\infty \frac{x_{t+s}}{(1+r)^{s+1}} + r a_t. \tag{4.17}$$

Equation (4.17) implies that consumption in period t is proportional to wealth. This solution for c_t is forward looking. It implies that an anticipated change in income in the future will have an immediate effect on current consumption. In general equilibrium, income is determined by labor and capital so this solution is similar to that for the basic model. This solution has been called the "life-cycle hypothesis" for reasons that will be explained later (see Modigliani and Brumberg 1954; Modigliani 1970). It has also been called the "permanent income hypothesis," as the present-value term in income can be interpreted as the amount of wealth that can be spent each period without altering wealth (see Friedman 1957). It also implies that temporary increases in wealth should be saved and temporary falls should be offset by borrowing.

In the special case where $x_{t+s} = x_t$ $(s \geqslant 0)$, the consumption function, equation (4.17), becomes

$$c_t = x_t + r a_t. \tag{4.18}$$

Thus, c_t is equal to total current income, i.e., income from savings, ra_t, plus income from other sources, x_t. Equation (4.18) can be interpreted as the familiar Keynesian consumption function. We note that it implies that the marginal and the average propensities to consume are unity.

We now have a complete description of consumption. The optimal path determines how consumption will behave in the future relative to current consumption. The consumption function determines today's consumption, and hence where the optimal path is located. The same information is contained in the consumption functions for periods $t, t+1, t+2, \dots$. Hence together they are an equivalent representation to current consumption and the optimal path.

4.2.5 Permanent and Temporary Shocks

In our discussion of the effects on consumption of changes in income and interest rates we have made no distinction between whether the changes are permanent or temporary. It is vital to make this distinction as the results are quite different. The policy implications of this are of great importance. First we consider shocks to income.

4.2.5.1 Income

If, for convenience, we assume that the real rate of interest is constant, then, in general, consumption is determined by equation (4.17). A permanent change in income in period t will affect $x_t, x_{t+1}, x_{t+2}, \dots$. If, again for convenience, we assume that $x_{t+s} = x_t$ ($s \geqslant 0$), then we can analyze the effect of a change in x. In this case the consumption function simplifies to become equation (4.18). It follows that both the marginal and the average propensities to consume following a permanent change in income are unity, i.e., none of the increase in income is saved. A permanent change in income is the prevailing state for an economy that is growing through time.

A temporary change in income is analyzed using equation (4.17). We rewrite the equation as

$$c_t = \frac{r}{1+r}x_t + \frac{r}{(1+r)^2}x_{t+1} + \cdots + ra_t.$$

It follows that a change in x_t (but not in $x_{t+1}, x_{t+2}, \dots$) has a marginal propensity to consume of only $r/(1+r)$. Hence, most of the increase in x_t is saved rather than consumed. We also note that an expected increase in x_{t+1} will also cause c_t to increase, but the marginal propensity to consume out of x_{t+1} is $r/(1+r)^2$, which is lower due to discounting income in period $t+1$ by $1/(1+r)$. The effect on consumption of a permanent shock to income is the discounted sum of current and expected future income effects.

Thus the Keynesian marginal propensity of unity implicitly assumes that the change in income is permanent, not temporary. The importance of this distinction for policy is clear. A policy change (such as a temporary cut in income tax) that is designed to affect income only temporarily will have little effect on consumption.

4.2.5.2 Interest Rates

First, we recall that the interest rate r_t is the real interest rate. A permanent change in real interest rates implies that $r_t, r_{t+1}, r_{t+2}, \dots$ all increase. This can be analyzed using equation (4.18) and treated as an increase in r. The effect on consumption depends on whether the household has net assets or net debts (i.e., on whether $a_t > 0$ or $a_t < 0$). If $a_t > 0$ then interest income—and hence total income—is increased permanently. The marginal propensity to consume from this increase is unity as none of the additional interest income is saved. But if $a_t < 0$ then debt service payments increase permanently and total income decreases. Consumption will therefore fall.

A temporary increase in interest rates will be treated by households as though it were a temporary change in income. For example, equation (2.1) shows that an increase in r_t just affects current interest earnings (or debt service payments). Hence, most of the additional interest income is saved, not consumed. In contrast, an expected increase in r_{t+1} requires us to use equation (4.16). It affects consumption by causing a substitution of consumption across time (i.e., an intertemporal substitution). This was analyzed earlier. We recall that the response of consumption depends on whether a_t is positive or negative. If it is negative, and hence the household is a net debtor, then there is an unambiguous decrease in c_t and increase in c_{t+1}. Thus, once again, there is an important difference between a permanent and a temporary change.

In practice, real interest rates tend to fluctuate about an approximately constant mean. This implies that a permanent increase in real interest rates is improbable. At best, it might prove a convenient way of analyzing a change in interest rates that is thought will last for many periods. The sustained rise in stock market returns in the 1990s is a possible example. This continued for so long that households may have treated it as more or less permanent. This may explain why the savings rate fell over this period and why consumption did not turn down when the stock market did. Perhaps consumers took the view that the fall in the stock market would be temporary and so they tried to maintain their level of consumption. Apart from relatively rare cases like this, it will usually prove more useful, especially for policy analysis, to treat the analysis of a change in real interest rates as being temporary, and to suppose that the effect of an increase in real interest rates will be to reduce current consumption.

4.2.5.3 Anticipated and Unanticipated Shocks to Income

Because consumption depends on wealth, and wealth is forward looking, unanticipated future changes in income and interest rates will have no effect on current consumption. But changes that are anticipated at time t will affect current consumption. The distinction between anticipated and unanticipated future changes in income helps to explain a confusion that prevailed for a time in the literature.

It was claimed that (4.16) was a rival consumption function to equation (4.17). As equation (4.16) appeared to suggest that consumption would be unaffected by income, it was seen as an inferior theory. For example, if $r_{t+1} = r = \theta$, then equation (4.16) implies that

$$c_{t+1} = c_t, \tag{4.19}$$

which does not involve income explicitly; c_{t+1} is just determined from knowledge of c_t.

To see why this interpretation is incorrect consider the effect on consumption of a permanent but unanticipated increase in income in period $t+1$. Thus there is an increase in $x_{t+1}, x_{t+2}, \ldots$. Taking the first difference of equation (4.18) and noting that the stock of assets is unchanged, the consumption function for period $t+1$ can be written

$$\begin{aligned} c_{t+1} &= x_{t+1} + r a_{t+1} \\ &= c_t + (x_{t+1} - x_t) + r(a_{t+1} - a_t) \\ &= c_t + (x_{t+1} - x_t). \end{aligned}$$

If income remains unchanged ($x_{t+1} = x_t$), then consumption would be unchanged too and would satisfy equation (4.19). But if $x_{t+1} > x_t$, then $c_{t+1} > c_t$. Thus consumption in period $t+1$ has responded to the unanticipated increase in income in period $t+1$. From equation (4.17), if the increase in x_{t+1} had been anticipated in period t, then c_t would have changed too. As a result, knowledge of c_t would be sufficient for determining c_{t+1} as in equation (4.19), and there would be no *additional* information possessed by income.

