# Introduction

A century ago, mathematics history began with the Greeks, then skipped a thousand years and continued with developments in the European Renaissance. There was sometimes a brief mention that the "Arabs" preserved Greek knowledge during the dark ages so that it was available for translation into Latin beginning in the twelfth century, and perhaps even a note that algebra was initially developed in the lands of Islam before being transmitted to Europe. Indian and Chinese mathematics barely rated a footnote.

Since that time, however, we have learned much. First of all, it turned out that the Greeks had predecessors. There was mathematics both in ancient Egypt and in ancient Mesopotamia. Archaeologists discovered original material from these civilizations and deciphered the ancient texts. In addition, the mathematical ideas stemming from China and India gradually came to the attention of historians. In the nineteenth century, there had been occasional mention of these ideas in fairly obscure sources in the West, and there had even been translations into English or other western languages of certain mathematical texts. But it was only in the late twentieth century that major attempts began to be made to understand the mathematical ideas of these two great civilizations and to try to integrate them into a worldwide history of mathematics. Similarly, the nineteenth century saw numerous translations of Islamic mathematical sources from the Arabic, primarily into French and German. But it was only in the last half of the twentieth century that historians began to put together these mathematical ideas and attempted to develop an accurate history of the mathematics of Islam, a history beyond the long-known preservation of Greek texts and the algebra of al-Khwarizmi. Yet, even as late as 1972, Morris Kline's monumental work *Mathematical Thought from Ancient to Modern Times* contained but 12 pages on Mesopotamia, 9 pages on Egypt, and 17 pages combined on India and the Islamic world (with nothing at all on China) in its total of 1211 pages.

It will be useful, then, to give a brief review of the study of the mathematics of Egypt, Mesopotamia, China, India, and Islam to help put this *Sourcebook* in context.

To begin with, our most important source on Egyptian mathematics, the Rhind Mathematical Papyrus, was discovered, probably in the ruins of a building in Thebes, in the middle of the nineteenth century and bought in Luxor by Alexander Henry Rhind in 1856. Rhind died in 1863 and his executor sold the papyrus, in two pieces, to the British Museum in 1865. Meanwhile, some fragments from the break turned up in New York, having been acquired also in Luxor by the American dealer Edwin Smith in 1862. These are now in the

Brooklyn Museum. The first translation of the Rhind Papyrus was into German in 1877. The first English translation, with commentary, was made in 1923 by Thomas Peet of the University of Liverpool. Similarly, the Moscow Mathematical Papyrus was purchased around 1893 by V. S. Golenishchev and acquired about twenty years later by the Moscow Museum of Fine Arts. The first notice of its contents appeared in a brief discussion by B. A. Turaev, conservator of the Egyptian section of the museum, in 1917. He wrote chiefly about problem 14, the determination of the volume of a frustum of a square pyramid, noting that this showed "the presence in Egyptian mathematics of a problem that is not to be found in Euclid." The first complete edition of the papyrus was published in 1930 in German by W. W. Struve. The first complete English translation was published by Marshall Clagett in 1999.

Thus, by early in the twentieth century, the basic outlines of Egyptian mathematics were well understood—at least the outlines as they could be inferred from these two papyri. And gradually the knowledge of Egyptian mathematics embedded in these papyri and other sources became part of the global story of mathematics, with one of the earliest discussions being in Otto Neugebauer's *Vorlesungen über Geschichte der antiken Mathematischen Wissenschaften* (more usually known as *Vorgriechische Mathematik*) of 1934, and further discussions and analysis by B. L. Van der Waerden in his *Science Awakening* of 1954. A more recent survey is by James Ritter in *Mathematics Across Cultures*.

A similar story can be told about Mesopotamian mathematics. Archaeologists had begun to unearth the clay tablets of Mesopotamia beginning in the middle of the nineteenth century, and it was soon realized that some of the tablets contained mathematical tables or problems. But it was not until 1906 that Hermann Hilprecht, director of the University of Pennsylvania's excavations in what is now Iraq, published a book discussing tablets containing multiplication and reciprocal tables and reviewed the additional sources that had been published earlier, if without much understanding. In 1907, David Eugene Smith brought some of Hilprecht's work to the attention of the mathematical world in an article in the *Bulletin of the American Mathematical Society*, and then incorporated some of these ideas into his 1923 *History of Mathematics*.

