Introduction

Continuous-time stochastic models have become a necessary ingredient of the finance student’s curriculum, due to their widespread applications to derivatives pricing. These methods, which were made famous by the seminal papers of Black and Scholes (1973) and Merton (1973, 1974), have proved so powerful that most (if not all) advanced programs in market finance include at least one course on continuous-time stochastic models that covers such technical tools as Itô calculus, parabolic partial differential equations, and the Feynman-Kac formula, which were initially developed for applications in other fields, such as physics.

This book is an introduction to another, equally important, field of application of the same methods (plus some additional tools), namely, corporate finance. It applies the rigor of the mathematical techniques mentioned above to concrete financial problems faced by corporations, for instance, how to finance their investments, when to distribute dividends or issue new securities and, possibly, when to default on their debt. As a result, the book combines the mathematical tools of stochastic calculus and the accounting measures that corporate managers follow actively, namely their profit and loss (P&L) accounts and balance sheets.

The target audience of the book includes PhD students in finance, economics and mathematics, as well as financial-industry professionals and academic researchers who want to get acquainted with recent research developments in continuous-time corporate finance, banking and insurance. The mathematical methodology is presented in a very practical manner: we do not prove any new theorems, but we give access to these powerful methods to the largest possible audience while preserving mathematical rigor.

The book’s prerequisites are limited to two ingredients: the standard, continuous-time, stochastic methods that are used in derivatives-pricing courses and a basic knowledge of accounting. After reading this book, everyone should be able to follow current frontier research in corporate finance, banking and insurance and, possibly, to develop original models on his or her own. Although highly technical, the mathematical methods presented in the book can be made accessible to a wide audience and used in a very intuitive way. This is the main
objective of this book.

Before the 1970s, the mathematical methods of finance were reduced to the cross-multiplication formula and the computation of discounted sums. The fantastic development of arbitrage-pricing methods has made Itô calculus and the Feyman-Kac formula part of the necessary toolbox in asset pricing. It would be a pity to be unable to also use them to solve corporate-finance, banking and insurance problems, only because their axiomatic foundations remain too technical for the majority of finance and economics students. Thus, we have striven to make these methods widely accessible.

Contents of the book

Chapter 1 starts by explaining the basic reason why the mathematical methods of option pricing are also powerful in corporate finance, banking and insurance. It is a simple consequence of a brilliant intuition of Black and Scholes (1973) and Merton (1974): given that the shareholders of a company are protected by limited liability, they have an option to default. Thus, the value of their shares is formally equivalent to the value of a call option on the assets of the company, with strike price equal to the nominal debt of the company. Shareholders have the option to repay the company’s debt, in which case they keep control of the assets of the company. Alternatively they may default on this debt, and lose everything.

Chapter 1 uses this far-reaching insight to derive formulas for pricing debt and equity contracts of corporations and to evaluate the risk premium that markets attach to their probability of default. The computation of the total value (equity plus debt) of a corporation gives rise to the celebrated Modigliani-Miller theorem: in the absence of frictions, such as tax exemptions on debt and liquidation costs, the total value of a company is independent of the way it is financed, i.e., the proportions of debt and equity in its liabilities, which is called the capital structure of the company. In the same vein, a world without frictions is a world where all financial decisions (when to distribute dividends and when to retain earnings, how to manage cash, when to issue new securities, etc.) would have no impact on the total value of the company. In that hypothetical world, corporate-finance decisions would be irrelevant.

Corporate finance becomes interesting only when frictions are introduced. This is the first departure from the standard option-pricing models, which typically rule out frictions. Several types of financial frictions are considered in the book: tax subsidies on debt and liquida-
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In the academic literature, all these costs were initially considered in one-period or discrete-time models, which provided valuable intuition on how these frictions impacted the financial decisions of corporations. However, discrete-time models are often inelegant and cannot be properly calibrated to real data. By contrast, the continuous-time setup that we use throughout this book allows us to get the best of two worlds: formal elegance as well as applicability to real-life problems. Derivative traders and risk managers use the same kind of models (with some simple twists) in their pricing and hedging decisions.

The fourth section of Chapter 1 presents the first attempt to overcome the Modigliani-Miller irrelevance result: when debt payments are tax deductible and asset liquidation involves deadweight losses, there is a trade-off between tax deductions and liquidation costs that leads to an optimal debt-to-equity ratio. This section presents one famous version of this “trade-off” theory in a continuous-time setting, namely, the Leland (1994) model.

Unfortunately, the predictions of the trade-off theory are not really consistent with what we observe in the real world. For reasonable calibrations of its parameters, the trade-off model predicts levels of corporate debt that are often too high. A vast academic literature has endeavored to enrich the trade-off model to bring its predictions closer to the observed behavior of firms. This literature is surveyed, among others, in the excellent works by Sundaresan (2013), Grasselli and Hurd (2010) and Bielecki and Rutkowski (2004).