This illustrates the limitations of working with the assumption of perfect foresight when, strictly, we should allow for uncertainty about the future. Accordingly, we should write equation (4.19) as

$$E_t c_{t+1} = c_t, \tag{4.20}$$

where E_t is the expectation conditional on information available up to and including period t. If expectations are rational, then the expectational error

$$e_{t+1} = c_{t+1} - E_t c_{t+1}$$

is unpredictable from information dated at time t, i.e., $E_t e_{t+1} = 0$. This would imply that consumption is a martingale process. (In the special case where $\mathrm{var}_t(\Delta c_{t+1})$ is constant the martingale process is given the more familiar name of a random walk.) Equation (4.20) was first derived by Hall (1978).

We have shown therefore that equation (4.19) is not an alternative theory of consumption, but is a description of the anticipated future behavior of consumption relative to today's consumption. Current consumption is given by the consumption function, equation (4.17) or, when income is expected to be constant, by equation (4.18). A complete description of consumption requires both equation (4.17) and equation (4.19). Equation (4.19) is not therefore a rival consumption function to equation (4.18).

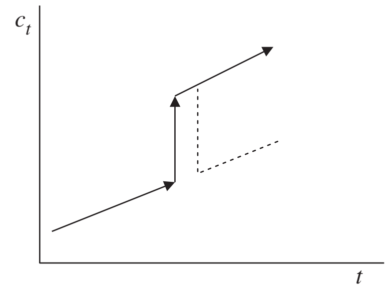

Figure 4.3. The effect on consumption of permanent and temporary shocks to income.

Figure 4.3 illustrates the argument. We plot the behavior of (log) consumption against time. It has two features: its slope and its location. Consider the lower segment. The movement of consumption along this segment is determined from the Euler equation; it is at a constant rate. The location of the segment (i.e., its level) is determined by the consumption function. Suppose that after a while there is a permanent positive shock to consumption due perhaps to an increase in income. This will cause consumption to jump to the higher segment. Consumption will then move along this segment, continuing to grow at the same rate as before because the Euler equation is unaffected by the jump. In contrast, a temporary positive shock to consumption would cause consumption to rise above the lower segment briefly before returning to it and then continuing along it at the old rate of growth, as shown by the dotted line. And if the permanent increase in income had been anticipated earlier, then consumption would have started to increase at that time. The path of consumption would then be above the lower segment of figure 4.3 from the date when the income change was anticipated and would join the upper segment smoothly when the increase in income takes place.

4.3 Savings

We have been considering consumption—now we briefly consider savings. We assume that the interest rate is the constant r. Savings are then

$$s_t = x_t + ra_t - c_t.$$

Eliminating c_t using equation (4.17) we obtain

$$\begin{aligned} s_t &= x_t - r\sum_{s=0}^{\infty}\frac{x_{t+s}}{(1+r)^{s+1}} \\ &= -\frac{r}{1+r}\sum_{s=1}^{\infty}\frac{x_{t+s}-x_t}{(1+r)^s} \\ &= -\sum_{s=1}^{\infty}\frac{\Delta x_{t+s}}{(1+r)^{s+1}}. \end{aligned}$$

This has a very interesting interpretation. It shows that saving is undertaken in order to offset (expected) future falls in income. These could be temporary—for example, due to spells of unemployment—or permanent—for example, due to retirement. Thus, abstracting from an economy that is growing, saving enables consumption to be kept constant throughout life.

4.4 Life-Cycle Theory

We have assumed so far that households are identical and live for ever. In fact, due to the finiteness of lives, the age of households is one of the main causes of differences in their behavior. A young household, possibly with dependants and many years of work before retirement, will have different consumption and savings patterns from old households, possibly already in retirement. Most obviously, a typical household with young children will have high expenditures relative to income, and so will have a low savings rate. A middle-aged household will usually save more in order to generate an income in retirement. An old household in retirement is likely to be dependent on past savings, such as a (contributed) pension, and to dissave. Clearly, the theory above does not capture all of these features. It can easily be modified, however, to reflect the main point that consumption and savings depend on age. Further, if the age distribution of the whole population is relatively stable, then, to a first approximation, we may be able to ignore age when analyzing aggregate consumption and savings.

4.4.1 Implications of Life-Cycle Theory

Before modifying the theory, we note how it can be interpreted to reflect some of these considerations. The key result is that consumption is in general a function of wealth (equation (4.17)). Since wealth is the discounted sum of expected future income over a person's life plus current financial assets, it may be expected to be fairly stable over time. This implies that consumption in each period would be stable too and would be independent of a person's age. Thus, fluctuations in income due to unemployment or retirement, when income from employment is zero, should not in theory affect current consumption. This is why the theory above is called the life-cycle theory; in principle, it automatically takes account of each household's position in its life cycle.

Life-cycle theory makes a number of strong assumptions. In particular, it assumes that the future can be anticipated reasonably accurately. Alternatively, we could make the strong assumption that households hold assets whose payoffs vary between good and bad times in such a way as to offset unexpected changes in income and, as a result, leave wealth unaffected.

Another critical assumption is that households are able to borrow to maintain consumption even when current income and financial assets are insufficient to pay for current consumption. In practice, a possibly substantial proportion of households face a borrowing constraint that prevents them from doing this.

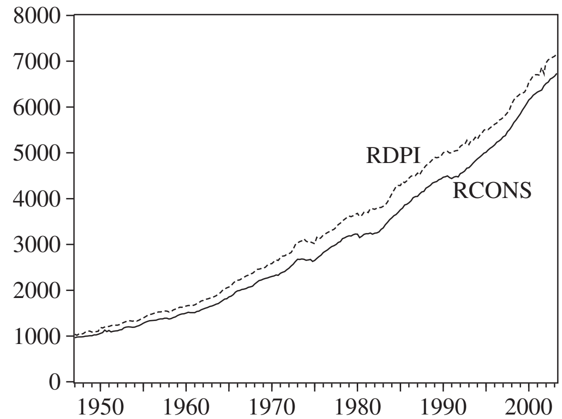

Figure 4.4. U.S. total real consumption and real disposable income 1947–2003.

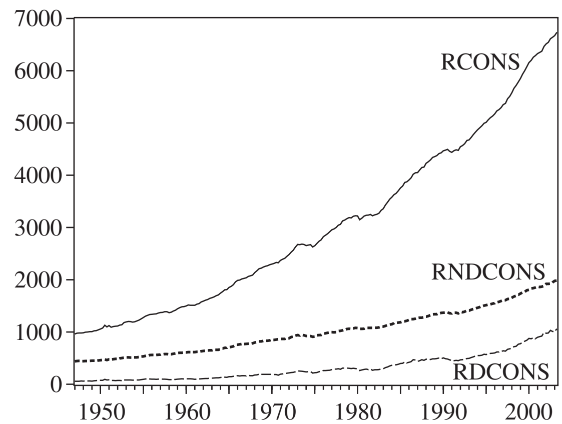

Figure 4.5. U.S. total, nondurable, and durable real consumption 1947–2003.

