the abacist versus the algorist

One afternoon in Rio de Janeiro, the Nobel Prize-winning physicist Richard Feynman was eating dinner in his favorite restaurant. It wasn't actually dinnertime yet, so the dining room was quiet … until the abacus salesman walked in. The waiters, who were presumably not interested in buying an abacus, challenged the salesman to prove that he could do arithmetic faster than their customer. Feynman agreed to the challenge.

At first, the contest wasn't even close. On the addition problems, Feynman wrote, the abacus salesman "beat me hollow." He would have the answer before Feynman even finished writing down the numbers. But then the salesman started getting cocky. He challenged Feynman to multiplication problems. Feynman still lost to the abacus, but not by as much. The salesman, not satisfied with his narrow margin of victory, challenged Feynman to harder and harder problems, and got more and more flustered. Finally he played his trump card. "*Raios cubicos*!" the salesman said. "Cube roots!"

Obviously, by this point the competition was more about pride than about selling an abacus. It's difficult to imagine why a restaurant manager would ever need to compute a cube root. But Feynman agreed, provided that the waiters, who were watching the competition and enjoying it immensely, would choose the number. The number they picked was 1729.03.

The abacist set to work with a passion, hunching over the abacus, his fingers flying too fast for the eye to follow. Meanwhile, Feynman writes, he was just sitting there. The waiters asked him what he was doing, and he tapped his head: "Thinking!" Within a few seconds, Feynman had written down five digits of the answer (12.002). After a while, the abacus salesman triumphantly announced "12!" and then a few minutes later, "12.0!" By this time Feynman had added several more digits to his answer. The waiters laughed at the salesman, who left in humiliation, beaten by the power of pure thought.

Like all good tales, Feynman's duel with the abacist has many layers of meaning. On the most superficial level, it is a story about genius; the Nobel Prize winner beating the machine. However, Feynman's intention when he told this story about himself was quite different. He was not a boastful man. In the context of his book, the point of the story was that *ordinary people*—not Nobel Prize winners, not geniuses—could do just the same thing as he did, with a little bit of number sense and mathematical knowledge. There were two secrets behind his seemingly magical feat. First, he needed to know that 1728 was a perfect cube: $12^3 = 1728$ (not common knowledge, perhaps, but it's something most physicists would be aware of, because a cubic foot is 12^3 or 1728 cubic inches.) And he needed to know a famous equation from calculus, called Taylor's formula—a very general approximation method that allows you to go from the exact equation:

$$\sqrt[3]{1728} = 12$$

to the approximate equation: $\sqrt[3]{1729.03} \approx 12.002$

Equations are the lifeblood of mathematics and science. They are the brush strokes that mathematicians use to create their art, or the secret code that they use to express their ideas about the universe. That is not to say that equations are the *only* tool that mathematicians use; words and diagrams are important, too. Nevertheless, when push comes to shove—for instance, when they have to compute the cube root of 1729.03—equations convey information with an economy and precision that words or abaci can never match.

The rest of the world, outside of science, does not speak the language of equations, and thus a vast cultural gap has emerged between those who understand them and those who do not. This book is an attempt to build a bridge across that chasm. It is intended for the reader who would like to understand mathematics on its own terms, and who would like to appreciate mathematics as an art. Surely we would not attempt to discuss the works of Rembrandt or Van Gogh without actually looking at their paintings. Why, then, should we talk about Isaac Newton or Albert Einstein without exhibiting their "paintings"? The following chapters will try to explain in words—even if words are feeble and inaccurate—what these equations mean and why they are justly treasured by those who know them.

Let's go back now to Richard Feynman and that abacus salesman, because there is more to say about them. In all likelihood, neither of them knew that they were playing out a scene that had already been enacted centuries before, when Arabic numerals first arrived in Europe.

When the new number system appeared around the beginning of the thirteenth century, many people were deeply suspicious of it. They had to learn nine new and unfamiliar symbols: 1, 2, 3, 4, 5, 6, 7, 8, and 9—or, to be more precise, they had to learn the somewhat distorted thirteenth-century versions thereof. The new symbols looked to some people like occult runes, instead of the nice solid Roman letters (I, V, X, etc.) they were accustomed to. To make things worse, they were *Arabic*—not even Christian—which made them appear even more suspicious to a deeply religious society. And finally, they included an innovation that was especially hard to grasp: the number zero, a something that meant nothing.

Nevertheless, Arabic numbers had an undeniable power. Unlike Roman numerals, which were useful for writing numbers but impractical for calculating with them, the decimal place-value system made it possible to do both. In a sense, Arabic numbers democratized mathematics. In many ancient societies, only a specially trained class of scribes could do arithmetic. With decimal notation, you did not need special training or special tools, only your brain and a pen.

The struggle between the old and new number systems went on for a very long time—well over two centuries. And, in fact, open competitions were held between abacists (people who used mechanical tools to do arithmetic) and algorists (people who used the new algorithmic methods). So Feynman and the abacus salesman were re-fighting a very old duel!

WE KNOW HOW the battle ended. Nowadays, everyone in Western society uses decimal numbers. Grade school students learn the algorithms for adding, subtracting, multiplying, and dividing. So clearly, the algorists won. But Feynman's story shows that the reasons may not be as simple as you think. On some problems, the abacists were undoubtedly faster. Remember that the abacus salesman "beat him hollow" at addition. But the decimal system provides a deeper insight into numbers than a mechanical device does. So the harder the problem, the better the algorist will perform. As science progressed during the Renaissance, mathematicians would need to perform even more sophisticated calculations than cube roots. Thus, the algorists won for two reasons: at the high end, the decimal system was more compatible with advanced mathematics; while at the low end, the decimal system empowered everyone to do arithmetic.

But before we start feeling too smug about our "superior" number system, the tale offers several cautionary lessons. First is a message that is far from obvious to most people: There are many different ways to do mathematics. The way you learned in school is only one of numerous possibilities. Especially when we study the history of mathematics, we find that other civilizations used different notations and had different styles of reasoning, and those styles often made very good sense for that society. We should not assume they are "inferior." An abacus salesman can still beat a Nobel Prize winner at addition and multiplication.

Feynman's tale exemplifies also how mathematical cultures have collided many times in the past. Often this collision of cultures has benefited both sides. For instance, the Arabs didn't invent Arabic numbers or the idea of zero—they borrowed them from India.

Finally, we should recognize that the victory of the algorists may be only temporary. In the present era, we have a new calculating device; it's called the computer. Any mathematics educator can see signs that our students' number sense, the inheritance bequeathed to us by the algorists, is eroding. Students today do not understand numbers as well as they once did. They rely on the computer's perfection, and they are unable to check its answers in case they type the numbers in wrong. We again find ourselves in a contest between two paradigms, and it is by no means certain how the battle will end. Perhaps our society will decide, as in ancient times, that the average person does not need to understand numbers and that we can entrust this knowledge to an elite caste. If so, the bridge to science and higher mathematics will become closed to many more people than it is today.