

Introduction

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The eighth volume of *The Best Writing on Mathematics* brings you a new collection of diverse, surprising, and well-written pieces, all published originally during 2016 in academic journals, scientific magazines, or mass media. In addition to the selection, at the end of the book I offer a copious reference section of notable writings and sources for those of you who want to find out more about mathematics on your own; that supplement is important for the goals of the series, serving the research needs of interested readers.

I hope that this series illustrates the versatility of mathematics and that of its interpretations; I also hope that the series helps readers gain a rich panoramic view of mathematics, as opposed to the impoverished parochial view promoted at all levels by our education system. The more facets of mathematics we discover, the more aware we are that mathematics has become a behemoth of human thought, with tentacles reaching into many of the ingenious innovations that fill our personal and collective lives with technological wonders and with deadly perils.

In a memorable line from the movie *Stand and Deliver* (1988), the mathematics teacher Jaime Escalante tells his students in a run-down Los Angeles community that “math is the great equalizer”—meaning, perhaps, that learning mathematics opens up life possibilities for achievement to everyone, regardless of their ethnicity, social condition, and family status. My own avatars teach me that, like everything else people say about mathematics, Escalante’s proclamation is both true *and* untrue, depending on the perspective one takes and on the life caprices one encounters. Against a long and parsimonious tradition that associates mathematics with recipes of ready-made clichés, I aim to show with this series that mathematics is more interesting than the most interesting writing about it—and, more, that this statement

remains valid even if we replace the attribute “interesting” with one of its antonyms, or even with most other attributes. Such a sweeping statement will sound disconcerting to the unaware mind, but it is intriguing to the inquisitive mind. Mathematics is a domain of clarity *and* obscurity, of enchantment *and* boredom, of unperturbed neatness *and* of puzzling paradox, of apodictic truth *and* of arguable interpretation. The pieces collected in this volume once again demonstrate the dynamic coexistence of opposite characteristics of mathematics—and show that mathematics is anything but the dull subject serviced by an increasingly powerful but stultifying educational bureaucracy unable to grasp, appreciate, promote, and teach the creative and imaginative sides of mathematics.

Overview of the Volume

In the same vein as the previous books in the series, this volume contains both expository and interpretive pieces on mathematics and aspects of life in the mathematical community, historical and contemporary.

To open the selection, Philip Davis sees mathematicians as producers and shows, with many examples from the past and from the present, that treating mathematical results as “products” is neither far-fetched nor outrageous; it is just an observation supported by abundant evidence but still denied by many mathematicians.

Evelyn Lamb explains why it is useful to know that certain big numbers are primes—and why people are finding primes among a variety of numbers of certain algebraic expression.

Kevin Hartnett describes the work of geometers and physicists who attempt to discover similarities among various random processes.

Siobhan Roberts glosses on the idiosyncratic mathematical achievements of a peculiar centenarian, the recently celebrated Richard Guy.

Lloyd Trefethen shows that the precision of mathematical statements obscures the multitude of contexts in which we can interpret such results.

Gerald Alexanderson reviews biographical contributions inspired by Srinivasa Ramanujan’s life and work, and he tells us the intriguing circumstances under which he acquired a bronze bust of the famous Indian mathematician.

Larry Riddle brings abstract algebra to the study of systems of functions to create beautiful fractal images.

Marc Frantz contributes with elements of projective geometry to the wider context (within optics and perception) of the moon tilt illusion.

Mohammadhossein Kasraei, Yahya Nourian, and Mohammadjavad Mahdavinejad study how the Persian architectural element *Girih* was used in the construction of three Iranian domes; they also analyze the relationship between dome curvature and the polygonal division of the dome's base circle.

Jo Boaler and Lang Chen summarize studies from several disciplines to conclude that children's degree of dexterity with "finger math" is important to their mathematical development.

Sinéad Breen and Ann O'Shea rethink the design of undergraduate mathematics education, proposing that the central role held by the pairing of content and techniques should be replaced by "threshold concepts," which they define and characterize in their piece.

John Mason pleads for a mathematics education attentive to the circumstantial elements that occasion learning—as opposed to the dogmatism of normative theories so popular with researchers.

Viktor Blåsjö exemplifies with a geometric-algebraic construction taken from Leibniz's work the changing meaning of mathematics and mathematics notation over the past few centuries.