Consequently, their consumption would be limited to their current income. Their consumption would therefore tend to fluctuate with their income, rather than be smoothed over time as life-cycle theory predicts. What does empirical evidence show? Figures 4.4 and 4.5 give a rough guide. Figure 4.4 plots disposable (after-tax) income against total consumption for the United States for the period 1947–2003. Figure 4.5 shows total, real nondurable, and durable consumption.

Figure 4.4 suggests that total consumption is somewhat smoothed but still fluctuates. The fluctuations in total consumption are not dissimilar to those in income. Figure 4.5 reveals that the fluctuations in total consumption are due much more to variations in durable consumption than to those in nondurable consumption. Table 4.1 gives the standard deviations of the growth rates.

The table reveals that the standard deviation of total consumption is 66% of that of disposable income, but the standard deviation of nondurable consumption expenditures is 54% of that of disposable income. Significantly, the

Table 4.1. Standard deviations in growth rates.

	%
Disposable income	1.12
Total consumption	0.74
Nondurables and services	1.22
Nondurables	0.60
Durables	17.13

standard deviation of nondurable consumption is only 3.5% of that of durable expenditures. We conclude that there is evidence of consumption smoothing, but only of nondurable expenditures. Neither service nor durable expenditures appear to be smoothed relative to income. We note, however, that unlike nondurable and service expenditures, durable expenditures are not a flow variable but a stock; it is the services from the durable stock that are a flow. Strictly speaking, therefore, the theory derived above does not apply to durables. We therefore reexamine the determination of durable consumption below.

4.4.2 Model of Perpetual Youth

A modification to life-cycle theory that explicitly recognizes the household's finite tenure on life is the Blanchard and Fischer (1989) and Yaari (1965) theory of perpetual youth. Each individual is assumed to have a constant probability of death in each period of ρ, which is independent of age. The probability of not dying in any period is therefore $1-\rho$. The probability of dying in s periods' time is the joint probability of not dying in the first $s-1$ periods multiplied by the probability of dying in period s, i.e.,

$$f(s) = \rho(1-\rho)^{s-1}, \quad s = 1, 2, \ldots .$$

Thus, expected lifetime is

$$E(s) = \sum_{s=1}^{\infty} s\rho(1-\rho)^{s-1} = \rho^{-1}.$$

Hence, in the limit as $\rho \to 0$, lifetime is infinite, as in the basic model. If newborns have the same probability of dying in each period, then, for the population size to be constant, births must exactly offset deaths. In practice, the average lifespan has increased over time, implying that ρ has been falling over time.

As the date of death is unknown, households must make consumption and savings decisions under uncertainty. We must therefore replace utility in period $t+s$ by the expected utility *given that the household will still be alive*. The probability of being alive in period $t+s$ is $(1-\rho)^s$. The present value of *expected* utility is then

$$\begin{aligned} V_t &= \sum_{s=0}^{\infty} \beta^s (1-\rho)^s U(c_{t+s}) \\ &= \sum_{s=0}^{\infty} \tilde{\beta}^s U(c_{t+s}), \end{aligned}$$

where $\tilde{\beta} = \beta(1-\rho)$. Thus, the household objective function has the same form as previously; only the discount rate has changed. We note that the optimal rate of growth of consumption is then given by

$$\frac{\Delta c_{t+1}}{c_t} = \frac{1}{\sigma}\left[1 - \frac{1}{\tilde{\beta}(1+r_{t+1})}\right] \simeq \frac{r_{t+1} - (\theta+\rho)}{\sigma(1+r_{t+1})}.$$

Thus, the prospect of death raises the minimum rate of return required to induce saving, cuts the optimal rate of consumption growth, and raises consumption levels. Since, in practice, the probability of death is not constant but increases with age, we may expect relatively higher consumption by older households, and net dissaving, especially among retired households.

To sum up, we can therefore proceed as before. We do not need to change the previous analysis, but we should be aware of what interpretation we give to the discount rate.

4.5 Nondurable and Durable Consumption

The key difference between nondurables and durables is that the former is a flow variable while the latter is a stock that provides a flow of services in each period. Moreover, the stock of durables depreciates over time due to wear and tear and obsolescence. We now modify our previous analysis of household consumption to incorporate these features.

In this section we denote real nondurable consumption by c_t, the stock of durables by D_t, and the total investment expenditures on durables by d_t. The accumulation equation for durables can be written as

$$\Delta D_{t+1} = d_t - \delta D_t, \tag{4.21}$$

where δ is the rate of depreciation. The household budget constraint is altered to reflect the fact that the household purchases both nondurables and durables. It is now written as

$$\Delta a_{t+1} + c_t + p_t^{\mathrm{D}} d_t = x_t + r_t a_t,$$

where p_t^{D} is the price of durables relative to nondurables and $p_t^{\mathrm{D}} d_t$ is the total expenditure on durables measured in terms of nondurable prices. Thus the budget constraint becomes

$$\Delta a_{t+1} + c_t + p_t^{\mathrm{D}}[D_{t+1} - (1-\delta)D_t] = x_t + r_t a_t. \tag{4.22}$$

Utility is derived by households from the services of nondurable and durable consumption through $U(c_t, D_t)$, where $U_c, U_D > 0$, $U_{cc}, U_{DD} \leqslant 0$. This reflects the fact that the greater the stock of durables, the greater the flow of services from durables. If $U_{cD} > 0$ nondurables and durables are complementary, and if $U_{cD} < 0$ they are substitutes.

The problem becomes that of maximizing

$$V_t = \sum_{s=0}^{\infty} \beta^s U(c_{t+s}, D_{t+s}),$$

with respect to $\{c_{t+s}, D_{t+s+1}, a_{t+s+1};\ s \geqslant 0\}$, subject to equation (4.22) and the durable accumulation equation (4.21). The Lagrangian is

$$\begin{aligned}\mathcal{L} = \sum_{s=0}^{\infty} \{\beta^s U(c_{t+s}, D_{t+s}) + \lambda_{t+s}[x_t + (1 + r_{t+s})a_{t+s} - c_{t+s} \\ - p^{\mathrm{D}}_{t+s} D_{t+s+1} + p^{\mathrm{D}}_{t+s}(1-\delta)D_{t+s} - a_{t+s+1}]\}.\end{aligned}$$

