PREFACE

After the publication of *A Mathematical Nature Walk*, my editor, Vickie Kearn, suggested I think about writing *A Mathematical City Walk*. My first reaction was somewhat negative, as I am a "country boy" at heart, and have always been more interested in modeling natural patterns in the world around us than man-made ones. Nevertheless, the idea grew on me, especially since I realized that many of my favorite nature topics, such as rainbows and ice crystal halos, can have (under the right circumstances) very different manifestations in the city. Why would this be? Without wishing to give the game away too early into the book, it has to do with the differences between nearly parallel "rays" of light from the sun, and divergent rays of light from nearby light sources at night, of which more anon. But I didn't want to describe this and the rest of the material in terms of a city *walk*; instead I chose to couch things with an "in the city" motif, and this allowed me to touch on a rather wide variety of topics that would have otherwise been excluded. (There are *seven* chapters having to do with traffic in one way or another!)

As a student, I lived in a large city—London—and enjoyed it well enough, though we should try to identify what is meant by the word "city." Several related dictionary definitions can be found, but they vary depending on the country in which one lives. For the purposes of this book, a city is a large, permanent settlement of people, with the infrastructure that is necessary to make that possible. Of course, the terms "large" and "permanent" are relative, and therefore we may reasonably include towns as well as cities and add the phrase "or developing" to "permanent" in the above definition. In the Introduction we will endeavor to expand somewhat on this definition from a historical perspective.

This book is an eclectic collection of topics ranging across city-related material, from day-to-day living in a city, traveling in a city by rail, bus, and car (the latter two with their concomitant traffic flow problems), population growth in cities, pollution and its consequences, to unusual night time optical effects in the presence of artificial sources of light, among many other topics. Our cities may be on the coast or in the heartland of the country, or on another continent, but presumably always located on planet Earth. Inevitably, some of the topics are multivalued; not everything discussed here is unique to the city—after all, people eat, garden, and travel in the country as well!

Why *X and the City*? In the popular culture, the letter X (or x) is an archetype of mathematical problem solving: "Find x." The X in the book title is used to introduce the topic in each subsection; thus "$X = t_c$" and "$X = N_{tot}$" refer, respectively, to a specific length of time and a total population, thereby succinctly introducing the mathematical topics that follow. One of the joys of studying and applying mathematics (and finding x), regardless of level, is the fact that the deeper one goes into a topic, the more avenues one finds to go down. I have found this to be no less the case in researching and writing this book. There were many twists and turns along the way, and naturally I made choices of topics to include and exclude. Another author would in all certainty have made different choices. Ten years ago (or ten years from now), the same would probably be true for me, and there would be other city-related applications of mathematics in this book.

Mathematics is a language, and an exceedingly beautiful one, and the applications of that language are vast and extensive. However, pure mathematics and applied mathematics are very different in both structure and purpose, and this is even more true when it comes to that subset of applied mathematics known as *mathematical modeling* (of which more below). I love the beauty and elegance in mathematics, but it is not *always* possible to find it outside the "pure" realm. It should be emphasized that the subjects are complementary and certainly not in opposition, despite some who might hold that opinion. I heard of one mathematician who referred to applied mathematics as "mere engineering"; this should be contrasted with the view of the late Sir James Lighthill, one of the foremost British applied mathematicians of the twentieth century. He wrote, somewhat tongue-in-cheek, that pure mathematics was a very important part of applied mathematics!

Applied mathematics is often elegant, to be sure, and when done well it is invariably useful. I hope that the types of problem considered in this book can be both fun and "applied." And while some of the chapters in the middle of the book might be described as "traffic engineering," it is the case that mathematics is the basis for all types of quantitative thought, whether theoretical or applied. For those who prefer a more rigorous approach, I have also included Chapter 17, entitled "The axiomatic city." In that chapter, some of the exercises require proofs of certain statements, though I have intentionally avoided referring to the latter as "theorems."

The subtitle of this book is *Modeling Aspects of Urban Life*. It is therefore reasonable to ask: what *is* (mathematical) modeling? Fundamentally, mathematical modeling is the formulation in mathematical terms of the assumptions (and their logical consequences) believed to underlie a particular "real world" problem. The aim is the practical application of mathematics to help unravel the underlying mechanisms involved in, for example, industrial, economic, physical, and biological or other systems and processes. The fundamental steps necessary in developing a mathematical model are threefold: (i) to formulate the problem in mathematical terms (using whatever appropriate simplifying assumptions may be necessary); (ii) to solve the problem thus posed, or at least extract sufficient information from it; and finally (iii) to interpret the solution in the context of the original problem. This may include validation of the model by testing both its consistency with known data and its predictive capability.

At its heart, then, this book is about just that: mathematical modeling, from "applied" arithmetic to linear (and occasionally nonlinear) ordinary differential equations. As a little more of a challenge, there are a few partial differential equations thrown in for good measure. Nevertheless, the vast majority of the material is accessible to anyone with a background up to and including basic calculus. I hope that the reader will enjoy the interplay between estimation, discrete and continuum modeling, probability, Newtonian mechanics, mathematical physics (diffusion, scattering of light), geometric optics, projective and three-dimensional geometry, and quite a bit more.

Many of the topics in the book are posed in the form of questions. I have tried to make it as self-contained as possible, and this is the reason there are several Appendices. They comprise a compendium of unusual results perhaps (in some cases) difficult to find elsewhere. Some amplify or extend material discussed in the main body of the book; others are indirectly related, but nevertheless connected to the underlying theme. There are also exercises scattered throughout; they are for the interested reader to flex his or her calculus muscles by verifying or extending results stated in the text. The combination of so many topics provides many opportunities for mathematical modeling at different levels of complexity and sophistication. Sometimes several complementary levels of description are possible when developing a mathematical model; in particular this is readily illustrated by the different types of traffic flow model presented in Chapters 8 through 13.

In writing this book I have studied many articles both online and in the literature. Notes identifying the authors of these articles, denoted by numbers in square brackets in the text, can be found in the references. A more general set of useful citations is also provided.