Carlo Séquin and Raymond Shiau examine a famous painting by Fra' Luca Pacioli to determine whether the plane rendering of a spatial geometric object is genuine, and they bring the topic to the present by offering a computerized version of that representation.

Jeremy Gray asks what would have passed as most valuable mathematical research, most worthy of award-winning consideration, a century and a half ago and examines in that context the work of several mathematicians prominent at the time.

Noson Yanofsky illustrates with an abundance of examples different types of mathematical and scientific limitations, from logical and physical to mental and practical.

Jean-Pierre Marquis defines abstraction and "levels" of abstraction in mathematics, distinguishing between the axiomatic method and the abstract method. Then he infers the philosophical consequences of using the latter.

Robert Bain considers pro and con arguments for the proposition that human reasoning, beliefs, and decision making actively adjust based on evidence and probability expectations.

Speaking of expectations, in the last piece of the anthology, Graham Southorn describes quantitative methods used in forecasting and explains why they never achieve certainty in our ever more complex world.

More Writings on Mathematics

Among recent books on mathematics deserving special mention are the following: the *Sourcebook in the Mathematics of Medieval Europe and North Africa* edited by Victor Katz; the path-opening *Visualizing Mathematics with 3D Printing* by Henry Segerman; the book-and-catalogue *Mathematics* edited by David Rooney for the Science Museum of London; and two interdisciplinary books reaching both close to and far away from mathematics, *The Oxford Handbook of Generality in Mathematics and the Sciences* edited by Karine Chemla, Renaud Chorlay, and David Rabouin, and the massive *Handbook of Geomathematics* edited by Willi Freeden, Zuhair Nashed, and Thomas Sonar.

Expository books on mathematics are *Elements of Mathematics* by John Stillwell, *The Circle* by Alfred Posamentier and Robert Geretschläger, *Algebra* by Peter Higgins, *Fractals* by Kenneth Falconer, *Combinatorics* by Robin Wilson, *Measurement* by David Hand, *Some Applications of Geometric Thinking* by Bowen Kerins et al., *Thinking Geometrically* by Thomas Sibley, *Geometry in Problems* by Alexander Shen, *Problem-Solving Strategies in Mathematics* by Alfred Posamentier and Stephen Krulik, *An Interactive Introduction to Knot Theory* by Inga Johnson and Allison Henrich, *Circularity* by Ron Aharoni, *Can You Solve My Problems?* by Alex Bellos, and *Summing It Up* by Avner Ash and Robert Gross.

Mathematics in life (including gambling and games) is described and interpreted in such books as *The Calculus of Happiness* by Oscar Fernandez, *Fluke* by Joseph Mazur, *Man vs. Mathematics* by Timothy Revell and Joe Lyward, *In Praise of Simple Physics* by Paul Nahin, *The Mathematics that Power Our World* by Joseph Khoury and Gilles Lamothe, *Living by Numbers* by Steven Connor, *The Perfect Bet* by Adam Kucharski, *The Joy of SET* by Liz McMahon and her coauthors, *That's Maths* by Peter Lynch, and *Math Squared* by Rachel Thomas and Maryanne Freiberger; Daniel Levitin takes a broad perspective in *A Field Guide to Lies*.

In the history of mathematics and biography, recently I noticed *A Brief History of Mathematical Thought* by Luke Heaton, *Infinite Series in a History of Analysis* by Hans-Heinrich Körle, *Turing* by Jack Copeland,

What Is the Genus? by Patrick Popescu-Pampu, *A Delicate Balance* edited by David Rowe and Wann-Sheng Horng, *The Early Period of the Calculus of Variations* by Paolo Freguglia and Mariano Giaquinta and a new edition of *Emmy Noether's Wonderful Theorem* by Dwight Neuenschwander.

Several recent books on mathematical connections with other disciplines are Dmitry Kondrashov's *Quantitative Life*, Richard Harris's *Quantitative Geography*, Youseop Shin's *Time Series Analysis in the Social Sciences*, and Ron Aharoni's *Mathematics, Poetry, and Beauty*. Collective volumes include *Big Data and Social Science* edited by Ian Foster and his collaborators, *Big Data in Cognitive Science* edited by Michael Jones, *Data Visualization* edited by Lauren Magnuson, *Handbook of Quantitative Methods for Detecting Cheating on Tests* edited by Gregory Cizek and James Wollock, *UK Success Stories in Industrial Mathematics* edited by Philip Aston, Anthony Mulholland and Katherine Tant, and *From Numbers to Words* by Susan Morgan, Tom Reichert, and Tyler Harrison. Two interdisciplinary volumes are *Quite Right* by Norman Biggs and *The Topological Imagination* by Angus Fletcher.