The first-order conditions are

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial c_{t+s}} &= \beta^s U_{c,t+s} - \lambda_{t+s}, && s \geqslant 0,\\
\frac{\partial \mathcal{L}}{\partial D_{t+s}} &= \beta^s U_{D,t+s} + \lambda_{t+s} p^{\mathrm{D}}_{t+s}(1-\delta) - \lambda_{t+s-1} p^{\mathrm{D}}_{t+s}, && s > 0,\\
\frac{\partial \mathcal{L}}{\partial a_{t+s}} &= \lambda_{t+s}(1 + r_{t+s}) - \lambda_{t+s-1} = 0, && s > 0.
\end{aligned}$$

The first and third equations give the usual Euler equation, but now defined in terms of nondurable consumption:

$$\frac{\beta U_{c,t+1}}{U_{c,t}}(1 + r_{t+1}) = 1.$$

From all three equations we obtain

$$U_{D,t+1} = U_{c,t+1} p^{\mathrm{D}}_{t+1}(r_{t+1} + \delta). \tag{4.23}$$

If, for example, utility is Cobb–Douglas and given by

$$U(c_t, D_t) = c_t^{\alpha} D_t^{1-\alpha},$$

then the Euler equation becomes

$$\beta\left(\frac{c_{t+1}/D_{t+1}}{c_t/D_t}\right)^{-(1-\alpha)}(1 + r_{t+1}) = 1 \tag{4.24}$$

and equation (4.23) can be written as

$$\frac{c_{t+1}}{p^{\mathrm{D}}_{t+1} D_{t+1}} = \frac{\alpha}{1-\alpha}(r_{t+1} + \delta). \tag{4.25}$$

Equation (4.25) implies that an increase in the real rate of interest reduces the value of the stock of durables relative to nondurable expenditures.

In steady state, when $\Delta c_{t+1} = \Delta D_{t+1} = 0$, we have $r_{t+1} = \theta$. Hence

$$\frac{c}{p^{\mathrm{D}}D} = \frac{\alpha}{1-\alpha}(\theta + \delta)$$

and the ratio of expenditures on nondurable consumption to durables in the long run is

$$\frac{c}{p^{\mathrm{D}}d} = \frac{\alpha}{1-\alpha}\frac{\theta + \delta}{\delta}.$$

Short-run behavior is affected by the lag in the adjustment of the stock of durables. Although nondurable and durable expenditures can instantly respond to a period t shock, the stock of durables is given in period t and cannot respond until period $t+1$. For example, a permanent increase in income from period t causes a permanent increase in expenditures on both nondurables and durables from period t. The relative response of nondurable to durable expenditures is obtained as follows. From equations (4.21) and (4.24) the expenditure on durables relative to nondurables is

$$\begin{aligned}
\frac{p_t^{\mathrm{D}}d_t}{c_t} &= \frac{c_{t+1}}{c_t}\frac{P_t^{\mathrm{D}}D_{t+1}}{c_{t+1}} - (1-\delta)\frac{P_t^{\mathrm{D}}D_t}{c_t} \\
&= \left[\frac{c_{t+1}}{c_t}\left(\frac{1+r_{t+1}}{1+\theta}\right)^{-1/(1-\alpha)} - 1 + \delta\right]\frac{P_t^{\mathrm{D}}D_t}{c_t} \\
&\simeq \left[\frac{\Delta c_{t+1}}{c_t} - \frac{1}{1-\alpha}(r_{t+1} - \theta) + \delta\right]\frac{P_t^{\mathrm{D}}D_t}{c_t}.
\end{aligned}$$

The effect on this relative expenditure of an increase in c_t, with D_t given, is therefore determined by the sign of the term in square brackets. If this is negative, then the relative expenditure on durables in period t is greater. This result seems to be supported by the evidence, which shows that the volatility of durables is larger than that of nondurables.

4.6 Labor Supply

So far we have focused on the consumption and savings decisions of households, taking noninterest income x_t as given. We now consider the household's labor-supply decision. This is a first step toward endogenizing noninterest income. This will be followed by a discussion of the demand for labor by firms and the coordination of these decisions in the labor market.

In the basic model of chapter 2 we assumed initially that households work for a fixed amount of time. In the extension to the basic model we distinguished between work and leisure, allowing a choice between the two. The wage rate was only included implicitly. We now assume an explicit wage rate w_t. Time spent in employment n_t generates labor income and therefore contributes to consumption c_t, but it is at the expense of leisure l_t, which is also assumed to be desirable to households. The total time available to households is unity, so $n_t + l_t = 1$. In contrast to our previous analysis of labor, we now write the instantaneous utility function as

$$U(c_t, l_t) = U(c_t, 1 - n_t),$$

where $U_c > 0$, $U_l < 0$, $U_{cc} \leqslant 0$, $U_{ll} \leqslant 0$, $U_{n,t} = -U_{l,t}$, and the household budget constraint is

$$\Delta a_{t+1} + c_t = w_t n_t + x_t + r_t a_t, \tag{4.26}$$

where w_t is the real-wage rate per unit of labor time, x_t is still treated as exogenous income but now excludes labor income, and r_t is the real rate of interest on net asset holdings a_t held at the beginning of period t.

The Lagrangian is

$$\begin{aligned}\mathcal{L} = \sum_{s=0}^{\infty} \{&\beta^s U(c_{t+s}, 1 - n_{t+s}) \\ &+ \lambda_{t+s}[w_t n_t + x_t + (1 + r_{t+s})a_{t+s} - c_{t+s} - a_{t+s+1}]\}.\end{aligned}$$

Maximizing with respect to $\{c_{t+s}, n_{t+s}, a_{t+s+1}; s \geqslant 0\}$ gives the first-order conditions

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial c_{t+s}} &= \beta^s U_{c,t+s} - \lambda_{t+s}, & s \geqslant 0,\\ \frac{\partial \mathcal{L}}{\partial n_{t+s}} &= -\beta^s U_{l,t+s} + \lambda_{t+s} w_{t+s}, & s \geqslant 0,\\ \frac{\partial \mathcal{L}}{\partial a_{t+s}} &= \lambda_{t+s}(1 + r_{t+s}) - \lambda_{t+s-1} = 0, & s > 0,\end{aligned}$$

along with the budget constraint.

Solving the first two conditions for $s = 0$ and eliminating λ_t gives

$$\frac{U_{l,t}}{U_{c,t}} = w_t. \tag{4.27}$$

Once consumption is determined, the supply of labor can be derived from this as a function of consumption and the wage rate.