We have decided to place most of our emphasis on a different type of frictions, namely issuance costs. We maintain the convenient (and not too unrealistic) assumption of perfect secondary markets, where existing stocks and bonds are traded with negligible transaction costs, but we explicitly take into account the frictions that exist in primary markets, where new stocks and bonds are issued. With issuance costs, which are sometimes sizable, it is important for companies to keep liquid reserves, and liquidity management becomes an important issue. Liquidity management is, therefore, a central topic of this book.

Chapter 2 presents the base liquidity-management model, where issuing costs are so high that companies never issue new securities and

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1As a matter of fact, the only sector in which firms borrow as much as predicted by the trade-off model is the financial sector, studied in Chapters 4 (banks) and 5 (insurance companies).
are liquidated when they run out of cash. Hence, the management of cash reserves becomes crucial, because it determines the company’s survival. This model was first studied (independently) by Jeanblanc-Picqué and Shiryaev (1995) and Radner and Shepp (1996). It will be our workhorse for Chapters 3 to 6.

Chapter 3 introduces the possibility of new equity issues into the base model. In the Leland model presented in Section 1.4, these new issues were considered costless, so firms could always use them to compensate their losses. In that case, cash reserves were useless. However, in practice, firms are reluctant to issue new equity because these issues are quite costly. In that context cash reserves are useful to absorb possible losses and limit the frequency of new equity issues. In this chapter we determine the optimal timing and size of new equity issues and study the implications of issuing costs on equity prices. Even with a stationary profitability (independent and identically distributed, or i.i.d., earnings), stock returns exhibit stochastic volatility. The assumption of constant volatility of stock returns (used by Black-Scholes and Merton) is only compatible with zero issuance costs.

Chapter 4 applies the base model of liquidity management to banks. Section 4.1 presents a simple model, due to Milne and Whalley (2001), that allows the study of the impact of regulatory capital requirements on the liquidity-management decisions of banks. Section 4.2 looks at portfolio-management decisions: the bank has a fixed amount of deposits and starts with some equity. As in the base model, it is too costly to issue new securities, which makes the management of cash reserves crucial. The bank’s manager decides how much to invest in some risky asset (loans or securities) with i.i.d. returns, and also how much to keep as cash reserves. This is reminiscent of the portfolio-management problem in Merton (1969), where the investor is characterized by some concave utility function and is, therefore, risk averse. Here the bank’s risk aversion is endogenous and stems from the financial frictions: even though shareholders are risk neutral or fully diversified, the bank behaves in a risk-averse way because unexpected losses may force it to liquidate assets or even stop operating. In Section 4.3, we present a model that deals with a richer liability side (short-term and unsecured long-term debt), in which we address issues of optimal bank funding through equity, and short- and long-term debts.

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Chapter 5 presents several applications of the base model to the insurance sector. Section 5.1 studies the impact of large losses on financial strategies of insurance companies. This is an extension of the base model where the firm’s net-earnings process also includes a Poisson component. Sections 5.2 and 5.3 deal with reinsurance strategies, which are crucial for insurance firms. In the former we study the reinsurance of small (Brownian) losses, whereas in the latter the (potential) losses to be reinsured are large (Poisson) ones.

Chapter 6 is dedicated to the determinants of corporate investment. Section 6.1 discusses the case without frictions: the $q$-theory of Tobin, as formalized in Hayashi (1982). Section 6.2 introduces external frictions and liquidity management into this model. Section 6.3 looks at the impact of external frictions on the decision to invest in a new technology.

Chapter 7 introduces agency frictions in a continuous-time framework. Although these frictions are very different from the external frictions studied so far, much of the methodology developed in Chapters 1–6 can be used. Section 7.1 presents an extension of the trade-off model of Section 1.4.1 where agency issues are present in the form of asset substitution problems. This means that shareholders may increase the firm’s exposure to risk after debt has been issued, thereby extracting additional value from debtholders. Section 7.2 solves the optimal-contracting problem between an entrepreneur and a group of financiers. There, the agency problem is due to the fact that the entrepreneur may divert part of the firm’s cashflows for private consumption. The optimal contract can be implemented through a combination of cash reserves, debt and equity. This allows us to discuss the firm’s optimal capital structure. Finally, we explore a similar setting where the agent may reduce his risk-prevention efforts, which exposes the firm to potentially large (Poisson) losses. The optimal contract uses downsizing of the firm after a large loss as a punishment device (the proverbial stick), whereas good performances lead to bonus payments to the manager (the proverbial carrot). There is now a large body of literature on agency models in continuous time, e.g. Cvitanic and Zhang (2013), Cadenillas et al. (2007) and the references therein.

Chapter 8 puts the base model into a general-equilibrium framework, where returns and prices are endogenized. Section 8.1 present the influential paper of Brunnermeier and Sannikov (2014) and showcases the complexities associated with the introduction of financial
frictions into a macroeconomic context. Section 8.2 presents a simpler model in the same spirit, which is very stylized, so as to keep it tractable and transparent. Section 8.3 presents a similar model that is specifically targeted at the insurance industry.

The last section of each chapter is devoted to suggesting further reading material. There are four appendixes. The first contains the mathematical proofs of the core results and other technical material; the second is devoted to the celebrated Modigliani-Miller “theorem.” The last appendix recalls useful mathematical results.