

Introduction

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This is the ninth volume in our series of remarkable writings on mathematics. The pieces you will read here were initially published during 2017 in various venues, including academic and other professional journals, book chapters, online publications, or newspapers. Except for a few technical mathematical notions required to understand select pieces, the book is accessible to the public that does not specialize in mathematics; yet the book will also interest mathematicians and scientists. Aiming to a wide audience has been, and remains, one of our goals when we prepare every volume.

The origins of *The Best Writing on Mathematics* series go back about fifteen years, to a time when my frustration with the clichés about mathematics I was reading and hearing made me curious to know opinions about mathematics not only from mathematicians but also from outsiders. I quickly discovered that a considerable literature on mathematics authored by mathematicians and by nonmathematicians exists and thrives. Despite its richness in ideas, it is mostly ignored in academic institutions, as if it did not exist and it had no instructional value. For several years, I mulled over the idea of editing such a series, and I attempted to start it; yet life difficulties and the disinterest toward my proposal from the publishers I approached stopped the project in its tracks.

Since 2010, the volumes in this series have contained more than two hundred pieces by authors with diverse backgrounds. These articles range in style from tightly argued theoretical positions on issues related to mathematics to bold speculations on the limits of the applicability of mathematics. Overall, the series is meant to convey to its readers the extraordinary ramifications of the influence of mathematics on contemporary mind, life, and society—and to stimulate connections we

usually overlook when we talk about mathematics. The ninth volume is no exception from the general profile of the series.

Overview of the Volume

In the first article in the book, Francis Su gives us an impassioned credo, which he delivered as president of the Mathematical Association of America. Su finds common threads between mathematics on one side and play, beauty, truth, justice, and love on the other side—and from these associations concludes that mathematics contributes to human flourishing.

Margaret Wertheim proposes that certain sentient organisms other than humans, and even nonliving artifacts, perform mathematics whenever they enact rigorous geometric or algebraic patterns—that is, they do so without conscious intelligence.

Robert Thomas considers prevalent views on the “beauty” of mathematics and argues that the quality of making us interested and curious to do mathematics is at least as valuable aesthetically as the quality of pleasing us.

Marijn Heule and Oliver Kullmann write that automated computer proofs are useful and meaningful even if we cannot understand them—and describe how such methods work, detailing one example of proof done by what they call brute reason.

Peter Denning explains how the growth of computational power enabled scientists to change their disciplines from within by adding simulation and information process analysis to the established practices of experimenting and theorizing.

Robbert Dijkgraaf points out that the dynamic of the long interaction between mathematics and physics is reversing; where traditionally mathematics influenced physics, lately physics branches such as string and quantum theories occasion breakthroughs in mathematics, possibly leading toward a new type of mathematics, which he tentatively calls “quantum mathematics.”

Erik Demaine and his coauthors describe a planar tangle toy, examine some of the topological configurations available through manipulating it, answer some of the mathematical questions it poses, and formulate a few open problems related to it.

James Grime shows us how to build subtly mischievous dice for playing slightly unfair games!

Arthur Benjamin, Joseph Kisenwether, and Ben Weiss consider another game: bingo. They observe and prove that, contrary to unexamined expectations, bingo winning patterns are asymmetrical, with completed horizontal rows occurring more frequently than completed vertical columns.

Peter Winkler summarizes a plethora of conflicting mathematical, psychological, semantic, and psychological arguments advanced over the past two decades in connection with a question of probability. He purposefully settles nothing, convinced that the controversy surrounding it will continue forever.

José Ferreirós delves into Eugene Wigner's intellectual biography and finds that Wigner's conception of mathematics and some of his consequential epistemological claims were influenced to a considerable degree by his professional friends and associates.

Chris Arney introduces us to the fundamentals of mathematical modeling, including to the necessary theoretical components of modeling, the current and potential scope of applications, and the essential bibliography specialized in mathematical modeling.

Nancy Emerson Kress offers precise advice to school students (and implicitly to teachers, but useful to everyone) on how to approach and to solve problems in mathematics.

On a topic that I have wanted to include in *The Best Writing on Mathematics* anthologies for a long time, Benjamin Braun and his coauthors define active classroom instruction, describe its benefits and variants, and prepare instructors inclined to adopt active teaching methods for some of the challenges and opportunities they are likely to experience.

In a brief note relating more detailed research, Daniel Mansfield and Norman Wildberger explain that in ancient Babylon the mathematicians (or perhaps the surveyors) of the time practiced a different trigonometry than we do today—simpler but precise, founded on the sexagesimal system of numeration.

Isabel Serrano, Lucy Odom, and Bogdan Suceavă examine the structure of mathematics in one of the most important encyclopedic works of premodern times, the *Etymologies* authored by Isidore of Seville in the seventh century.

Michael Barany traces the current awareness of the importance of mathematics in society to developments that started shortly before, but especially during, the Second World War; Barany tells the story of the people who accomplished the major shift in research funding from a long tradition of private sponsorship to one supported in considerable part by the state.

Finally, Caroline Yoon argues that writing and doing mathematics have more in common than we usually admit—and encourages mathematicians (as well as budding mathematicians) to transfer their mathematical skills into writing competencies. Her advice coincides with one of the main goals of *The Best Writing on Mathematics* series.

More Writings on Mathematics

Every year we offer further reading suggestions on mathematics, chosen from recent publications. Toward the end of the book you can find a long list of pieces I considered for selection in this volume. Here, I list some of the books that have come to my attention lately.

I make special mention of the graphically exceptional collection *The Arts of Ornamental Geometry*, edited by Gülru Necipoğlu and the collection of interviews with Russian mathematicians edited by Andrei Sobolevski under the title *Mathematical Walks*.