Consumption is derived much as it was before. From the first and third conditions we obtain the same Euler equation as before, namely equation (2.12), which for convenience we repeat:

$$\frac{\beta U_{c,t+1}}{U_{c,t}}(1 + r_{t+1}) = 1. \tag{4.28}$$

This is then combined with the intertemporal budget constraint associated with the new instantaneous budget constraint (4.26). Assuming that the interest rate is constant, we can show that

$$c_t = \frac{r}{1+r} W_t = r \sum_{0}^{\infty} \left[\frac{w_{t+s} n_{t+s}}{(1+r)^{s+1}} + \frac{x_{t+s}}{(1+r)^{s+1}} \right] + r a_t, \tag{4.29}$$

where wealth is

$$W_t = \sum_0^\infty \left[\frac{w_{t+s}n_{t+s}}{(1+r)^s} + \frac{x_{t+s}}{(1+r)^s}\right] + (1+r)a_t.$$

Hence, wealth includes discounted current and future labor income. Equations (4.27) and (4.29) and the labor constraint are three simultaneous equations in $\{c_{t+s}, l_{t+s}, n_{t+s}\}$, from which the optimal levels of consumption and the supply of labor can be obtained.

Consider the special case where $w_{t+s} = w_t$ and $x_{t+s} = x_t$ ($s \geqslant 0$). The consumption function then becomes

$$c_t = w_t n_t + x_t + ra_t. \tag{4.30}$$

Thus c_t is once more equal to total current income, i.e., income from labor $w_t n_t$ as well as from savings ra_t and x_t. The supply of labor is derived from equations (4.27) and (4.30).

To illustrate, suppose that instantaneous utility is the separable power function

$$U(c_t, l_t) = \frac{c_t^{1-\sigma} - 1}{1-\sigma} + \ln l_t,$$

where $\sigma > 0$, then

$$U_{c,t} = c_t^{-\sigma} \quad \text{and} \quad U_{l,t} = \frac{1}{l_t}.$$

Equation (4.27) becomes

$$\frac{1/(1-n_t)}{c_t^{-\sigma}} = w_t,$$

implying that the supply of labor is

$$n_t = 1 - \frac{c_t^\sigma}{w_t}. \tag{4.31}$$

Consequently, given c_t, an increase in w_t will increase labor supply and hence total labor income. However, from (4.29) an increase in labor income will increase consumption, and from (4.31) an increase in consumption will reduce the labor supply. This implies that the sign of the net effect of an increase in the wage rate on the labor supply is not determined. This can also be shown by combining equations (4.30) and (4.31) to eliminate c_t and give the labor-supply function:

$$n_t = N^s(w_t, x_t, r, a_t).$$

It follows that

$$\frac{\partial n_t}{\partial w_t} = \frac{1}{w_t}\left(\frac{1}{\sigma c_t^{\sigma-1} + 1} - n_t\right),$$

where the sign is still not clear. However, the smaller consumption is, the more likely it is that the sign will be positive. In contrast, an increase in x_t, a_t, or r will cause an unambiguous increase in c_t, and hence a fall in the labor supply.

4.7 Firms

Next we consider the decisions of the representative firm. Firms make decisions on output, factor inputs (capital and labor), and product prices. They also determine their financial structure—that is, whether to use equity or debt finance—and the proportion of profits to disburse as dividends. We assume that the representative firm seeks to maximize the present value of current and future profits by a suitable choice of output, investment, the capital stock, labor, and debt finance. In effect we are assuming that firms use debt, rather than equity finance, and borrow from households. Consequently, firm debts are household assets.

First we consider the problem in the absence of costs of adjustment of labor. We then examine the effects of including labor costs of adjustment. We recall that in chapter 2 we considered the cost of adjustment of capital in the centralized model of the economy.

4.7.1 Labor Demand without Adjustment Costs

The present value of the stream of real profits discounted using a constant real interest rate r is

$$\mathcal{P}_t = \sum_{s=0}^{\infty}(1+r)^{-s}\Pi_{t+s}, \tag{4.32}$$

where the firms's real profits (net revenues) in period t are

$$\Pi_t = y_t - w_t n_t - i_t + \Delta b_{t+1} - r b_t,$$

where w_t is the real-wage rate, n_t is labor input, and b_t is the stock of outstanding firm debt at the beginning of period t, i.e., b_t is corporate debt and it is held by households. As we are still working in real terms we have set the price level to unity.

The production function depends on two factors of production,

$$y_t = F(k_t, n_t),$$

and capital is accumulated according to

$$\Delta k_{t+1} = i_t - \delta k_t.$$

Thus the net revenue of the firm is

$$\Pi_t = F(k_t, n_t) - w_t n_t - k_{t+1} + (1-\delta)k_t + b_{t+1} - (1+r)b_t.$$

Firms seek to maximize the present value of their profits with respect to $\{n_{t+s}, k_{t+s+1}, b_{t+s+1};\ s \geqslant 0\}$. Hence they maximize

$$\mathcal{P}_t = \sum_{s=0}^{\infty}(1+r)^{-s}\{F(k_{t+s}, n_{t+s}) - w_{t+s}n_{t+s} - k_{t+s+1} \\ + (1-\delta)k_{t+s} + b_{t+s+1} - (1+r)b_{t+s}\}.$$

The first-order conditions are

$$\begin{aligned}
\frac{\partial \mathcal{P}_t}{\partial l_{t+s}} &= (1+r)^{-s}\{F_{n,t+s} - w_{t+s}\} = 0, && s \geqslant 0,\\
\frac{\partial \mathcal{P}_t}{\partial k_{t+s}} &= (1+r)^{-s}[F_{k,t+s} + 1 - \delta] - (1+r)^{-(s-1)} = 0, && s > 0,\\
\frac{\partial \mathcal{P}_t}{\partial b_{t+s}} &= (1+r)^{-s}(1+r) - (1+r)^{-(s-1)} = 0, && s > 0.
\end{aligned}$$

The demand for labor is obtained from the usual condition that the marginal product of labor equals the real wage,

$$F_{n,t} = w_t,$$

and will depend on the stock of capital. For a given stock of capital and a given technology, an increase in the wage rate will reduce the demand for labor.

The demand for capital is derived from

$$F_{k,t+1} = r + \delta$$

using the inverse function $F^{-1}_{k,t+1}(r+\delta)$. Hence gross investment is

$$i_t = F^{-1}_{k,t+1}(r+\delta) - (1-\delta)k_t.$$

Consequently, an increase in the rate of interest reduces investment. An increase in the marginal product of capital due, for example, to a permanent technology shock raises the optimal stock of capital and investment. We note that this solution implicitly assumes that there are no lags of adjustment in investment. Investment and the capital stock instantaneously achieve their optimal levels for each period. If there are additional costs to investing, as in Tobin's q-theory, then firms will prefer to take more time to adjust their capital stock to the long-run desired level. As we saw in chapter 2, this introduces additional dynamics into the investment and capital accumulation decisions, and through these into the economy as a whole.

In the short term, the firm chooses the capital stock so that the net marginal product of capital equals the cost of financing. This is also the opportunity cost of holding a bond instead. In the long term (i.e., in general equilibrium), households will be willing to save until the return to savings falls to the household rate of time preference θ. At this point $F_{k,t+1} - \delta = \theta$, the same result we obtained for the basic centralized model.

