INTRODUCTION ROBERT CLIFFORD GUNNING

Mathematics is one of the greatest human accomplishments. It has long been an important human activity, from its early use in surveying and in construction in Babylonian and ancient Egyptian times, through the present phenomenal expansion of its use and study. By the period in which Greek mathematics flourished, it was not just important for its uses but it was also a major intellectual endeavor, and already it had developed the abstraction and rigor that continue to be a principal characteristic.

One of the striking aspects of mathematics is its cumulative nature; it is perhaps the only truly cumulative human activity. The axiomatic approach familiar from Euclidean geometry, and indeed the body of geometry the Greeks developed, remain essential parts of mathematics today. Euclid's proof that there are infinitely many prime numbers and the recognition by Pythagoras that π is irrational are as valid now as ever and are still the standard results taught to young students of mathematics. The method of calculating volumes by slicing, as developed by Archimedes, was extended and included in Cavalieri's principle, applying the tools of calculus developed by Newton and Leibniz to the same problems. That method was extended further to Fubini's theorem when the more general theories of measure and integration, developed by Lebesgue and others early in the twentieth century, provided new and more powerful tools for the calculation.

The ancient Greeks appreciated the anomalous nature of the parallel postulate and had begun research on the true nature of that postulate; this research was continued through the development of non-Euclidean geometry in the nineteenth century and the more extensive development of differential geometry since then. This cumulative nature, the fact that nothing once proved is really lost, although much may be forgotten from time to time, means that the body of mathematics is amazingly extensive. The only way in which the mass of results developed over the years can be recalled and understood effectively is by combining a vast number of individual results and observations into a more general, and more abstract, structure that can be grasped and used as a tool for further work. And these abstract tools, when they too are extended and analyzed by continuing research, in turn only can be recalled and understood effectively by combining them in even more abstract and general structures.

This extensive and cumulative nature of mathematics does mean that it is difficult to convey to nonmathematicians a good deal of the true nature of modern mathematics. Of course everyone uses and understands a good deal of mathematics. Business and commerce and construction and so many other fundamental activities rely on mathematical tools to be useful or even to work at all, and some of these mathematical tools themselves can be quite abstract and general, even if the abstraction and generality are effectively hidden by being embedded in computers and electronics. The charm and true fun of mathematics are familiar to anyone who is fascinated by Sudoku or Rubik's Cube. Easily stated puzzles such as the Catalan conjecture (that $1 = 9 - 8 = 3^2 - 2^3$ is the only instance of two nontrivial powers of integers that differ by 1) or Fermat's last theorem (that the equation $x^n + y^n = z$ has no nontrivial integer solutions when n is an integer strictly greater than 2) also indicate the intriguing and delightful possibilities in mathematics, even when the solutions of these puzzles can vary from being rather tricky and not altogether obvious to being quite extraordinarily difficult. However, it is very hard to convey a true appreciation of the absolutely fascinating and absorbing beauty of mathematics, the grandeur of its structures, the delight in recognizing what common elements really underlie a number of puzzles, and the joy of completing an extremely involved and challenging calculation and of being the first to do so or the first to come up with a new and significant mathematical structure. A deep appreciation of mathematics really requires an understanding that can come only by knowing enough of the vast structure of mathematics to be able to follow the proofs and arguments. Imagine the difficulty in teaching a true appreciation of music if the only way to enjoy the late quartets was the way Beethoven himself did, by hearing them in his mind after reading the scores as written. Even to convey the significance of some of the problems that guide current research in mathematics — such as the Clay Mathematical Institute's seven Millennium Prize Problems, each of which has a million-dollar reward for the first accepted and published solution — is challenging. Books have been written to give an indication of the nature of the Poincaré conjecture, a characterization of three-dimensional spheres, the first Millennium Prize Problem with an apparent solution and one that at least can be described vaguely in everyday terms; any description of some of the others, like the Hodge conjecture or the Navier–Stokes equation or the Birch and Swinnerton-Dyer conjecture, really requires a considerable background to appreciate, if not even to understand.

That does not mean, though, that mathematicians have given up attempts to explain the nature and joys of mathematics to everyone else in the world. Courses in "mathematical appreciation" or on "mathematics for poets" are common in many universities, and indeed are quite popular in some; but even those do require a level of commitment and work that is more than can be expected from many people in such busy and challenging times as the present. Mathematical puzzles and puzzle books abound, and popular accounts of particular mathematical topics appear with great regularity but often involve only a vague and superficial understanding of what really is going on.

This book is intended to provide another approach to spread more widely an appreciation of the nature of mathematics through an idea that occurred to Brandon Fradd and that he has effectively pushed through to completion. Even as an undergraduate mathematics major at Princeton University in the 1980s, he was concerned with the ways in which mathematics is taught, both to prospective mathematicians and to the general population. When he came across the elegant book of photographs of scientists by Mariana Cook, he conceived the plan of creating a comparable book focused on mathematicians, containing both photographs of current practitioners of mathematics and some brief descriptions of their lives, thoughts, and motivations to do mathematics. In Ms. Cook he found a superb photographer who could not only create perceptive records of the individuals she talked to but could also bring out some of the aspects of their personalities that might indicate the sort of people who find mathematics an overwhelming delight and challenge and what motivates them in this really rather arduous and compelling activity. The result of this collaboration is the current volume. It focuses on some ninety-two mathematicians around the world who have pursued a wide variety of mathematics with an equally wide variety of motivations and gives not only a photographic study of each, but also some indication, in the words of each, of what drives them to do mathematics. The selection is not intended as a list of the "top" current mathematicians but rather is somewhat random. The project began with some of the mathematicians who were Brandon Fradd's teachers and friends at Princeton and expanded with suggestions from those individuals of others throughout the world who could convey a sense of the variety of people who currently practice mathematics. The hope of its creators is that this book might be a way of indicating that the pursuit of mathematics is a continuing activity that attracts a wide variety of delightful, individualistic, and devoted men and women, and might give at least some indication of what motivates and inspires these mathematicians.