Many more books than I can mention are published each year in mathematics education. Some recent volumes that came to my attention are *Creativity and Giftedness* edited by Roza Leikin and Bharath Sriraman, *Posing and Solving Mathematical Problems* edited by Patricio Felmer, Erkki Pehkonen, and Jeremy Kilpatrick, *Teaching School Mathematics* by Hung-Hsi Wu, *Task Design in Mathematics Education* edited by Anne Watson and Minoru Ohtani, *Semiotics as a Tool for Learning Mathematics* edited by Adalira Sáenz-Ludlow and Gert Kadunz, *Psychometric Methods in Mathematics Education* edited by Andrew Izsák, Janine Remillard, and Jonathan Templin, *Mathematics Education and Language Diversity* edited by Richard Barwell et al., *Putting Essential Understanding of Geometry and Measurement into Practice* edited by Kathryn Chval, and *The Second Handbook of Research on the Psychology of Mathematics Education* edited by Ángel Gutiérrez, Gilah C. Leder, and Paolo Boero.

To conclude, I enumerate several recent titles in the philosophy of mathematics and some logic essays: *Making and Breaking Mathematical Sense* by Roi Wagner, *Essays on Paradoxes* by Terence Horgan, *Bolzano's Logical System* by Ettore Casari, *Talking about Numbers* by Katharina Felka, *Resonance* by Krzysztof Burdzy, *Plural Logic* by Alex Oliver and Timothy Smiley, a new volume in Yuki Hiroshi's *Math Girls* series, as well as the collective volumes *Logical Modalities from Aristotle to Carnap* edited by Max

Cresswell, Edwin Mares, and Adriane Rini, and *Cultures of Mathematics and Logic* edited by Shier Ju, Benedikt Löwe, and their collaborators. A sociological viewpoint underpins *The Quantified Self* by Deborah Lupton.



I hope that you, the reader, will enjoy reading this anthology at least as much as I did while working on it. I encourage you to send comments, suggestions, and materials I might consider for (or mention in) future volumes to Mircea Pitici, P.O. Box 4671, Ithaca, NY 14852; or send electronic correspondence to mip7@cornell.edu.

Books Mentioned

- Aharoni, Ron. *Circularity: A Common Secret to Paradoxes, Scientific Revolutions, and Humor*. World Scientific, Singapore: 2016.
- Aharoni, Ron. *Mathematics, Poetry, and Beauty*. Singapore: World Scientific, 2016.
- Ash, Avner, and Robert Gross. *Summing It Up: From One Plus One to Modern Number Theory*. Princeton, NJ: Princeton University Press, 2016.
- Aston, Philip J, Anthony J. Mulholland and Katherine M. M. Tant. (Eds.) *UK Success Stories in Industrial Mathematics*. New York: Springer Science+Business Media, 2016
- Barwell, Richard, et al. (Eds.) *Mathematics Education and Language Diversity*. Cham, Switzerland: Springer Science+Business Media, 2016.
- Bellos, Alex. *Can You Solve My Problems? A Casebook of Ingenious, Perplexing, and Totally Satisfying Puzzles*. London, UK: Guardian Books, 2016.
- Biggs, Norman. *Quite Right: The Story of Mathematics, Measurement, and Money*. Oxford, UK: Oxford University Press, 2016.
- Burdzy, Krzysztof. *Resonance: From Probability to Epistemology and Back*. London: Imperial College Press, 2016.
- Casari, Ettore. *Bolzano's Logical System*. Oxford, UK: Oxford University Press, 2016.
- Chemla, Karine, Renaud Chorlay, and David Rabouin. (Eds.) *The Oxford Handbook of Generality in Mathematics and the Sciences*. Oxford, UK: Oxford University Press, 2016.
- Chval, Kathryn. (Ed.) *Putting Essential Understanding of Geometry and Measurement into Practice in Grades 3–5*. Reston, VA: The National Council of Teachers of Mathematics, 2016.
- Cizek, Gregory J., and James A. Wollack. (Eds.) *Handbook of Quantitative Methods for Detecting Cheating on Tests*. Abingdon, UK: Routledge, 2016.
- Connor, Steven. *Living by Numbers: In Defense of Quantity*. London: Reaktion Books, 2016.
- Copeland, Jack. *Turing: Pioneer of the Information Age*. Oxford, UK: Oxford University Press, 2014.
- Cresswell, Max, Edwin Mares, and Adriane Rini. (Eds.) *Logical Modalities from Aristotle to Carnap: The Story of Necessity*. Cambridge, UK: Cambridge University Press, 2016.
- Falconer, Kenneth. *Fractals: A Very Short Introduction*. Oxford, UK: Oxford University Press, 2013.
- Felka, Katharina. *Talking about Numbers: Easy Arguments for Mathematical Realism*. Frankfurt, Germany: Klostermann, 2016.