The condition $\partial \mathcal{P}_t/\partial b_{t+s} = 0$ is independent of b_t, and hence is satisfied for all values of b_t, including zero. Since any value of debt is consistent with maximizing profits, the firm can choose between using debt finance or its profits (i.e., retained earnings) when financing new investment. This is a version of the Modigliani–Miller theorem (see Modigliani and Miller 1958).

4.7.2 Labor Demand with Adjustment Costs

In effect, we have assumed that labor consists of the number of hours worked per worker. A generalization of this would be to decompose labor into the number of workers and the hours they work. Individuals may choose whether or not to work (the participation decision), and how many hours to work. In practice, household choice may be constrained by firms to working full time, part time, and/or overtime. Thus, it is firms that dominate the balance between the number of workers employed and the hours worked. The indivisible labor model of Hansen (1985) assumes that hours of work are fixed by firms and individuals simply decide whether or not to participate in the labor force. We modify this by assuming that households can choose whether or not to work, but if they do decide to participate in the labor force, the number of hours is chosen by firms.

Since working more hours often involves having to pay a premium overtime hourly wage rate, and hiring and firing entails additional costs, when adjusting labor input, firms must trade off the cost of changing the workforce against that of altering the number of hours worked. Intuitively, it may be less costly to meet a temporary increase in labor demand by raising the number of hours worked, while it may be cheaper to meet a permanent increase in labor demand by increasing the number of workers. We construct a simple model of the firm that illustrates how one might incorporate these features of the labor market. For convenience we abstract from the capital decision and firm borrowing.

We assume that the firm's production function is

$$y_t = F(n_t, h_t),$$

where n_t is the numbers of workers and h_t the number of hours each person works, and $F_n, F_h > 0$, $F_{nn}, F_{hh} \leqslant 0$, and $F_{nh} \geqslant 0$. Wages for each person are $W(h_t)$ with $W' \geqslant 0$, $W'' \geqslant 0$ to reflect the need to pay higher hourly wage rates the greater the number of hours worked by each person.

We also assume that there are costs to hiring and firing. The change in the workforce can be written as

$$n_t = v_t - q_t + n_{t-1},$$

where v_t represents total new hires and q_t total quits. There are costs associated both with taking on new employees and with firing existing workers. If $v_t = q_t$ there is no change in the labor force, yet there may still be hiring and firing costs due to turnover in each period. These could vary over time. For example, during a boom more workers may quit to find a better job and this may result in further hires to replace them. For convenience, however, we simply assume that there is a cost to changes in the total workforce, whether the workforce is increasing or decreasing, and we ignore the problem of turnover. Accordingly, we assume that the firm maximizes present value $\mathcal{P}_t$, equation (4.32), where the firms's net revenues in period t are given by

$$\Pi_t = F(n_t, h_t) - W_t(h_t)n_t - \tfrac{1}{2}\lambda(\Delta n_{t+1})^2.$$

The last term reflects the cost of hiring and firing during period t. Our discussion of turnover could be addressed by allowing λ to be higher when there is a lot of labor turnover in the boom phase of the business cycle and lower in recession when there is less turnover, but we assume that λ is constant.

The first-order conditions for maximizing $\mathcal{P}_t$ with respect to $\{n_{t+s}, h_{t+s}; s \geqslant 0\}$ are

$$\frac{\partial \mathcal{P}_t}{\partial n_{t+s}} = (1+r)^{-s}(F_{n,t+s} - W_{t+s} + \lambda \Delta n_{t+s+1}) + (1+r)^{-(s-1)} \lambda \Delta n_{t+s} = 0,$$
$$\frac{\partial \mathcal{P}_t}{\partial h_{t+s}} = (1+r)^{-s}(F_{h,t+s} - W'_{t+s} n_{t+s}) = 0.$$

From the second first-order condition,

$$\frac{F_{h,t}}{n_t} = W'_t. \tag{4.33}$$

This implies that the marginal product of an extra hour per worker is equal to the marginal hourly wage. We note that adjustment is instantaneous in equation (4.33). Given the number of workers, the equation gives their number of hours of work. If there is only a single hourly wage rate, then W'_t does not depend on h_t.

The second first-order condition gives the level of employment and can be written as

$$\Delta n_t = \frac{1}{1+r} \Delta n_{t+1} + \frac{1}{\lambda(1+r)} (F_{n,t} - W_t) \tag{4.34}$$
$$= \frac{1}{\lambda(1+r)} \sum_{s=0}^{\infty} (1+r)^{-s} (F_{n,t+s} - W_{t+s}). \tag{4.35}$$

Equation (4.35) shows that there will be an increase in the number of employees if the marginal product of workers exceeds their total wages either today or in the future. In steady state we have $\Delta n_t = 0$ when $F_{n,t} = W_t$. This is the usual marginal productivity condition for labor, i.e., each worker is paid their marginal product for the total number of hours worked.

Equation (4.34) can also be written in terms of the level of employment as

$$n_t = \frac{1}{2+r} n_{t+1} + \frac{1+r}{2+r} n_{t-1} + \frac{1}{\lambda(2+r)} (F_{n,t} - W_t). \tag{4.36}$$

This shows that the firm's adjustment of its number of employees takes place over time. The greater the turnover of workers, the higher λ is and the slower the adjustment of employment is.

Consider now the response of hours and employment to a permanent increase in labor demand as measured by an increase in the marginal products $F_{n,t}$ and $F_{h,t}$. To make matters clearer, assume that the production function can be expressed in terms of total hours as $F(n_t h_t)$. Thus, an increase in the number of total hours is required. Noting that $F_{h,t} = F'_t n_t$ and $F_{n,t} = F'_t h_t$, in the long run

$$\frac{F_{h,t}}{n_t} = F'_t = W'_t,$$
$$F_{n,t} = F'_t h_t = W_t.$$

Due to the convexity of the wage function, $W'_t \geqslant W_t/h_t$. Hence, it is more costly to raise the number of hours per worker than to increase the number of workers. Thus, in the long run, hours will stay constant and employment will increase. But in the short run, as employment takes time to adjust, there will be a temporary increase in the number of hours. The response of hours and employment to a temporary increase in labor demand depends on the cost of hiring and firing relative to the cost of increasing hours worked.

4.8 General Equilibrium in a Decentralized Economy

General equilibrium is attained through markets coordinating the decisions of households and firms. The goods market coordinates households' consumption decisions and firms' output and investment decisions. The labor market coordinates firms' demand for labor and households' supply of labor with the real-wage rate equating labor demand and supply. Financial markets coordinate households' savings decisions and firms' borrowing requirements through the real interest rate. The bond market coordinates the savings in financial assets of households and the borrowing by firms. In the absence of considerations of risk, the price of bonds is determined by equating their rate of return in the long run to the rate of time preference of households. And the stock market prices the capital of firms so that its rate of return is the same as that on bonds.