Among books on mathematics in daily life, including puzzles and games, you can pick from *Gladiators, Pirates and Games of Trust* by Haim Shapira, *Chancing It* by Robert Matthews, *We Are Data* by John Cheney-Lippold, *A Survival Guide to the Misinformation Age* by David Helfand, *The Math Behind . . .* by Colin Beveridge, and *The Power of Networks* by Christopher Brinton and Mung Chiang.

You will find plenty more expository and exciting mathematics at an accessible level in *Foolproof* by Brian Hayes, *Mathematics Rebooted* by Lara Alcock, *Arithmetic* by Paul Lockhart, and even in (the now second volume of) *The Mathematics of Various Entertaining Subjects* edited by Jennifer Beineke and Jason Rosenhouse. More challenging for the mathematically uninitiated, yet appealing to the interested learner with a solid background in school mathematics are *Prime Numbers and the Riemann Hypothesis* by Barry Mazur and William Stein, *Introduction to Experimental Mathematics* by Søren Eilers and Rune Johansen, *Modern Cryptography and Elliptic Curves* by Thomas Shemanske, *A Conversational*

Introduction to Algebraic Theory by Paul Pollack, and the very original and eye-pleasing presentation in *An Illustrated Theory of Numbers* by Martin Weissman.

In the history of mathematics literature, we have two works on China: Daniel Morgan's *Astral Sciences in Early Imperial China*, and Tina Su Lyn Lim and Donald Wagner's *The Continuation of Ancient Mathematics*. For mathematics on other meridians, we cite Jason Fagone's *The Woman Who Smashed Codes* [Elizebeth Smith Friedman], Christopher Graney's *Mathematical Disquisitions* [on Galileo], and Johnny Ball's *Wonders beyond Numbers*. Also in the field of history are *Mathematical Cultures*, edited by Brendan Larvor, *The Mathematics of Secrets* by Joshua Holden, Edward Watts's *Hypatia*, and Michel Serres's *Geometry*. An adventurous autobiography is Ted Hill's *Pushing Limits*.

On statistics, probability, and data, you can learn about *Ten Great Ideas about Chance* from Persi Diaconis and Brian Skyrms, and about *The Tao of Statistics* from Dana Keller. Jeffrey Stanton authored *Reasoning with Data*, and David Carlson published *Quantitative Methods in Archaeology Using R*.

Mathematics is applied in many disciplines, and the literature dedicated to applications is huge. Here are several new titles: *Models, Mathematics, and Methodology in Economic Explanation* by Donald Katzner and *How Much Inequality Is Fair?* by Venkat Venkatasubramanian in economics; *Narrative and Numbers* by Aswath Damodaran, *The Money Formula* by Paul Wilmott and David Orrell, *The Spider Network* by David Enrich, and *Analysing Quantitative Survey Data for Business and Management Students* by Jeremy Dawson in business and finance; *The Birth of Physics* by Michel Serres, *Reality Is Not What It Seems* by Carlo Rovelli, and *It's about Time* by Roger Cooke in physics; *Computational Modeling of Cognition and Behavior* by Simon Farrell and Stephan Lewandowsky, *How Our Days Became Numbered* by Dan Bouk, and *Political Games* by Macartan Humphreys in psychology and other social sciences. A wide-ranging collection of contributions on computing and its history, especially related to Alan Turing's work, is *The Once and Future Turing*, edited by Barry Cooper and Andrew Hodges. In *New Lines*, Matthew Wilson applies mathematical methods to geography and mapmaking, while Paul Charbonneau does so for a multitude of contexts in *Natural Complexity*. An eminently accessible introduction to the mathematics of basic models is *Algebra with Models* by Marian Anton and Karen Santoro. Finally in this category,

an idiosyncratic use of mathematics is given by Ben Blatt in *Nabokov's Favorite Word Is Mauve*.

In the philosophy of mathematics, recent titles include *Rethinking Knowledge* by Carlo Cellucci, *Gödel's Disjunction* by Leon Horsten and Philip Welch, *The Ethics of Technology* by Martin Peterson, *Objectivity, Realism, and Proof*, edited by Francesca Boccuni and Andrea Sereni, and *Categories for the Working Philosopher* by Elaine Landry. In the topic of logic are *Bolzano's Conception of Grounding* by Stefan Roski and Kurt Gödel's *Logic Lectures*. A wide-ranging book difficult to categorize is John Stillwell's *Reverse Mathematics*.

Many interdisciplinary books contain mathematical arguments at their hearts. This type of literature is expanding fast. Here are some titles: *Convergence* by Peter Watson, *The Mathematical Corporation* by Josh Sullivan and Angela Zutavern, *Observation and Experiment* by Paul Rosenbaum, *Numbers and the Making of Us* by Caleb Everett, *Mathematics as a Tool* edited by Johannes Lenhard and Martin Carrier, *Autonomous Nature* by Carolyn Merchant, *Scale* by Geoffrey West, *Do the Math!* by John White, *The Probabilistic Foundations of Rational Learning* by Simon Huttegger, and *Game Changers* by Rudolf Taschner.

A book that bears the qualification “dictionary” but in fact is a collection of relevant place descriptions, is *Mathematical Berlin* by Iris and Martin Grötschel. Almost exclusively visual but full of interesting information is *The Book of Circles* by Manuel Lima.



I hope that you, the reader, will enjoy reading this anthology at least as much as I did working on it. I encourage you to send comments, suggestions, and materials I might consider for (or mention in) future volumes to Mircea Pitici, P.O. Box 4671, Ithaca, NY 14852; or send electronic correspondence to mip7@cornell.edu.

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