Meanwhile, other archaeologists were adding to Hilprecht's work and began publishing some of the Mesopotamian mathematical problems. The study of cuneiform mathematics changed dramatically, however, in the late 1920s, when François Thureau-Dangin and Otto Neugebauer independently began systematic programs of deciphering and publishing these tablets. In particular, Neugebauer published two large collections: *Mathematische Keilschrift-Texte* in 1935–37 and (with Abraham Sachs) *Mathematical Cuneiform Texts* in 1945. He then summarized his work for the more general mathematical public in his 1951 classic, *The Exact Sciences in Antiquity*. Van der Waerden's *Science Awakening* was also influential in publicizing Mesopotamian mathematics. Jens Høyrup's survey of the historiography of Mesopotamian mathematics provides further details.

Virtually the first mention of Chinese mathematics in a European language was in several articles in 1852 by Alexander Wylie entitled "Jottings on the Science of the Chinese: Arithmetic," appearing in the *North China Herald*, a rather obscure Shanghai journal. However, they were translated in part into German by Karl L. Biernatzki and published in *Crelle's Journal* in 1856. Six years later they also appeared in French. It was through these articles that Westerners learned of what is now called the Chinese Remainder problem and how it was initially solved in fourth-century China, as well as about the ten Chinese classics and the Chinese algebra of the thirteenth century. Thus, by the end of the nineteenth century, European historians of mathematics could write about Chinese mathematics, although, since they did not have access to the original material, their works often contained errors.

The first detailed study of Chinese mathematics written in English by a scholar who could read Chinese was *Mathematics in China and Japan*, published in 1913 by the Japanese scholar Yoshio Mikami. Thus David Eugene Smith, who co-authored a work solely on Japanese mathematics with Mikami, could include substantial sections on Chinese mathematics in his own *History* of 1923. Although other historians contributed some material on China during the first half of the twentieth century, it was not until 1959 that a significant new historical study appeared, volume 3 of Joseph Needham's *Science and Civilization in China*, entitled *Mathematics and the Sciences of the Heavens and the Earth*. One of Needham's chief collaborators on this work was Wang Ling, a Chinese researcher who had written a dissertation on the *Nine Chapters* at Cambridge University. Needham's work was followed by the section on China in A. P. Yushkevich's history of medieval mathematics (1961) in Russian, a book that was in turn translated into German in 1964. Since that time, there has been a concerted effort by both Chinese and Western historian of mathematics to make available translations of the major Chinese texts into Western languages.

The knowledge in the West of Indian mathematics occurred much earlier than that of Chinese mathematics, in part because the British ruled much of India from the eighteenth century on. For example, Henry Thomas Colebrooke collected Sanskrit mathematical and astronomical texts in the early nineteenth century and published, in 1817, his *Algebra with Arithmetic and Mensuration from the Sanscrit of Brahmegupta and Bhascara*. Thus parts of the major texts of two of the most important medieval Indian mathematicians were available in English, along with excerpts from Sanskrit commentaries on these works. Then in 1835, Charles Whish published a paper dealing with the fifteenth-century work in Kerala on infinite series, and Ebenezer Burgess in 1860 published a translation of the *Sūrya-siddhānta*, a major early Indian work on mathematical astronomy. Hendrik Kern in 1874 produced an edition of the *Āryabhaṭīya* of Aryabhata, while George Thibaut wrote a detailed essay on the *Śulbasūtras*, which was published, along with his translation of the *Baudhāyana Śulbasūtra*, in the late 1870s. The research on medieval Indian mathematics by Indian researchers around the same time, including Bāpu Deva Sāstrī, Sudhākara Dvivedī, and S. B. Dikshit, although originally published in Sanskrit or Hindi, paved the way for additional translations into English.