4.8.1 Consolidating the Household and Firm Budget Constraints

Before examining general equilibrium in further detail, we consider what the results so far imply for the various constraints on households and firms, and how they can be combined, or consolidated. This enables us to determine a number of the variables defined above, such as exogenous income x_t, household assets a_t, firm debt b_t, and profits Π_t.

The national income identity is

$$y_t = c_t + i_t = F(k_t, n_t).$$

The household budget constraint is

$$\Delta a_{t+1} + c_t = w_t n_t + x_t + r_t a_t.$$

This includes labor income, exogenous income, and interest income. Combining these with the capital accumulation equation

$$\Delta k_{t+1} = i_t - \delta k_t$$

gives

$$x_t = F(k_t, n_t) - w_t n_t - \Delta k_{t+1} - \delta k_t + \Delta a_{t+1} - r a_t. \tag{4.37}$$

For the firm, profits are

$$\Pi_t = F(k_t, n_t) - w_t n_t - \Delta k_{t+1} - \delta k_t + \Delta b_{t+1} - r b_t. \tag{4.38}$$

Subtracting equation (4.38) from equation (4.37) gives

$$x_t - \Pi_t = \Delta(a_{t+1} - b_{t+1}) - r(a_t - b_t).$$

Since households' financial assets are firms' debts, $a_t = b_t$. This is the condition for equilibrium in the bond market. (If firms issue no debt, then $a_t = 0$.) It then follows that $x_t = \Pi_t$. Consequently, instead of x_t being exogenous, as has been assumed so far, we have shown that x_t is the distributed profit of the firm, the profits being distributed in the form of dividends.

As $F_{n,t} = w_t$ and $F_{k,t} = r + \delta$ for all t (as r and δ are constant) we can rewrite firm profits as

$$\begin{aligned} \Pi_t &= F(k_t, n_t) - F_{n,t} n_t - \Delta k_{t+1} - (F_{k,t+1} - r) k_t + \Delta b_{t+1} - r b_t \\ &= F(k_t, n_t) - F_{n,t} n_t - F_{k,t} k_t - \Delta(k_{t+1} - b_{t+1}) + r(k_t - b_t). \end{aligned}$$

If the production function has constant returns to scale (or, alternatively, approximating using a Taylor series expansion about $n_t = k_t = 0$),

$$F(k_t, n_t) = F_{n,t} n_t + F_{k,t} k_t.$$

Hence,

$$\Pi_t = -(k_{t+1} - b_{t+1}) + (1 + r)(k_t - b_t), \tag{4.39}$$

where $k_t - b_t$ can be interpreted as the net value of the firm.

From equation (4.39), the net value of the firm can be rewritten as the forward-looking difference equation

$$k_t - b_t = \frac{\Pi_t + (k_{t+1} - b_{t+1})}{1 + r}.$$

As $1/(1 + r) < 1$ we solve this equation forwards to obtain

$$k_t - b_t = \sum_{s=0}^{\infty} \frac{\Pi_{t+s}}{(1 + r)^{s+1}},$$

where we assume that the transversality condition

$$\lim_{s \to \infty} \frac{k_{t+s} - b_{t+s}}{(1 + r)^s} = 0$$

holds, implying that the discounted net value of the firm tends to zero. Thus the value of the firm is the discounted value of current and future profits. If profits are constant, and equal to Π, then this simplifies to

$$k_t - b_t = \frac{\Pi}{r}.$$

In other words, the net value of the firm, $k - b$, equals the present value of profits, Π/r. Further, as $x = \Pi$ and $a = b$, total household asset income is $x + ra = rk$.

If all profits are distributed as dividends and not retained to finance investment, then $k - b$ also equals the present value of total dividend payments. Dividing $k - b$ and Π/r by $n_{s,t}$, the number of shares in existence at time t, gives the standard formula for the value of a share, namely, the present value of current and expected future profits (commonly called earnings) per share. If all profits are distributed as dividends, and assuming that dividends per share d_t are expected to be the same in the future, the value of a share can also be written as

$$\frac{k_t - b_t}{n_{s,t}} = \frac{d_t}{r}. \tag{4.40}$$

Thus, $d_t = r((k_t - b_t)/n_{s,t})$, implying that dividend income is the permanent income provided from the net value of the firm. The rate of return to capital is $r = F_k - \delta$, the net marginal product of capital. r is also the rate of return to bonds and, as we have seen, in the long run this is equal to θ, the rate of time preference of households, which limits their willingness to save, i.e., to lend to firms. Financial markets therefore equate the rate of return on all forms of capital and determine the income flows from assets. Later, in chapter 10, we examine other aspects of the determination of asset prices and returns such as risk considerations and the concept of no-arbitrage.

If some profits are retained for investment, then the value of a share will also depend on the discounted net value of the firm at some point $T > t$ in the future and can be written

$$\frac{k_t - b_t}{n_{s,t}} = \sum_{s=0}^{T-1} \frac{d_{t+s}}{n_{s,t}(1+r)^{s+1}} + \frac{k_T - b_T}{n_{s,t}(1+r)^{T+1}}. \tag{4.41}$$

If we assume that all profits are eventually distributed as dividends, then, as $T \to \infty$, equation (4.41) reduces to equation (4.40).

4.8.2 The Labor Market

Abstracting from labor market adjustment costs and a nonlinear wage function, the demand and supply for labor are determined, respectively, from

$$F_{n,t} = w_t,$$
$$U_{n,t} = -w_t U_{c,t}.$$

Real wages adjust to clear the market so that

$$w_t = F_{n,t} = \frac{U_{n,t}}{U_{c,t}}.$$

This is a partial-equilibrium solution for labor as the marginal product of labor and the two marginal utilities will, in general, depend upon other endogenous variables, i.e., variables that are also determined within the economy.

The full general equilibrium solution requires that each endogenous variable is determined in terms of the exogenous variables.

This may be clearer if we employ particular functional forms. If, for example, the production function is Cobb–Douglas with constant returns to scale, then

$$y_t = Ak_t^{\alpha} n_t^{1-\alpha}$$

and so

$$F_{n,t} = (1-\alpha)A\left(\frac{k_t}{n_t}\right)^{\alpha},$$

implying that the demand for labor is

$$n_t^d = \left[\frac{w_t}{(1-\alpha)A}\right]^{-\alpha} k_t,$$

where k_t is given at time t. If the utility function is that considered previously, namely,

$$U(c_t, l_t) = \frac{c_t^{1-\sigma} - 1}{1-\sigma} + \ln(1-n_t),$$

then the labor supply is

$$n_t^s = 1 - \frac{c_t^{\sigma}}{w_t}.$$

Thus, the supply of labor depends on an endogenous variable, c_t. The equilibrium quantity of labor is

$$n_t = \left[\frac{w_t}{(1-\alpha)A}\right]^{-\alpha} k_t = 1 - \frac{c_t^{\sigma}}{w_t}.$$