- Felmer, Patricio, Erkki Pehkonen, and Jeremy Kilpatrick. (Eds.) *Posing and Solving Mathematical Problems: Advances and New Perspectives*. Cham, Switzerland: Springer Nature, 2016.
- Fernandez, Oscar E. *The Calculus of Happiness: How a Mathematical Approach to Life Adds Up to Health, Wealth, and Love*. Princeton, NJ: Princeton University Press, 2017.
- Fletcher, Angus. *The Topological Imagination: Spheres, Edges, and Islands*. Cambridge, MA: Harvard University Press, 2016.
- Foster, Ian, et al. (Eds.) *Big Data and Social Science: A Practical Guide to Methods and Tools*. Boca Raton, FL: CRC Press, 2016.
- Freeden, Willi, M. Zuhair Nashed, and Thomas Sonar. (Eds.) *Handbook of Geomathematics*. Heidelberg, Germany: Springer Verlag, 2015.
- Freguglia, Paolo, and Mariano Giaquinta. *The Early Period of the Calculus of Variations*. Cham, Switzerland: Springer Science+Business Media, 2016.
- Gutiérrez, Ángel, Gilah C. Leder, and Paolo Boero. (Eds.) *The Second Handbook of Research on the Psychology of Mathematics Education: The Journey Continues*. Rotterdam, Netherlands: Sense Publishers, 2016.
- Hand, David J. *Measurement: A Very Short Introduction*. Oxford, UK: Oxford University Press, 2016.
- Harris, Richard. *Quantitative Geography: The Basics*. Los Angeles: Sage, 2016.
- Heaton, Luke. *A Brief History of Mathematical Thought*. Oxford, UK: Oxford University Press, 2017.
- Higgins, Peter M. *Algebra: A Very Short Introduction*. Oxford, UK: Oxford University Press, 2015.
- Hiroshi, Yuki. *Math Girls: Gödel's Incompleteness Theorem*. Austin, TX: Benton Books, 2016.
- Horgan, Terence. *Essays on Paradoxes*. Oxford, UK: Oxford University Press, 2017.
- Izsák, Andrew, Janine T. Remillard, and Jonathan Templin. (Eds.) *Psychometric Methods in Mathematics Education: Opportunities, Challenges, and Interdisciplinary Collaborations*. Reston, VA: The National Council of Teachers of Mathematics, 2016.
- Johnson, Inga, and Allison Henrich. *An Interactive Introduction to Knot Theory*. Mineola, NY: Dover Publications, 2016.
- Jones, Michael N. (Ed.) *Big Data in Cognitive Science*. New York: Routledge, 2017.
- Katz, Victor J. (Ed.) *Sourcebook in the Mathematics of Medieval Europe and North Africa*. Princeton, NJ: Princeton University Press, 2016.
- Kerins, Bowen, et al. *Some Applications of Geometric Thinking*. Providence, RI: American Mathematical Society, 2016.
- Khoury, Joseph and Gilles Lamothe. *The Mathematics that Power Our World: How Is It Made?* Singapore: World Scientific, 2016.
- Kondrashov, Dmitry A. *Quantitative Life: A Symbiosis of Computation, Mathematics, and Biology*. Chicago: Chicago University Press, 2016.
- Körle, Hans-Heinrich. *Infinite Series in a History of Analysis: Stages Up to the Verge of Summability*. Berlin: de Gruyter, 2015.
- Kucharski, Adam. *The Perfect Bet: How Science and Math Are Taking the Luck Out of Gambling*. New York: Basic Books, 2016.
- Leikin, Roza, and Bharath Sriraman. (Eds.) *Creativity and Giftedness: Interdisciplinary Perspectives from Mathematics and Beyond*. Basel, Switzerland: Springer, 2016.
- Levitin, Daniel J. *A Field Guide to Lies: Critical Thinking in the Information Age*. New York: Dutton, 2016.
- Lupton, Deborah. *The Quantified Self: A Sociology of Self-Tracking*. Cambridge, UK: Polity Press, 2016.
- Lynch, Peter. *That's Maths*. Dublin, Ireland: M. H. Gill & Co., 2016.
- Magnuson, Lauren. (Ed.) *Data Visualization: A Guide to Visual Storytelling for Libraries*. Lanham, MD: Rowman & Littlefield, 2016.