Despite the availability of some Sanskrit mathematical texts in English, it still took many years before Indian contributions to the world of mathematics were recognized in major European historical works. Of course, European scholars knew about the Indian origins of the decimal place-value system. But in part because of a tendency in Europe to attribute Indian mathematical ideas to the Greeks and also because of the sometimes exaggerated claims by Indian historians about Indian accomplishments, a balanced treatment of the history of mathematics in India was difficult to achieve. Probably the best of such works was the *History of Indian Mathematics: A Source Book*, published in two volumes by the Indian mathematicians Bibhutibhusan Datta and Avadhesh Narayan Singh in 1935 and 1938. In recent years, numerous Indian scholars have produced new Sanskrit editions of ancient texts, some of which have never before been published. And new translations, generally into English, are also being produced regularly, both in India and elsewhere.

As to the mathematics of Islam, from the time of the Renaissance Europeans were aware that algebra was not only an Arabic word, but also essentially an Islamic creation. Most early algebra works in Europe in fact recognized that the first algebra works in that continent were translations of the work of al-Khwārizmī and other Islamic authors. There was also some awareness that much of plane and spherical trigonometry could be attributed to Islamic authors. Thus, although the first pure trigonometrical work in Europe, *On Triangles* by Regiomontanus, written around 1463, did not cite Islamic sources, Gerolamo Cardano noted a century later that much of the material there on spherical trigonometry was taken from the twelfth-century work of the Spanish Islamic scholar Jābir ibn Aflaḥ.

By the seventeenth century, European mathematics had in many areas reached, and in some areas surpassed, the level of its Greek and Arabic sources. Nevertheless, given the continuous contact of Europe with Islamic countries, a steady stream of Arabic manuscripts, including mathematical ones, began to arrive in Europe. Leading universities appointed professors of Arabic, and among the sources they read were mathematical works. For example, the work of Ṣadr al-Ṭūsī (the son of Naṣīr al-Dīn al-Ṭūsī) on the parallel postulate, written originally in 1298, was published in Rome in 1594 with a Latin title page. This work was studied by John Wallis in England, who then wrote about its ideas as he developed his own thoughts on the postulate. Still later, Newton's friend, Edmond Halley, translated into Latin Apollonius's *Cutting-off of a Ratio*, a work that had been lost in Greek but had been preserved via an Arabic translation.

Yet in the seventeenth and eighteenth centuries, when Islamic contributions to mathematics may well have helped Europeans develop their own mathematics, most Arabic manuscripts lay unread in libraries around the world. It was not until the mid-nineteenth century that European scholars began an extensive program of translating these mathematical manuscripts. Among those who produced a large number of translations, the names of Heinrich Suter in Switzerland and Franz Woepcke in France stand out. (Their works have recently been collected and republished by the Institut für Geschichte der arabisch-islamischen Wissenschaften.) In the twentieth century, Soviet historians of mathematics began a major program of translations from the Arabic as well. Until the middle of the twentieth century, however, no one in the West had pulled together these translations to try to give a fuller picture of Islamic mathematics. Probably the first serious history of Islamic mathematics was a section of the general history of medieval mathematics written in 1961 by A. P. Yushkevich, already mentioned earlier. This section was translated into French in 1976 and published as a separate work, *Les mathématiques arabes* (*VIII^e^–XV^e^ siècles*). Meanwhile, the translation program continues, and many new works are translated each year from the Arabic, mostly into English or French.

By the end of the twentieth century, all of these scholarly studies and translations of the mathematics of these various civilizations had an impact on the general history of mathematics. Virtually all recent general history textbooks contain significant sections on the mathematics of these five civilizations. As this sourcebook demonstrates, there are many ideas that were developed in these five civilizations that later reappeared elsewhere. The question that then arises is how much effect the mathematics of these civilizations had on what is now world mathematics of the twenty first-century. The answer to this question is very much under debate. We know of many confirmed instances of transmission of mathematical ideas from one of these cultures to Europe or from one of these cultures to another, but there are numerous instances where, although there is circumstantial evidence of transmission, there is no definitive documentary evidence. Whether such will be found as more translations are made and more documents are uncovered in libraries and other institutions around the world is a question for the future to answer.

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