The equilibrium real wage can be derived from this. It does not have a closed-form solution, but will depend on c_t and k_t. If the equilibrium real wage is

$$w_t = w(c_t, k_t),$$

then the equilibrium quantity of labor is

$$\begin{aligned} n_t &= \left[\frac{w(c_t, k_t)}{(1-\alpha)A}\right]^{-\alpha} k_t \\ &= n(c_t, k_t). \end{aligned}$$

It also follows that, in equilibrium, labor income is

$$\begin{aligned} w_t n_t &= w_t - c_t^{\sigma} \\ &= f(c_t, k_t). \end{aligned}$$

4.8.3 The Goods Market

Equilibrium in the goods market requires that aggregate demand equals aggregate supply. Aggregate demand is

$$\begin{aligned} y_t^d &= c_t + i_t \\ &= c_t + k_{t+1} - (1-\delta)k_t \\ &= c_t + F_{k,t}^{-1}(r+\delta) - (1-\delta)k_t, \end{aligned}$$

where we have used the result that consumption is proportional to wealth, the net marginal product of capital $F_{k,t} - \delta = r$, and hence k_{t+1} is the inverse function of $r + \delta$. Assuming a Cobb–Douglas production function,

$$F_{k,t}^{-1}(r+\delta) = \left[\frac{\alpha A}{r+\delta}\right]^{1/(1-\alpha)} n_t.$$

Hence,

$$y_t^d = c_t + \left[\frac{\alpha A}{r+\delta}\right]^{1/(1-\alpha)} n_t - (1-\delta)k_t.$$

We note that c_t is proportional to wealth and so could be substituted.

Aggregate supply is obtained from the production function. Accordingly, goods-market equilibrium can be expressed as

$$Ak_t^{\alpha} n_t^{1-\alpha} = c_t + \left[\frac{\alpha A}{r+\delta}\right]^{1/(1-\alpha)} n_t - (1-r-\delta)k_t.$$

In steady state we have shown that

$$\begin{aligned} c_t &= w_t n_t + x_t + r a_t \\ &= w_t n_t + \Pi_t + r a_t \\ &= w_t n_t + r k_t. \end{aligned}$$

Aggregate supply is obtained from the production function. Consequently, goods-market equilibrium becomes

$$Ak_t^{\alpha} n_t^{1-\alpha} = \left\{w_t + \left[\frac{\alpha A}{r+\delta}\right]^{1/(1-\alpha)}\right\} n_t - (1-r-\delta)k_t.$$

This involves three variables: k_t, n_t, and w_t. It can be solved together with the three equations

$$\begin{aligned} w_t &= w(c_t, k_t) = w^*(k_t, n_t, r), \\ n_t &= n(c_t, k_t) = n^*(k_t, n_t, r), \\ k_t &= \left[\frac{\alpha A}{r+\delta}\right]^{1/(1-\alpha)} n_t. \end{aligned}$$

We then have the complete solution.

4.9 Comparison with the Centralized Model

We may summarize the similarities between the basic centralized model and the decentralized model as follows. In the basic centralized model labor was not included explicitly, although, in effect, there was a single unit of labor. Despite this, the capital stock is determined in both the basic centralized model and the decentralized model from the marginal product of capital, investment is derived from the capital accumulation equation, and consumption is obtained from the national income identity.

In the centralized model capital is determined from the condition

$$F'(k_{t+1}) = \theta + \delta.$$

In the decentralized solution it is obtained from

$$F_{k,t+1} = r + \delta.$$

As there is just one unit of labor in the basic model, $n_t = 1$. Hence either $F_{k,t+1}$ does not depend upon labor or, equivalently, $F'(k_{t+1})$ includes labor implicitly. Further, in steady state, $r = \theta$ as households will continue to save, and firms will continue to accumulate capital, until the return obtained falls to θ, when households will not save any more and firms will no longer wish to accumulate more capital. Thus, in the centralized model, there is an implicit real interest rate, which is given by the net marginal product of capital:

$$r = F'(k_{t+1}) - \delta.$$

Although there is no explicit wage rate in the basic model, there is an implicit wage rate. As there is just one unit of labor in the basic model, the wage rate is also equal to the total cost of labor. From

$$F(k_t, n_t) = F_{n,t} n_t + F_{k,t+1} k_t$$

and from the condition that the marginal product of labor is the real wage, and also as $n_t = 1$, we obtain the following expression for the implicit real wage in the basic model:

$$w_t = F(k_t) - F'(k_{t+1}) k_t. \tag{4.42}$$

In the basic centralized model there are no debts or financial assets; there is only capital, which is equity. An implicit measure of profits in the basic model can be obtained from the definition of profits in the decentralized model. If we define w_t as in equation (4.42), set $n_t = 1$, and $b_t = 0$, then firm profits are

$$\Pi_t = F(k_t) - [F(k_t) - F'(k_{t+1}) k_t] - \Delta k_{t+1} - \delta k_t.$$

Substituting $F'(k_{t+1}) = \theta + \delta$ gives

$$\Pi_t = -k_{t+1} + (1 + \theta) k_t.$$

Consequently, the value of the capital stock (equity) in the basic model is

$$\begin{aligned} k_t &= \frac{\Pi_t + k_{t+1}}{1 + \theta} \\ &= \sum_{s=0}^{\infty} \frac{\Pi_{t+s}}{(1 + \theta)^{s+1}}, \end{aligned}$$

namely, the discounted value of current and future profits. If profits are denoted by the constant Π, then

$$k_t = \frac{\Pi}{\theta}.$$

Consumption is obtained in the basic model from the resource constraint:

$$c_t = F(k_t) - k_{t+1} + (1 - \delta) k_t.$$

When $n_t = 1$, it is also obtained from this equation in the decentralized model.

4.10 Conclusions

We have now seen how decisions can be decentralized and how markets, particularly labor and financial markets, coordinate decisions. This generalization has added useful detail to the basic centralized model and it has allowed us to include further variables such as saving, financial assets, the interest rate, labor, and the real-wage rate, and it has made it easier to examine a number of issues in greater depth. The analysis has also shown that the essential insights of the basic centralized model are unchanged. As it is often easier to analyze general equilibrium by using the basic centralized model than by using a decentralized model, which tends to lead to an increase in detail without altering the main conclusions, when it is convenient and the results are little affected, we will revert to using a centralized model in preference to a decentralized model. Further, we note that including labor caused only minor changes to the previous results; consequently, we shall also exclude labor where appropriate and feasible.

The decentralized general equilibrium model provides a benchmark against which later models may be compared. This is not to say that the model is suitable for analyzing every situation. We have still not introduced government, money, nominal values, or taken account of economic transactions with the rest of the world. Moreover, we have assumed that households and firms have perfect foresight and that there are no market imperfections due, for example, to monopoly power or frictions. It is the presence of these features that causes most of the complications in setting monetary and fiscal policy: inflation control and macroeconomic stabilization. Without these it is debatable whether active monetary and fiscal policy would even be required.