- Mazur, Joseph. *Fluke: The Math and Myth of Coincidence*. New York: Basic Books, 2016.
- McMahon, Liz, Gary Gordon, Hannah Gordon, and Rebecca Gordon. *The Joy of SET: The Many Mathematical Dimensions of a Seemingly Simple Card Game*. Princeton, NJ: Princeton University Press, 2017.
- Morgan, Susan E., Tom Reichert, and Tyler R. Harrison. *From Numbers to Words: Reporting Statistical Results for the Social Sciences*. New York: Routledge, 2017.
- Nahin, Paul J. *In Praise of Simple Physics: The Science and Mathematics behind Everyday Questions*. Princeton, NJ: Princeton University Press, 2017.
- Neuenschwander, Dwight E. *Emmy Noether's Wonderful Theorem*. Baltimore: The Johns Hopkins University Press, 2017.
- Oliver, Alex, and Timothy Smiley. *Plural Logic*, 2nd ed. Oxford, UK: Oxford University Press, 2016.
- Popescu-Pampu, Patrick. *What Is the Genus?* Cham, Switzerland: Springer Nature, 2016.
- Posamentier, Alfred S., and Robert Geretschläger. *The Circle: A Mathematical Exploration beyond the Line*. Amherst, NY: Prometheus Books, 2016.
- Posamentier, Alfred S., and Stephen Krulik. *Problem-Solving Strategies in Mathematics: From Common Approaches to Exemplary Strategies*. Singapore: World Scientific, 2015.
- Revell, Timothy, and Joe Lyward. *Man vs. Mathematics: Understanding the Curious Mathematics That Powers Our World*. London: Aurum Press, 2016.
- Rooney, David. (Ed.) *Mathematics: How It Shaped Our World*. London: Scala Arts & Heritage Publishers and The Science Museum, 2016.
- Rowe, David, and Wann-Sheng Horng. (Eds.) *A Delicate Balance: Global Perspectives on Innovation and Tradition in the History of Mathematics*. Bern, Switzerland: Birkhäuser, 2016.
- Sáenz-Ludlow, Adalira, and Gert Kadunz. (Eds.) *Semiotics as a Tool for Learning Mathematics: How to Describe the Construction, Visualization, and Communication of Mathematical Concepts*. Rotterdam, Netherlands: Sense Publishers, 2016.
- Segerman, Henry. *Visualizing Mathematics with 3D Printing*. Baltimore: The Johns Hopkins University Press, 2016.
- Shen, Alexander. *Geometry in Problems*. Providence, RI: American Mathematical Society, 2016.
- Shier Ju, Benedikt Löwe, Thomas Müller, and Yun Xie. (Eds.) *Cultures of Mathematics and Logic*. Bern, Switzerland: Birkhäuser, 2016.
- Shin, Yuseop. *Time Series Analysis in the Social Sciences: The Fundamentals*. Oakland, CA: University of California Press, 2017.
- Sibley, Thomas Q. *Thinking Geometrically: A Survey of Geometries*. Washington, DC: Mathematical Association of America, 2015.
- Stillwell, John. *Elements of Mathematics: From Euclid to Gödel*. Princeton, NJ: Princeton University Press, 2016.
- Thomas, Rachel, and Maryanne Freiberg. *Math Squared: 100 Ideas You Should Know*. London: Metro, 2016.
- Wagner, Roi. *Making and Breaking Mathematical Sense: Histories and Philosophies of Mathematical Practice*. Princeton, NJ: Princeton University Press, 2016.
- Watson, Anne, and Minoru Ohtani. (Eds.) *Task Design in Mathematics Education*. Cham, Switzerland: Springer Science+Business Media, 2016.
- Wilson, Robin. *Combinatorics: A Very Short Introduction*. Oxford, UK: Oxford University Press, 2016.
- Wu, Hung-Hsi. *Teaching School Mathematics: Pre-Algebra*. Providence, RI: American Mathematical Society, 2016.